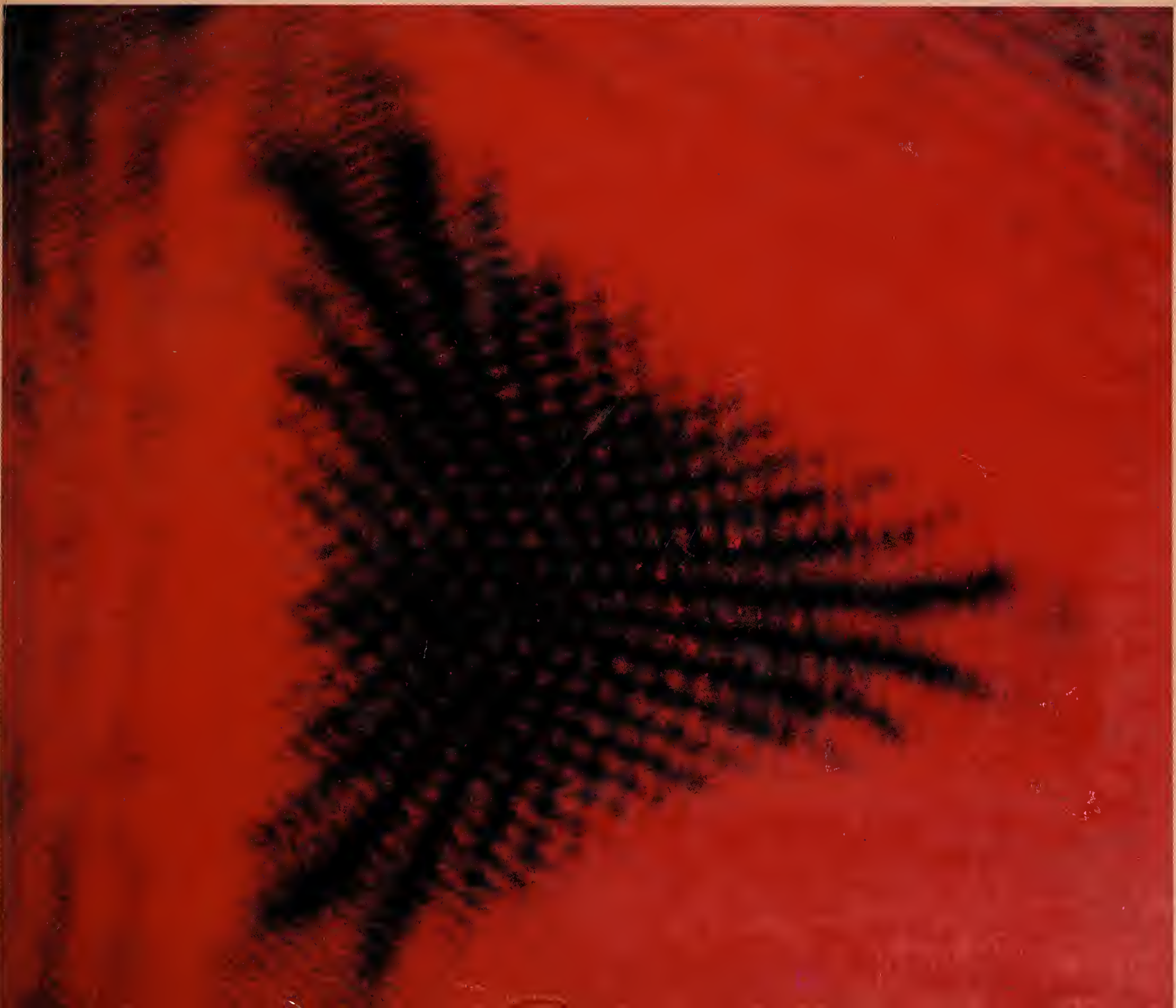
Volume III

# SCIENTIFIC PAPERS OF C V RAMAN

## OPTICS



C. V. Raman (1888-1970) who was awarded the Nobel Prize for Physics (1930) was a master in the field of optics.

*Volume III of the Scientific Papers of C V Raman* contains 63 of his publications in Optics. His celebrated Baroda Lectures (1941) reprinted here is a masterly elucidation of many optical phenomena reflecting forty years of living with light. Here may be found expositions on the geometric theory of Fresnel diffraction, the diffraction by a sphere and a circular disc, the study of haloes and coronae, and his discovery of the speckle phenomenon (1919) from observations of the radiant spectrum seen by the eye. The surprising observations on conical refraction, the wavelike character of periodic precipitates, the propagation of light in polycrystalline media, and the remarkable study of mirages of new relevance today in the context of cosmic mirages formed by gravitational lenses, are to be found in this collection.

This volume also has in it the celebrated Raman–Nath papers on the diffraction of light by ultrasonic waves introducing ideas which have found application decades later in many new fields like the dynamical theory of electron diffraction.

Scientific Papers of

# C V RAMAN

## Volume III

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Edited by

S Ramaseshan

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## TABLE OF CONTENTS

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These volumes could not have been brought out without the invaluable support of the publication staff of the Indian Academy of Sciences and the staff of the Raman Research Institute Library. I wish to record my indebtedness to them and also to many others who so willingly helped. Special thanks are due to Thomson Press (I) Ltd for their efficient printing and prompt delivery of the volumes.

Thanks are also due to the Jawaharlal Nehru Foundation for a Fellowship during the tenure of which much of the preliminary planning of these volumes was done and to Rajaram Nityananda of the Raman Research Institute for his continual help.

*S RAMASESHAN*

# INTRODUCTION

## The background

The third volume of the *Scientific Papers* contains 65 papers covering many fields in optics which interested Raman: diffraction, theory and experiment; Huyghens' principle applied to a variety of phenomena including total internal reflection; the "radiant spectrum" and the speckle phenomenon; a variety of lovely optical interference effects; a heroic experiment to measure the so-called "convection of light" in moving air; a remarkable effect in conical refraction; the diffraction of light by ultrasonic waves; elegant experiments to establish the wavelike nature of periodic precipitates and the theory of the propagation of light in polycrystalline media.

This volume also contains the monograph entitled *Lectures on Physical Optics*, known more familiarly as the Baroda Lectures, since they were delivered in that city in 1941. This was originally planned in two volumes of which the first was ready for printing in 1943. Clearly other matters must have intervened, for, the volume was published only in 1959 and the promised sequel never came. But even as an incomplete fragment, the *Lectures* deserve to be read for the masterly exposition of many interference and diffraction phenomena with a uniquely physical (and personal!) perspective. The book draws on forty years of living with light and is written in an inimitable style.

This introduction will have served its purpose if it guides the reader through the diverse aspects of optics in which Raman worked and whets his appetite for the original papers. His first paper was published in the *Philosophical Magazine* when he was eighteen. During his itinerant period, serving as a Finance Officer of the Government of India in Nagpur, Rangoon and Calcutta, it appears that his optical researches were sporadic as he was more interested in acoustics. But once he settled down at Calcutta, the floodgates opened. The summaries which follow are by no means exhaustive but may bring out some of the significant phases of Raman's optical researches spread over more than sixty years.

## Diffraction

*The obliquity factor*: The first impression one gains from even the earliest papers is that Raman was completely familiar, right from the beginning, with the literature of the subject and kept a sharp lookout for new phenomena which

would not fit in with existing concepts. His very first paper on asymmetric diffraction by an oblique aperture led him to a direct study of the obliquity factor which had entered diffraction theory through the work of Kirchoff. The geometry chosen by Raman led to a direct determination of this factor to which experiments in the usual geometry were rather insensitive.

*The sphere and the disc*: Because of its easy availability it was common practice to use a spherical ball instead of the circular disc to demonstrate the famous bright spot in the centre of a circular shadow (in Fresnel diffraction) and to assume that the sphere and the disc gave identical results. Raman and his brilliant student, K S Krishnan, suspected that this was not so because when the eye is placed at the central spot, it was noticed that in the case of the sphere the intensity of the boundary radiation was much less and the polarisation was also much less pronounced. This together with the fact that the radiation can only emanate from a smaller circle suggested that the creep of the wave over the surface must be taken into account.

They made accurate measurements of the intensity of the central spot with a sphere and a disc of the same diameter. The spot inside the shadow of the sphere was much feebler than that of the disc at short distances, the two becoming identical at large distances. Using a simple geometric theory they could explain these observations.

## The geometric theory of Fresnel diffraction

It is clear from reading these early optical papers that Raman was fully aware of the more rigorous electromagnetic treatments of the phenomena he studied but treasured the physical insight and heuristic power provided by Huyghens' principle, Fresnel's zones and Young's edgewaves. Raman felt that the rigorous procedures used to calculate Fresnel diffraction patterns afford no insight into the relationship between the forms of the obstacle (or aperture) and the character of the diffraction pattern.

He was greatly influenced by the discovery by Gouy in 1886 of the reality of Young's "edgewaves"—that a straight sharp metallic edge held in a pencil of light appears luminous and the strongly polarised light is diffracted through large angles. Raman used this concept of radiation from the boundary to explain his observation that for an elliptic aperture there is a concentration of luminosity on and within the evolute of the elliptic boundary in remarkable analogy to the caustics of geometrical optics.

Raman (and Y V Kathavate) further noted that when the eye is moved away from the centre of the geometric shadow of the disc, the bright luminous circular boundary collapses to two bright narrow regions ("poles") on the boundary, at the ends of the diameter parallel to the line joining the eye to the centre. This was

the beginning of the formulation of a geometric theory of Fresnel diffraction in which the intensity at any point in the shadow can be considered as arising due to the interference of the radiation from such poles. With the poles easily located geometrically, the delicate and yet intricate tracery of Fresnel patterns exhibited even by simple geometric shapes formed by straight edges (like the square, the rectangle or the triangle) could be deduced. Indeed with similar simple rules, patterns for much more complex shapes with curved and straight edges could also be easily obtained.

### The fascination for haloes and coronae

Raman was obsessed with the beauty of the coronae and haloes that appear around the sun and moon; an obsession that never left him. He studied the corona around the sun when he was in Nagpur in 1910—the lunar coronae around 1922 in Calcutta and the corona around Venus early in the morning just a year before he died. He took delight in producing and looking at them in the laboratory as these artificial coronae are more striking in colour than those seen in nature. It is also remarkable how much science Raman could get from his study and contemplation of this one phenomenon.

The discovery of the speckle phenomenon in 1919; the theory of X-ray diffraction of the liquids propounded by him and Ramanathan in 1923; the derivation for the first time of the structure factor of atoms by considering X-rays to be scattered by a cloud of electrons and the subsequent classical derivation of the Compton Effect formula (which led him on to the discovery of the Raman Effect) all seem to have come out of this singleminded interest in coronae.

### The radiant spectrum and the speckle phenomenon

In 1919 Raman discovered what is now called "The speckle phenomenon". (Exener and de Haas also made the same discovery at about the same time tackling the same problem!) When a small source of light is viewed against a dark background one sees a "radiant spectrum" or coloured streamers appearing to diverge from the source. Raman deduced that this was essentially due to the diffraction corona formed by imperfections in the refractive medium of the eye—only that the interference effects due to the different particles must be superposed on to the diffraction effect. Because of this the maxima of the halo will exhibit violent fluctuations of intensity and appear "mottled". (These are the speckles one has become familiar with after lasers were invented.) When the light is white each mottle or speckle becomes a coloured streamer. Since the diffracting particles are randomly distributed, the intensity distribution of the mottles will be decided by the random walk process or the Rayleigh Law. Many of these deductions were

verified by Raman and his distinguished student G N Ramachandran in haloes formed by lycopodium powder on a glass plate. Raman conjectured that the "diffracting screen" in the eye may be a set of minute regions with small difference in refractive indices with respect to the surrounding medium acting as a random phase grating, the Fourier transform of which gives the mottled image.

### Wavelike character of periodic precipitates

Raman was intrigued by the formation of periodic precipitates in nature—the Liesegang phenomenon. He spent hours at the Indian Museum in Calcutta examining geological specimens manifesting this phenomenon, and he also acquired many beautiful specimens from all over the world for his personal collection. The similarity between wave patterns and periodic precipitates had been noticed by many and the physical basis for this had been suggested as the Ostwald diffusion wave in three dimensions.

However, to Raman, such analogies would be without physical content unless the real distinguishing character of a true wave—i.e. the existence of a phase relationship, could be observed in the form of interference and diffraction effects.

An exploratory investigation (with K Subba Ramaiah) to ascertain the preferred orientation of the crystallites in such precipitates produced hundreds of magnificent microphotographs with thousands of Liesegang rings displaying what appeared to be inexplicable features. The main problem arises because while in a wavetrain the disturbance may be positive or negative, the density of the precipitate is necessarily a positive quantity. The first step was to recognise those features which could obviously reveal such effects unambiguously as effects due to waves. For example, a phase difference of half a wavelength must exist across a line of zero disturbance produced when two waves cross at an angle. This revealed itself by the staggering of the precipitates on either side of a line where no precipitation occurred. Gross and very fine structures of the precipitates seen under high magnification were identified as due to the recondite interplay of interference effects due to individual waves and groups of waves.

### Conical refraction

Raman's first inroad into this subject was when he proved that the equipment usually available in physics laboratories (in those days) "to demonstrate internal conical refraction" actually demonstrated external conical refraction! This was followed by the discovery of a very remarkable optical phenomenon. A luminous object when observed through the singular direction of a parallel plate of aragonite forms a bright, well-defined, real, erect image of *unit magnification* with the unexpected property that the *image is continuous*, i.e. it may be observed

anywhere along that line (and not at a single point as in the case of a lens). The light is also unpolarised. Raman showed that the image formation is due to the dimpled form of the wavefront and the intense concentration of luminosity which occurs on the singular point of the wave surface. Raman and his collaborators also made extended studies with naphthalene crystals (grown by the Bridgman method for Raman Effect studies) in which the angles of internal and external conical refraction are ten times larger than the traditional aragonite.

### Einstein's aberration experiment

Two short papers in this volume reveal Raman in the role of a critic. In 1926, Einstein proposed an aberration experiment to distinguish between the quantum and (semi) classical views of light. On the wave picture, a moving source emits a frequency which depends on direction due to the Doppler Effect. The suggestion was that such a wavefront propagating in a dispersing medium like carbon disulphide would undergo an additional tilt proportional to the frequency gradient across it and hence would give rise to a shifted image. On the quantum picture, Einstein believed that no such shift would take place.

Setting aside the behaviour in the quantum theory, Raman gave a brief and incisive analysis of the wave aspects and showed that no shift was to be expected in any case. He pointed out that Michelson's determination of the velocity of light with a rotating mirror effectively involved a moving source and drew attention to Gibbs' work in which the effect of a dispersing medium on the waves from such a source was analysed. The essential point was that one is dealing with a wave packet and the tilt of the wavefronts due to dispersion is inevitably accompanied by a drift through the envelope. The direction of the energy flow as defined by the wavefront at the centre of the group remains unchanged. As if this were not enough; Raman gives even more basic arguments starting from his favourite topic, the Huyghens' principle which leads to the same conclusion. From a modern point of view, one would say that the proposed experiment involves only second order coherence and no difference between the statistical predictions of the quantum and semiclassical pictures is to be expected.

### The Fresnel–Fizeau drag in gases

It may be worthwhile to draw attention to an interference experiment performed on an impressively large scale with air blown through two hundred-foot long pipes in an attempt to measure the Fresnel–Fizeau drag for gases. The preparations for this experiment involved fairly large scale civil works and equipment and the final expected effect was just a shift of one tenth of a fringe. While the results were marginal, Raman's energy and enthusiasm as well as the

confidence he must have inspired in those who supported him stand out. The experiment would probably be considered ambitious even today since efforts on a similar scale are under way all over the world to combine light from optical telescopes separated by large distances.

## The Raman–Nath theory

Almost seven years after the discovery of the Raman Effect, Raman turned his attention to a remarkable optical phenomenon occurring when a parallel beam of light is diffracted by ultrasonic waves. This volume includes, as did Volume II on Acoustics, the very well-known series of papers by Raman and Nath on this subject. The original experiments in this field by Debye and Sears and by Lucas and Biquard showed a surprisingly large number of diffraction orders and an apparently irregular distribution of intensity among them, neither of which was satisfactorily explained by the ideas current at that time. The Raman–Nath papers introduced the physical concept of a corrugated (i.e. strongly phase-modulated) wavefront and the associated mathematical tool—a set of first order differential-difference equations for the amplitudes of the various diffraction orders taking multiple scattering into account. Once these ideas were introduced the observations fell into place at one stroke. It may be worth remarking that the actual formula for the amplitudes involves Bessel functions in which Debye was an acknowledged expert!

Even more significant than the specific problem which gave rise to the theory were the ideas which found applications in other fields decades later. The multiple-beam dynamical theory of electron diffraction turns out (in retrospect) to be modelled on the Raman–Nath theory and it plays an important role in the interpretation of electron microscope images. The idea of an equation which is first order in the direction of propagation and second order in the transverse direction (the parabolic equation approximation) is contained in the Raman–Nath papers and now plays a significant role in the theory of wave propagation in a random medium. These applications involve going beyond the approximation of pure phase modulation and including the amplitude variations which this produces—a step which Raman and Nath had already taken in the later papers of the series.

## Light in polycrystalline media

Raman's mineral collection contained a multitude of beautiful specimens like marble, alabaster, gypsum, feldspar, moonstones, labradorites, jades, opals, etc, which displayed striking optical effects. He felt that many of their properties were essentially due to fine crystallites of one phase imbedded (with or without

preferred orientation) in an isotropic or birefringent medium. He wished to use his old technique of employing a beam of light as a probe to explore the secondary structure of these minerals and to understand their optical properties. This explains renewed interest in the beautiful Christiansen phenomenon and the propagation of light in polycrystalline media.

Raman was, of course, aware that to solve this problem one has to generalise the Raman–Nath theory for random phase gratings taking into account polarisation states. However, for practical reasons he contented himself with developing simple theories using which he could extract a considerable amount of useful information. The results of the experimental studies on these minerals are included in the *Scientific Papers* Volume IV.

### Total internal reflection and the mirage

A strong common theme underlying many of the papers in this volume is Huyghens' principle and the notion of secondary waves. For example, the phenomenon of evanescent waves occurring in the rarer medium in total internal reflection was given by a beautiful graphical interpretation in terms of a change in topology of the Fresnel zones. Raman backed this up by a detailed analytical calculation making contact with the usual theory. The value of this viewpoint is that it suggested further novel experiments involving diffraction by the evanescent waves both in the far field and near field. Errors and omissions in the literature on total reflection, by authors as distinguished as Kelvin and Schuster, were corrected politely but forcefully.

The interplay of geometrical optics and the wave theory is seen in many of Raman's papers but perhaps nowhere more clearly than in the paper on the optics of the mirage written in 1959 with his outstanding student Pancharatnam. This paper is noteworthy for the elegant way in which a stable layer of air with the desired refractive index gradient was produced and the beautiful pictures which the authors obtained with this arrangement. On the theoretical side, the authors comment on the conceptual difficulties of purely ray-optic treatments of the problem. A solution of the wave equation is given which accounts for the observed fringe pattern. (A similar solution had been given by workers in the field of ionospheric radiowave propagation some years earlier.) The associated wavefront is characterised (without a figure!) as made up of three sheets joined at cusps which travel along a caustic surface. Consequently, three images are to be expected in general and pictures showing their positions and parities are given. The principles elucidated by Raman and Pancharatnam for the terrestrial mirage were reintroduced many years later in the context of cosmic mirages. This phrase refers to the formation of multiple images of distant quasars formed by the gravitational bending of light by intervening masses. In efforts to model these so-called "gravitational lenses", the odd number of images, their parities, cusped

wavefronts and caustics play a prominent role. The Raman–Pancharatnam paper should therefore be remembered not only for the wave optical treatment of the mirage but also for the clarification of the associated geometrical optics limit.

These optical papers of Raman are understandably less well known than his work on light-scattering on the molecular scale. Nevertheless, they amply repay the attention of anyone interested in the subject even today. In addition, the papers give a fascinating picture of how their author approached the study of light which was certainly one of his lifetime preoccupations.

*RAJARAM NITYANANDA*
*G S RANGANATH*
*S RAMASESHAN*

## CONTENTS

# THE PAPERS

*Philos. Mag.* **12** 494–498 (1906)

# Unsymmetrical diffraction-bands due to a rectangular aperture

C V RAMAN
(Demonstrator in Physics at the Presidency College, Madras)*

When a pencil of monochromatic light coming from a slit in the focal plane of a collimating lens falls upon the object-glass of a telescope in front of which a narrow rectangular aperture is placed with its sides parallel to the luminous slit of the collimator, the diffraction-pattern seen in the focal plane of the telescope consists of a series of bright and dark bands symmetrically arranged on either side of the geometrical image of the slit, provided that the light falls normally upon the aperture. If, on the contrary, the aperture is held inclined to the incident pencil—its sides being still parallel to the slit—the diffraction-pattern is *not* necessarily symmetrical. The symmetry is not, however, *sensibly* departed from, unless the incidence be very oblique. The case in which this unsymmetrical pattern was first seen in this: place a prism on the table of a spectrometer and observe the image formed by the light reflected at very oblique incidence from one of the faces of the prism. With a prism of face-width 4·5 cm and an incidence of 85°, the diffraction-pattern seen in the field is sensibly symmetrical, and the minima of illumination equidistant from one another. If the incidence is greater than 87°, this is no longer true. The bands are wider on one side of the pattern than on the other, those on the side towards the direct image of the slit being broader. This asymmetry increases greatly as the angle of incidence approaches 90°, and at the same time the number of bands on one side of the pattern—the side where they are broader—becomes smaller and smaller till at last they disappear altogether.

The facts can be explained quite easily. Let $a$ be the width of the face of the prism and $\lambda$ the wavelength of the light, and $(\pi/2) - \theta$ the angle of incidence. Then, in any direction making an angle $(\pi/2) - \phi$ with the normal to the face of the prism, there is no illumination provided

$$a(\cos\theta - \cos\phi) = \pm n\lambda, \tag{1}$$

where $n$ is any whole number.

If $n = 0$, $\theta = \phi$, and we have the position of the light reflected according to the

*Communicated by the author.

usual law. If $\theta$ is not small,

$$2a \sin\frac{\theta+\phi}{2} \sin\frac{\phi-\theta}{2} = \pm n\lambda$$

$$\phi - \theta = \pm \frac{n\lambda}{a \sin\theta}; \tag{2}$$

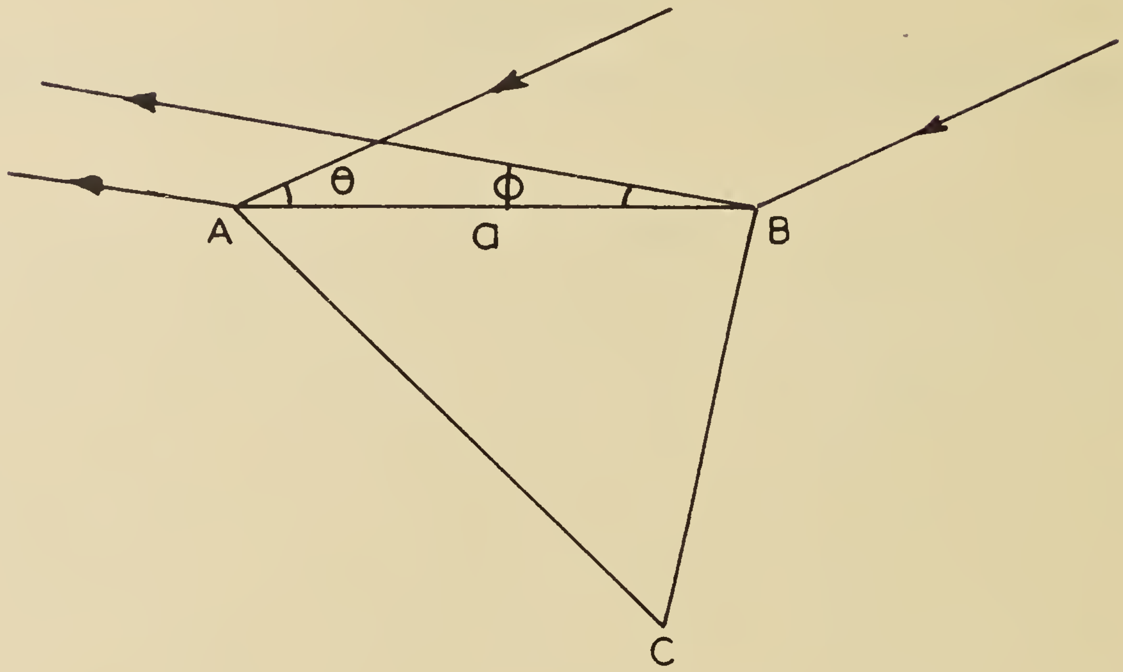


**Figure 1**

and the diffraction-pattern is identical with that produced by the effective aperture of the prism-face, and is symmetrical. If $\theta$ is small, (2) is no longer true, and a reference to the tables shows that if the angle is small, for equal increments of its cosine, the increments of the angle are large and by no means equal. This shows that the bands are fairly broad, and that the minima are not at equal angular distances from one another.

I give this example worked out from the following data:

$$a = 3\,\text{cm.} \quad \lambda = 7000\,\text{A.U.} \quad \theta = 1°\,9'.$$

| Angular distance from | |
|---|---|
| The 4th minimum to the 3rd | 202″ |
| 3rd minimum to the 2nd | 212″ |
| 2nd minimum to the 1st | 223″ |
| 1st minimum to the central band | 233″ |
| Central band to the 1st minimum | 247″ |
| 1st minimum to the 2nd | 266″ |
| 2nd minimum to the 3rd | 287″ |
| 3rd minimum to the 4th | 311″ |

Further, the smallest value of $\phi$ admissible is zero. There is therefore a limit to the number of bands possible on one side of the pattern. There can be one or two

or more, the number being the greatest integer in

$$\frac{a(1-\cos\theta)}{\lambda}.$$

This fact can be put in another way. If AB be the face of the prism and BP the incident wave-front, the limit to the diffraction-pattern is set by the direction BA, for points on the surface AB obviously cannot send out wavelets in the direction AG.

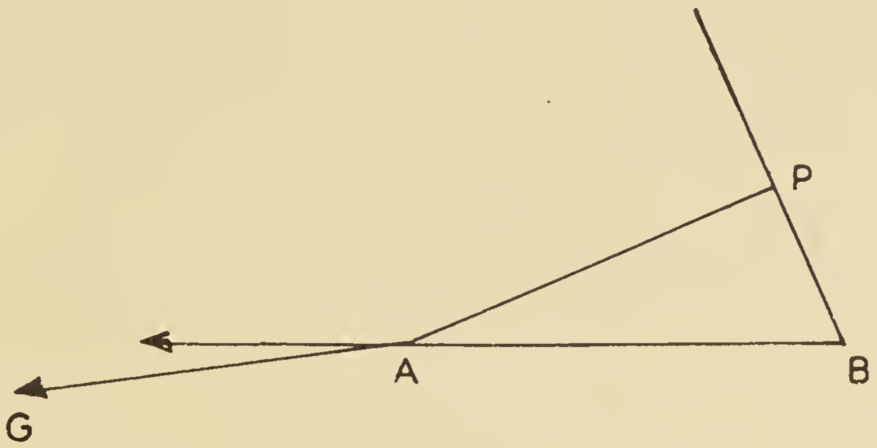


**Figure 2**

Measurements were made on the diffraction-pattern by means of a micrometer, in order to test the theory. The measurements given below were made at an incidence where the asymmetry was not very marked yet sufficient to be easily seen.

$$a = 4{\cdot}57\,\text{cm.} \quad \lambda = 6500\,\text{A.U.} \quad \theta = 1°24'55''.$$

| Angular distance from | Observed | Calculated |
|---|---|---|
| The 5th minimum to the 4th | 110″ | 108″ |
| 4th minimum to the 3rd | 105″ | 110″ |
| 3rd minimum to the 2nd | 111″ | 113″ |
| 2nd minimum to the 1st | 118″ | 115″ |
| 1st minimum on one side to the 1st on the other | 234″ | 237″ |
| 1st minimum to the 2nd | 125″ | 123″ |
| 2nd minimum to the 3rd | 127″ | 126″ |
| 3rd minimum to the 4th | 132″ | 130″ |
| 4th minimum to the 5th | 132″ | 134″ |

For the same angle of incidence and with approximately homogeneous light of mean wavelength 7100, the observed width of the central band was 261″ ± 2″, the calculated value being 260″. Theory and observation agree as to the number of bands on one side of the pattern, if it is not more than six or seven.

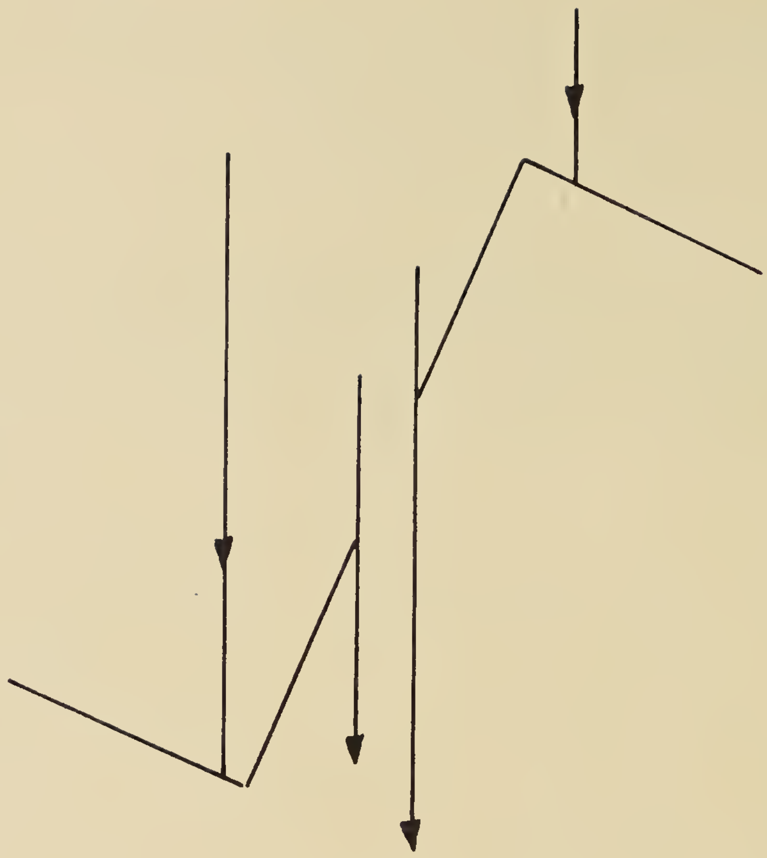

**Figure 3**

The facts described above suggest that by holding a fairly wide rectangular aperture very obliquely in the pencil of light, we should get an identical system of diffraction-fringes. This was verified by experiment. An aperture 2 cm wide was cut in a thin sheet of zinc, which was then bent into the shape shown (figure 3). The side-vanes served to cut off the light from the other portions of the collimator and to support the sheet on the table of the spectrometer. It was found that the diffraction-bands were sensibly symmetrical when the incidence was moderate, and asymmetrical when it approached 90°. The minima in this case are of course given by the usual equation

$$a(\cos\theta - \cos\phi) = \pm n\lambda.$$

The angle $\theta$ could not be measured; only the relative positions of the bands could be determined. The table gives some measurements.

$$a = 2\,\text{cm} \quad \lambda = 6500\,\text{A.U.} \quad \theta \text{ unknown.}$$

From

| | |
|---|---|
| The 2nd minimum to the 1st | 306″ |
| 1st minimum to the central band | 322″ |
| Central band to the 1st | 364″ |
| 1st minimum to the 2nd | 422″ |

The intensity of illumination at any point of the diffraction-pattern is given by

the expression

$$I \frac{\sin^2 \dfrac{\pi a}{\lambda}(\cos\theta - \cos\phi)}{\left\{\dfrac{\pi a}{\lambda}(\cos\theta - \cos\phi\right\}^2}.$$

It is perhaps worth noting that Bridge's theorems do not hold for this unsymmetrical system; i.e. the scale as well as the relative distribution of the maxima and minima in the unsymmetrical pattern is altered by altering the width of the face of the prism or the wavelength of the light, the incidence being constant. If, however, the ratio $a/\lambda$ is kept unchanged, the diffraction-pattern is not altered.

The experiments and observations recorded in this note were made at the Presidency College Physical Laboratory.

*Nature (London)* **76** 637 (1907)

# Newton's rings in polarised light

An erroneous statement regarding the above-mentioned. subject is made in Preston's 'Theory of Light' (p. 363, 1901 edition) and also in Edser's 'Light' (p. 519, 1902 edition). As the error is a rather serious one, it seems worth while to point it out.

When the rings are seen between two lenses of the same substance, by light polarised perpendicularly to the plane of incidence, reflected at an angle greater than the polarising angle of the substance, it is stated that the centre of the rings is bright. That this is wrong can be seen. For:

(1) Stokes has shown from the principle of reversibility that, whatever be the nature of light, the centre of the rings seen between lenses of identical refractive indices is black at all incidences of the light.

(2) Since the centre of the rings is black *at all incidences* for common light and for light polarised in the plane of incidence, it follows by resolution that it is also black when the light is polarised in a perpendicular plane.

(3) When the angle of incidence is less than the polarising angle, the coefficients of reflection *in glass* and *in air* at the bounding surfaces of the two media are opposite in sign. It is argued that, on increasing the incidence, the coefficient of reflection *in air* changes sign as the polarising angle is passed, and therefore at such incidences the two coefficients agree in sign, and destructive interference no longer takes place. Really, however, it appears from Fresnel's formula (coefficient $= -\tan(i-r)/\tan(i+r)$ that both the coefficients change sign as the incidence passes through the polarising angle, and therefore continue to differ in sign, as can be directly shown from the principle of reversibility. Destructive interference does, therefore, take place.

(4) I have shown by experiment that the statement is not true.

(5) Airy has shown (Lloyd's 'Wave Theory,' p. 178, and Jamin's 'Optique Physique,' p. 503) that when the two lenses differ in refractive index, the centre of the rings seen in light polarised perpendicularly to the plane of incidence is white only when the incidence lies between the angles of polarisation of the two media. Outside these limits the centre is dark.

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Science Association Laboratory, Calcutta
12 September

*Nature* (*London*) **78** 55–56 (1908)

# Secondary waves of light

It has hitherto been held that, so long as the diffraction apertures used (cut in perfectly opaque or perfectly reflecting screens) are large compared with the wavelength of light, Fresnel's expression for the amplitude of the disturbance due to a surface-element gives us a close approximation to the observed diffraction effects, and that the exact value for the obliquity factor is of little importance (e.g. see Schuster's 'Optics,' sec. 48). That this is true only in the special case in which the apertures are held normal to the waves of light, and not in other cases, is shown by some new diffraction phenomena that I have made the subject of study.

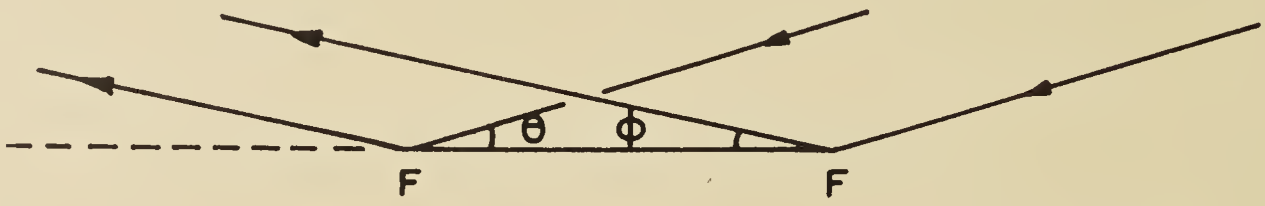
The only experiment so far known which might seem to show effects due to the obliquity factor is the well known one with the circular disc, but it is really inconclusive. The observed fact, that the illumination along the axis of the disc decreases as the disc is approached, is more or less entirely due to minute irregularities in the rim of the disc, and not, as is sometimes stated, to the increasing obliquity of the secondary waves producing the illumination.

The theoretical grounds on which my experiments were based were these: If diffraction bands are produced and observed in a direction in which the amplitude of the disturbance in the secondary waves varies rapidly from point to point, we might expect effects due to varying obliquity. Such effects would obviously not occur if the diffraction aperture or mirror is, as is usual, held normal to the waves of light incident on it, but might if it be held obliquely.

In the *Philos. Mag.* for November, 1906, I showed that the diffraction bands due to a rectangular aperture held very obliquely are not equidistant, that the band-width increases progressively from one side of the pattern to the other, and that the number of bands on one side of the pattern is limited. They are most easily seen on the spectrometer if the image of the slit of the instrument formed by light reflected very obliquely from the face of a prism is observed. The positions of the minima of illumination, actually observed, are closely in agreement with those calculated from the usual formula ($\cos\theta - \cos\phi = \pm n\lambda/a$), $\theta$, $\phi$ being the complements of the angles of incidence and diffraction. Further observations have elicited the following: from the expression for the intensity of the illumination in the pattern deduced by the ordinary method

$$I = a^2 \sin^2 \frac{\pi a}{\lambda}(\cos\theta - \cos\phi) \Big/ \frac{\pi^2 a^2}{\lambda^2}(\cos\theta - \cos\phi)^2,$$

it would appear that the maxima of illumination in corresponding bands on either side of the middle one should be of equal brightness. This is flatly contradicted by observation, both visual and photographic. It is found that the bands on one side are considerably fainter than those on the other, and this difference becomes very large as the light approaches grazing incidence. The illumination in the diffraction pattern (with a given angle of incidence) decreases and dies away as we approach the limiting plane of the fringes, which is the plane of the reflecting surface (FF in the diagram).




This effect is inexplicable if the question of the variation of the amplitude in different directions of the secondary waves, supposed to be sent out by the elements of the reflecting surface FF, is not taken into account. It must be remembered that we are not dealing with apertures small compared with the wavelength; both the aperture and its projection are large compared with $\lambda$, and there are no polarisation effects observed. The question may be attacked analytically, and it can be shown that an element of the surface of a reflecting body is equivalent in its effect to a double source of appropriate intensity which, it is known, produces zero effect in its equatorial plane and a maximum along its axis. The effect of an element of the surface FF is therefore zero along the line FF, and in other directions increases as we move away from the line FF. Remembering that the elements are not in the same phase, and integrating their separate effects, we get an explanation of the phenomenon observed.

A fuller discussion and a mathematical investigation will be published in due course. I found that similar effects are observed when the transmitting aperture is used. Some experiments with coarsely ruled gratings are in progress which seem to point in the same direction.

*C V RAMAN*

Science Association Laboratory, Calcutta
2nd April

*Philos. Mag.* **17** 495 (1909)

# Historical note on the discovery of the ultra-microscopic method

## To the editors of the Philosophical Magazine

Gentlemen,

It is essential that, when the work of earlier investigators is found to have anticipated the 'discoveries' of later workers, due justice be done to the former. That is the reason why I venture to draw the attention of the readers of the *Philos. Mag.* to the forgotten pages of the '*Indian Engineering*', a weekly journal, in the numbers for April 7th, 14th, and 21st, 1888, of which were published three papers by Mr G Dubern. In these papers was described a method devised by him for observing and studying the properties of particles too small to be visible under microscopes even of the highest powers, when used with the ordinary arrangements for illumination. The method briefly described was the following: The particles were held on a glass plate under a microscope; a powerful beam of convergent light entered the glass plate obliquely through a polished slant-end-surface, and being incident at an angle greater than the critical angle on the surface of the plate, suffered a series of total internal reflexions inside the glass plate. Under these circumstances, the ultra-microscopic particles appeared as tiny bright specks on a dark field. Mr G Dubern's arrangement is identical in all particulars with Mr Cotton's modification of the ultra-microscopic method of Siedentopf and Szigmondy, and a comparison between the figures illustrating Mr Dubern's arrangement in the '*Indian Engineering*', and figure 306 on page 491 of Prof. Wood's 'Optics' representing Mr Cotton's apparatus, will convince even the sceptical that Mr Dubern's apparatus is fully entitled to the name 'Ultra-microscope,' and that he must be regarded as having more or less completely anticipated Siedentopf and Szigmondy.

I have the honour to be, Gentlemen,

Your most obedient Servant,
*C V RAMAN*

Science Association Laboratory, Calcutta
17 December 1908

*Philos. Mag.* 17 204–216 (1909)

# The experimental study of Huygens's secondary waves

C V RAMAN, M.A.*

In the *Philos. Mag.* for Nov. 1906 (pp. 495–498), I published a note on the diffraction-bands formed when a rectangular aperture is held very obliquely in a parallel beam of light. I showed that the bands cease to be of the same symmetrical type as the fringes formed when a rectangular aperture is held normally. They are not equidistant, the band-width increasing progressively from one side of the pattern to the other. Further, the number of bands visible on one side of the pattern is limited. The photographs of the effect published with this paper (plate I) exhibit these features.

Further observation of the diffraction-bands on the spectrometer, made by the methods I described in the paper referred to (i.e. of observing through the telescope the image of the slit of the instrument formed by light reflected very obliquely at the face of a prism, or by light passing through a rectangular aperture cut in a thin sheet of metal and held very obliquely on the table), elicited the following: it was found that the bands on one side of the pattern were fainter than those on the other, the difference becoming very large as grazing incidence was approached. This feature is visible on all the three photographs in the plate. The effect is inexplicable on the ordinary (non-analytical) theory of diffraction.

The illumination at any point in the pattern is, as deduced by the ordinary method, proportional to

$$\sin^2 \frac{\pi a}{\lambda}(\sin i - \sin \theta) \bigg/ \frac{\pi^2 a^2}{\lambda^2}(\sin i - \sin \theta)^2,$$

$a$ being the aperture, $\lambda$ the wavelength, and $i$, $\theta$ being the angles of incidence and diffraction respectively. Plotting this expression against $\theta$, it is seen to be the ordinary symmetrical curve $\sin^2 x/x^2$ with its abscissae distorted but its ordinates the same: (1) the maxima of illumination in corresponding bands on either side of the central one ($i = \theta$) are equal: (2) the illuminations at corresponding points on either side of the diffraction-pattern

$$(\overline{\sin i - \sin \theta_1} = \overline{\sin \theta_2 - \sin i}),$$

are equal: (3) as the largest value of $\theta$ admissible is $\pi/2$, it follows that the curve of

*Communicated by the author.

illumination at this point drops suddenly to zero; in other words, there is a discontinuity in the illumination-curve at this point. *All three results are contradicted by observation.* As has been stated above, the bands on the side nearer to the limiting plane $\theta = \pi/2$ were found to be fainter than those on the other side and the illumination at points in the diffraction-pattern decreased to zero as the limiting plane was approached.

The diffraction fringes were observed through a nicol; there was no relative change in the illumination at different points in the pattern as the nicol was rotated, and at very oblique incidences no change at all.

An explanation of the effect was sought for on the following lines: each element of the reflecting surface may be supposed to send out hemispherical secondary

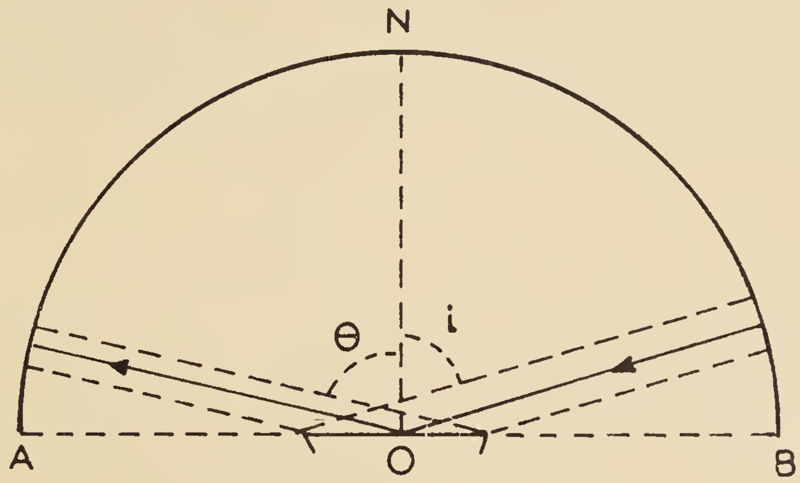



**Figure 1**

wavelets (figure 1) and the illumination in the diffraction-pattern may be determined by integrating the effects of these secondary waves. If the amplitude of the disturbance in a secondary wave in the direction in which the diffraction-pattern is formed varies rapidly with the obliquity, such variation would have to be taken into account and we should have an explanation of the difference between the observed result and that predicted by the ordinary theory. In no diffraction-experiment so far known, with apertures of ordinary size, has the variation of the amplitude with the obliquity, in a secondary wave, manifested itself. In the well-known Fresnel–Arago Circular-disk experiment, the fact that as the disk is approached the illumination along the axis of the disk decreases, cannot be taken to be an obliquity effect, it being more or less entirely due to the large increase in the effect of minute irregularities in the rim of the disk or of minute inaccuracies in its setting, as the latter is approached. Some authorities have gone so far as to deny that an obliquity-effect is possible at all. The present paper will show that this last view is erroneous. In the case considered in this paper, if we assume that the effect of an element at O (figure 1) of the reflecting surface is zero at points on the lines OA, OB and a maximum in the direction ON, the rate of variation with respect to $\theta$ of the amplitude in the secondary wave ANB would be a maximum in either of the directions OA, OB, and zero in the

direction ON. If plane waves of light are incident on the reflecting surface at a very oblique angle, the diffraction-pattern (as observed in a telescope focussed for infinity) is formed in the neighbourhood of the direction OA, and the variation of the effect of an element with $\theta$, the angle of diffraction, would have large effects. The intensity in the diffraction-pattern would be zero in the direction OA, and at a point at which $\theta > i$, would be less than at the corresponding point at which $\theta < i$.

The point will now be investigated mathematically. Take the case of an aperture of any shape and of dimensions large compared with $\lambda$, cut in a thin perfectly reflecting sheet of infinite extent, and let parallel waves of light be incident on the aperture at an angle $i$. The light passing through the aperture falls upon the object-glass (focal length $f$) of a telescope focussed for infinity. Let $x, y$ be coordinates in the plane of the aperture and $\xi$, $\eta$ in the focal plane of the telescope, $y$ and $\eta$ being parallel, and assume that the effect of an element $\mathrm{d}x$, $\mathrm{d}y$ of the aperture is, in the focal plane, equal to

$$-\frac{A}{\lambda f}\,\mathrm{d}x\,\mathrm{d}y\,F(i,\theta)\sin\frac{2\pi}{\lambda}(Vt - c - P),$$

where $A$ is the amplitude in the incident wave, $c$ is a constant, $\theta$ the angle of diffraction, and $P$ the path-difference of the effects at $\xi, \eta$, of the element $\mathrm{d}x\,\mathrm{d}y$ and of an element at the origin in the plane of the aperture. The rigorous expression for $P$ is a little complicated, but if the angle of incidence is not so large that $(\sin i - \sin\theta)$ cannot be put equal to $\sin(i-\theta)\cos i$, $P$ can be shown to be equal to

$$-\frac{x\xi\cos i + y\eta}{f}.$$

The amplitude of the effect at any point in the diffraction-pattern is therefore

$$-\iint\frac{A}{\lambda f}\,\mathrm{d}x\,\mathrm{d}y\,F(i,\theta)\sin\frac{2\pi}{\lambda}\left(Vt + \frac{x\xi\cos i + y\eta}{f} - c\right),$$

the double integral being taken over the whole of the aperture. Putting $x\cos i = p$ and $y = q$, $p$ and $q$ being therefore the projections of $x$ and $y$ on the wave-front, the above given expression reduces to

$$-\iint\frac{A}{\lambda f}\,\mathrm{d}p\,\mathrm{d}q\cdot\frac{F(i,\theta)}{\cos i}\sin\frac{2\pi}{\lambda}\left(Vt + \frac{P\xi + q\eta}{f} - c\right),$$

the integral being taken over the whole of the projection of the aperture on the wave-front. Now, the quantity of energy passing, per unit of time, through the obliquely held aperture, must, at any rate approximately, be equal to the quantity that would pass through an aperture identical with the projection of the first on the wave-front, cut in a screen held parallel to the waves. From this it follows that, *provided we assume $F(i,\theta)$ does not vary sensibly* throughout the diffraction-

pattern, it is equal to $\cos i$*. This, on the assumptions made, agrees with the expression deduced by Kirchhoff in his 'rigorous' formulation of Huygens's principle

$$-\frac{A}{2R\lambda}(\cos i + \cos\theta)\sin\frac{2\pi}{\lambda}(Vt - R)\mathrm{d}x\,\mathrm{d}y,$$

for in the direction of the wave-normal, $\cos i = \cos\theta$ and the expression reduces to

$$-\frac{A}{R\lambda}\cos i\sin\frac{2\pi}{\lambda}(Vt - R)\mathrm{d}x\,\mathrm{d}y. \tag{1}$$

The question now to be discussed is, whether we can always assume $F(i, \theta)$ to be appreciably the same in all the directions with which we are concerned, i.e. throughout the diffraction-pattern. Taking Kirchhoff's formula, the approximation clearly becomes inadmissible as $i$ approaches $\pi/2$. In this case $\cos i$ and $\cos\theta$ are both small and a variation in $\theta$ affects the value of the expression very largely. The value of $i$ at which the approximation ceases to represent matters fairly well depends upon the size of the aperture. $\theta$ will, in the diffraction-pattern, range from $\pi/2$ to $i$ and less. The value of the factor $(\cos i + \cos\theta)$ will vary from $\cos i$ to $2\cos i$ and more. Kirchhoff's formulation of Huygens's principle thus leads us to expect that at oblique incidences we should observe some phenomena due to the variation of the obliquity, which are inappreciable in the case of normal incidence. But though Kirchhoff's formula is able to indicate this, it itself does not hold at such incidences. It will be remembered that his formula is a purely mathematical deduction holding rigorously only in the case in which the wave-surfaces are not limited by screens of any kind. When they are limited by screens

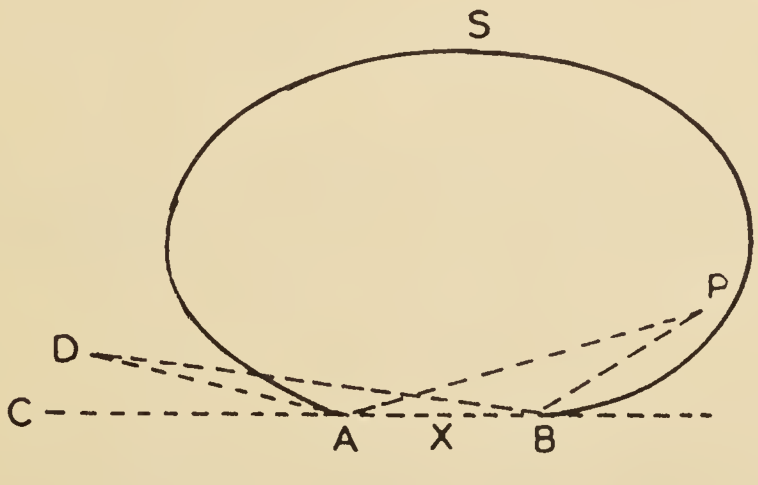



**Figure 2**

*From this and the integral given above, it appears that the diffraction-bands due to an aperture held obliquely are, at moderate incidences, identical with those due to its projection held normally: a proposition that might seem otherwise obvious, were it not for the fact that it is not true for very oblique incidences. As an instance, it can be shown experimentally that the diffraction-pattern observed when a circular aperture is held obliquely in front of a telescope directed at a point-source consists of a system of ellipses.

and mirrors, Kirchhoff's formula does not entirely meet the physical circumstances of the case and at oblique incidences leads to results widely differing from the truth. For example, let P be a point-source inside a surface S, such as that in the diagram, which is closed everywhere except over the opening AB. Take a point D inside the plane ABC at this point, the intensity ought obviously to be zero, D being inside the plane of the aperture. But Kirchhoff's formula leads to a different result. An element of the surface AB at X would have an effect proportional to

$$(\sin\underline{|P \times B} - \sin\underline{|D \times A}),$$

and if the angle PAB be not very large, the path-difference PA + AD − PB − BD would not be very large compared with $\lambda$ and therefore there ought to be a finite effect at D. This absurd result discredits the applicability of Kirchhoff's formula to experiment, and further shows that the investigation of the correct obliquity factor has an actual tangible relation to experiment.

We now proceed to obtain a solution of the general differential equation of wave-motion in the form of a surface-integral which satisfies the requisite boundary conditions. Let S be the surface over which the integration is to be effected and $r$ be the distance of a point in the space around from an element dS of the surface.

$$\exp(-ikr)/r,$$

is a solution of the symbolical equation

$$(\nabla^2 + k^2) = 0.$$

If d$n$ be an element of the normal to the element dS, then

$$\frac{d}{dn}\exp(-ikr)/r,$$

is also a solution of the differential equation. If $\phi$ is an expression which is a function of the position of dS on the surface but does not contain $r$, then

$$\frac{1}{2\pi}\iint \frac{d}{dn}\left(\frac{\exp(-ikr)}{r}\right)\phi dS, \tag{a}$$

is a solution of the equation. This integral is the well-known expression for the potential of a sheet of double sources of sound, provided one of the directions of the normal $n$ be regarded as positive and the other negative. The value of the integral at the surface itself is, on one side of it $+\phi$, on the other $-\phi$: for, regarding one of the directions of the normal as positive, the value of the integral at a point indefinitely near the positive side of the surface

$$= -\int \frac{d}{dr}\left(\frac{\exp(-ikr)}{r}\right)\phi r dr \cos\theta$$

$$= -\phi N \int_N^\infty \frac{d}{dr}\left(\frac{\exp(-ikr)}{r}\right)dr, \tag{b}$$

where $N$ is the indefinitely small distance of the point from the nearest element of the surface and $\theta$ is the angle between $n$ and $r$.

$$= Lt_{N=0}\phi N \frac{\exp(-ikN)}{N}.$$

$$= \phi.$$

In the particular case in which S is an infinite plane and $\phi$ is constant over the whole of it, the expression ($b$), instead of being an approximation true in the limit ($N = 0$), is perfectly rigorous for all values of $N$ and expression ($a$) reduces to

$$\pm \phi N \frac{\exp(-ikN)}{N} = \pm \phi \exp(-ikN),$$

which is the velocity potential of two sets of aerial waves proceeding from the infinite plane, on opposite sides of it, to an infinite distance.

If the surface over which the integration is effected is part only of an infinite plane, the integral ($a$) can be written as

$$-\frac{1}{2\pi}\iint \frac{d}{dr}\left(\frac{\exp(-ikr)}{r}\right)\phi dS \cos\theta. \tag{c}$$

At points on that part of the infinite plane over which the integration is *not* effected, the integral ($c$) is zero, for (figure 3) $\theta = \pi/2$ and $\cos\theta = 0$ for all such

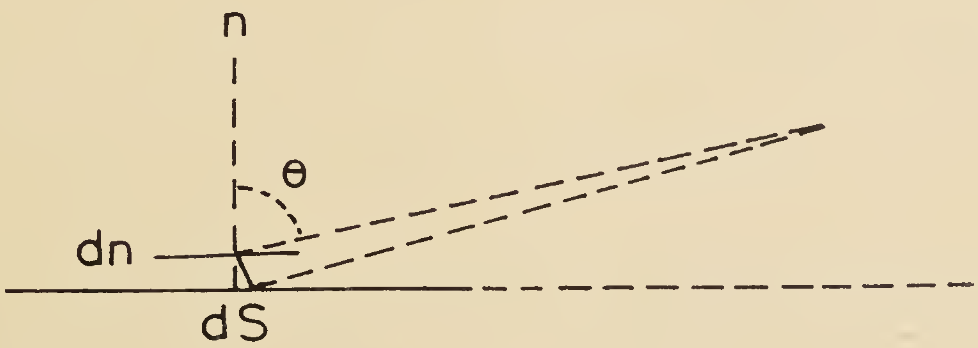



**Figure 3**

points. This part of the infinite plane is therefore one of 'silence.' This is true whether $\phi$ is or is not constant over the whole of the surface of integration.

We shall now apply the solution ($a$) of the general differential equation to the following problems: it is understood that in each case the reflecting surface or transmitting aperture is of dimensions large compared with the wavelength.

*Reflexion of plane aerial waves at an infinite rigid plane*, whose position is given by $x = 0$.

Let the plane waves be incident at an angle $i$ on the positive of the plane, and the

velocity-potential of the incident waves be the real part of

$$A \exp[ik(Vt + x\cos i - y\sin i)].$$

Superpose upon this, the value of the integral

$$\frac{A}{2\pi}\int\int\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\exp(-ikr)}{r}\right)\exp[ik(Vt - Y\sin i)]\,\mathrm{d}Y\mathrm{d}Z, \qquad (d)$$

taken over the whole of the infinite plane, $Y$ and $Z$ being the $y$ and $z$ coordinates at any point on the plane.

The value of the integral at any point in the space on the positive side of the infinite plane is

$$A \exp[ik(Vt - x\cos i - y\sin i)]$$

and on the negative side

$$-A \exp[ik(Vt + x\cos i - y\sin i)].$$

The resultant disturbance on the positive side is

$$\phi = A\exp[ik(Vt + x\cos i - y\sin i)]$$
$$+ \exp[ik(Vt - x\cos i - y\sin i)]$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}x} = Aik\cos i\exp[ik(Vk - y\sin i)]\exp(ikx\cos i)$$
$$-\exp(-ikx\cos i), \qquad (e)$$

which when $x = 0$, is equal to zero.

On the negative side

$$\phi = 0 \quad \text{and} \quad \frac{\mathrm{d}\phi}{\mathrm{d}x} = 0.$$

The necessary conditions are thus satisfied. The normal velocity at the plane is zero on both sides of it, and the region on the negative side of the plane is entirely screened from disturbance. The same is true even if the reflecting surface occupies only part of the plane $x = 0$, for at points close to the plane, on either side of it, the result of the integration ($d$) is practically the same as if it were extended over infinity.

In the case last mentioned of a finite reflecting surface, the effect of the reflected waves at points not near the reflecting plane and on the positive side of it, is given by the integral ($d$) taken over the whole of the reflecting area. This integral may be written as

$$-\frac{A}{2\pi}\int\int\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\exp(-ikr)}{r}\right)\exp[ik(Vt - Y\cos i)]\cos\theta\,\mathrm{d}Y\mathrm{d}Z, \qquad (f)$$

where $\theta$ is the obliquity of the point of observation viewed from the element $dYdZ$. The point of interest is, that for all points on the continuation of the reflecting sheet, $\theta$ being equal to $\pi/2$, the value of the integral is rigorously zero and the plane is therefore one of silence. This becomes of importance experimentally when the angle of incidence is such that the diffraction-pattern is formed near this plane of silence. It is not difficult to understand why the elements of a reflecting surface should be equivalent to double sources and not to simple sources of sound: for, if the reflecting plane be replaced by an *indefinitely thin* sheet in the same position, every point of which instead of being kept fixed is obliged to follow the vibration in the incident waves, then it is obvious that these waves would be transmitted without disturbance to the far side of the plane. The effect of the reflecting plane must therefore be equivalent to that of the reversed motion of the thin sheet in a medium entirely at rest. This involves periodic *compressions* and rarefactions on one side of the plane, and simultaneous *rarefactions* and compressions on the other i.e., periodic *introductions* and abstractions of fluid on one side, and simultaneous *abstractions* and introductions on the other. This is the equivalent of a sheet of double sources.

## Passage of plane aerial waves through an aperture in a thin plate

This can be seen to be directly deducible from the preceding: the integral (*d*) would have to be taken over the reflecting plate (excluding the aperture), and the expression for the velocity potential of a system of plane aerial waves passing through an infinite medium superposed upon it. On the positive side of the reflecting sheet, the integral (*d*) gives the effect of the reflected waves, and since in the integration the area of the aperture is excluded, the effect of an element of the aperture in any direction on the positive side of the sheet is zero. On the negative side of the plate, the disturbance passing through the aperture appears as the difference of two quantities: if the integral (*d*) is taken over the whole of the reflecting sheet, including the aperture in it, this difference is zero. It follows therefore that the disturbance passing through the aperture is given by the integral (*d*) taken over the area of the aperture alone, with its sign reversed. It follows therefore, that considerations such as those that apply to the case of reflexion at oblique incidences apply to this also.

*Light incident on a perfectly reflecting screen*, the waves being polarized in a plane at right angles to the plane of incidence.

The magnetic vector $\zeta$ in the incident waves is parallel to the axis of $z$ and to the reflecting screen. Since $\zeta$ satisfies the general differential equation and also, at the screen, the condition $(d\zeta/dx) = 0$, the expression for $\zeta$ in the secondary waves is exactly the same as that for $\phi$ in the preceding paragraphs. As for the other components of the magnetic vector in the secondary waves, they are zero at all

points in the plane of incidence and we need not at present trouble about them.

*Light incident on a perfectly reflecting screen*, the waves being polarized in the plane of incidence.

The electric vector $\zeta$ in the incident waves is parallel to the axis of $z$ and to the reflecting screen. The expression for the vector $\zeta$ in the secondary waves can still be deduced from the integral ($d$), if the sign of this is changed and the operand $\mathrm{d}/\mathrm{d}x$ is replaced by $\mathrm{d}/\mathrm{d}n$, where $\mathrm{d}n$ is an element of the normal to the reflecting surface, both directions of this being regarded as positive. On the side of the screen on which the waves are incident and at points close to it,

$$\zeta = A \exp[ik(Vt + x\cos i - y\sin i)] - A\exp[ik(Vt - x\cos i - y\sin i)],$$

which is zero if $x = 0$.

On the other side, at points close to the reflecting sheet

$$\begin{aligned}\zeta &= A\exp[ik(Vt + x\cos i - y\sin i)] - A\exp[ik(Vt + x\cos i - y\sin i)]\\ &= 0.\end{aligned}$$

The other components of the electric vector in the secondary waves need not be considered here.

## Apertures in perfectly-reflecting plates

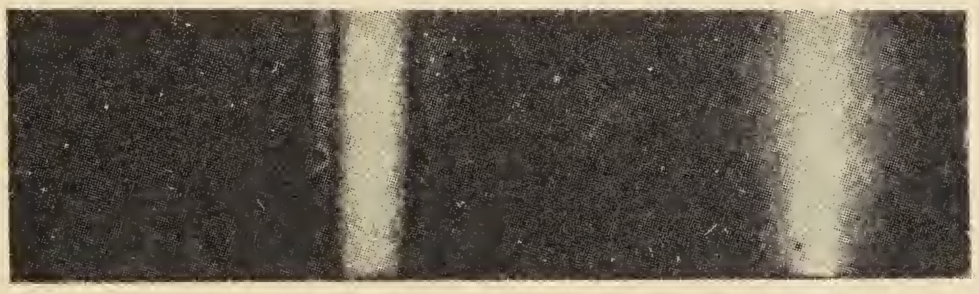
The results for these cases can be deduced from the expressions for reflexion, in exactly the same way as was done for aerial waves. On the side on which the waves are incident, the effect of an element of the aperture is zero. On the other side, the $z$ component of the electric or (as the case may be) magnetic vector is given by the integral ($f$) taken over the aperture and with its sign reversed.

From the integral ($f$) it can be seen that the obliquity-factor, for the $z$ component of the light-vector in the secondary waves, is simply the cosine of the obliquity and is independent of the angle of incidence of the waves on the reflecting screen or aperture. Differentiating and realizing the integral ($f$), it can be seen that for moderate incidences it is equivalent to the expression (1), for at such incidences, the diffraction-pattern is formed in a direction in which $\cos\theta$ does not vary rapidly with $\theta$ and may therefore be put equal to the mean value $\cos i$. At oblique incidences this is no longer true. The integral of ($f$) in the case of a rectangular aperture (sides $l, m$) held obliquely in a parallel beam of light, in front of a telescope, gives for the illumination in the diffraction-pattern

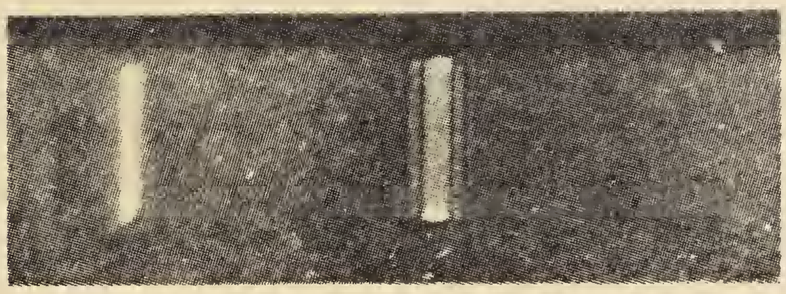
$$A^2\frac{l^2m^2}{\lambda^2 f^2}\cos^2\theta\,\frac{\sin^2\dfrac{\pi l}{\lambda}(\sin i - \sin\theta)\sin^2\left[\dfrac{\pi m}{\lambda}\sin\psi\right]}{\left[\dfrac{\pi l}{\lambda}(\sin i - \sin\theta)\right]^2\left[\dfrac{\pi m}{\lambda}\sin\psi\right]^2}, \qquad (g)$$

$\theta$ and $\psi$ being the angles of diffraction, in other words, the angles made by the diffracted 'ray' with the two planes normal to the plane of the aperture. The expression deduced from the non-analytical theory is

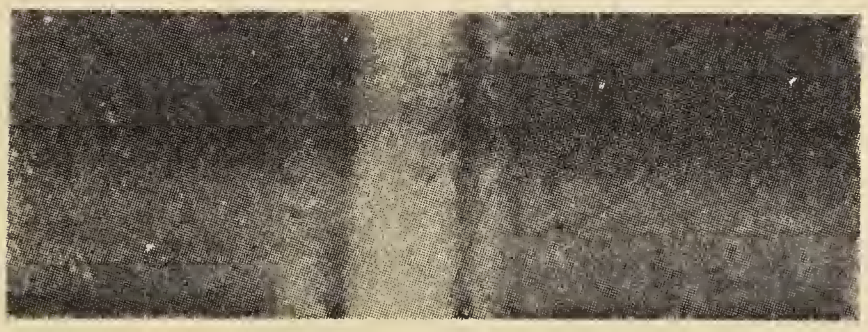
$$A^2 \frac{l^2 m^2}{\lambda^2 f^2} \cos^2 i \frac{\sin^2 \frac{\pi l}{\lambda}(\sin i - \sin\theta) \sin^2\left[\frac{\pi m}{\lambda}\sin\psi\right]}{\left[\frac{\pi l}{\lambda}(\sin i - \sin\theta)\right]^2 \left[\frac{\pi m}{\lambda}\sin\psi\right]^2}. \tag{h}$$



(1)



(2)



(3)

**Plate I**

These two expressions give, for moderate incidences, practically identical results. At all incidences, they give the same positions for the minima of illumination in the diffraction-pattern. But as regards the distribution of illumination in the pattern, and as a minor point, as regards the positions of the maxima of illumination, they give at oblique incidences very different results. The expression ($h$), as has already been mentioned, makes it out that the illuminations at corresponding points on either side of the central band should be equal, and that at the limiting plane $\theta = \pi/2$ the curve of illumination should drop discontinuously from a finite to zero value, both of which results are contradicted by experiment. The expression ($g$) shows that the illumination at points in the pattern on the side of the central band, nearer to the limiting plane $\theta = \pi/2$, is less than at the corresponding points on the farther side: and that since $\cos^2 \theta$ is zero if $\theta = \pi/2$, the curve of illumination falls continuously to zero at the limiting plane. Both these results are verified by experiment. Photographs (1) and (2) (see plate I of the diffraction-bands formed by reflexion at the face of a prism, and photograph (3) by transmission through an aperture, all exhibit these effects.

The investigation of the intensity at different points on a secondary wave, given above, is for the case of the incidence of light on a perfectly reflecting screen. It can be shown that by a suitable modification of the integral ($a$) it can be made to cover the case of the reflexion and refraction of light-waves at a dielectric medium, and that the obliquity-factor in these cases, as given by theory and as applicable to experiment, is $\cos \theta$. As a matter of fact the photographs (1) and (2) in the plate are of the diffraction-bands formed by reflexion at a *glass* prism.

Similar obliquity-effects should be obtained with other forms of aperture, for example with a reflexion or transmission grating, *with wide rulings*, if it is held obliquely and if points in the field of view receive no light from the *grooves* of the grating, the only effective parts being plane portions of the grating, these being parallel to, or coincident with, its general surface. These and other considerations will form the subject of a future paper.

## Summary and conclusion

Each element of a reflecting surface, may, when waves are incident upon it, according to Huygens, be supposed to send out into each of the two media meeting at the surface, hemispherical secondary wavelets. The amplitude of the disturbance at points on these secondary waves may be investigated mathematically, as in this paper, and shown to be a maximum at the pole of the hemisphere and zero at points on its equator. A similar result for the case of an aperture in a screen can be deduced therefrom. Observation of diffraction-phenomena at oblique incidences confirms this result. The law of variation of the amplitude of the principal component of the light-vector is the cosine of the obliquity. This may be subjected to experimental investigation, and it is hoped that if the

necessary instruments are available, photometrical measurements to test this can be made.

The experiments and observations recorded in this note were made at the Physical Laboratory of the Indian Association for the Cultivation of Science, Calcutta, and formed the subject of a demonstration held at a Special Meeting of that body on the 18th of January last.

P.S. dated the 26th of Nov.—Photometrical measurements in verification of the above have been carried out. These will be dealt with in a future communication.

*Nature* (*London*) **82** 69 (1909)

# The photometric measurement of the obliquity factor of diffraction

In vol. lxxviii of *Nature* (*London*) (May 21, 1908, p. 55) was published a note on 'Secondary Waves of Light,' in which I described the diffraction effects produced by an obliquely held rectangular aperture or reflecting surface, and pointed out that the observed distribution of illumination in the pattern was not in accordance with that deduced in the ordinary way. I indicated an explanation of the discrepancy, that it was due to the variation of the obliquity factor of diffraction within the limits of the pattern.

The interest of the observations lay in the fact that such an effect had never been noticed before, and that the observations enabled us actually to trace the variation of the amplitude of vibration from point to point on Huygens's secondary waves. A full description of the effect and a mathematical investigation were published in the *Philos. Mag.* for January.

The effect observed was that the intensities of illumination in corresponding bands on opposite sides of the central band in the unsymmetrical pattern were unequal. A photometric investigation of this difference in illumination has been carried out. The method was to use revolving sectors to reduce the illumination in one of the two bands to be compared, so as to make them both of equal brightness. The following table illustrates the comparisons made:

| No. of expt. | Ratio of illumination according to ordinary theory | Ratio of illumination actually determined | Ratio of illumination calculated from obliquity |
|---|---|---|---|
| 1 | 1·00 | 1·66 | 1·61 |
| 2 | 1·00 | 1·81 | 1·98 |
| 3 | 1·00 | 2·66 | 2·43 |
| 4 | 1·00 | 3·25 | 3·27 |

The obliquity law demonstrated by these measurements is that, in the hemispherical wavelets emitted by each element of a transmitting aperture or reflecting surface upon which waves are incident at any angle, the amplitude of the light vector is, at any point in the plane of incidence, proportional to the cosine of the angle made by the line joining that point and the element, with the normal to the plane of the element.

*C V RAMAN*

Post Box 59, Rangoon

*Philos. Mag.* **21** 618–626 (1911)

# The photometric measurement of the obliquity factor of diffraction*

C V RAMAN, M.A.†

In an earlier paper on 'The experimental study of Huygens's secondary waves,' published in this Journal (*Philos. Mag.* Jan. 1909), I showed that the study of diffraction-patterns formed at oblique incidences is of particular interest, as the phenomena observed throw a fresh light on Huygens's principle and on the theory of secondary waves. Quantitative measurements of the effects described in that paper have since been made with the rectangular aperture, this being the one which lends itself most readily to the work. The present paper deals with the detailed and quantitative verification of the explanation suggested by me to account for the effects observed: its net result is to show that it is possible actually to observe and measure the variation of the amplitude of vibration on a Huygens's secondary wave.

My first observations on oblique diffraction by a rectangular aperture or reflecting surface were published in a note in the *Philos. Mag.* for Nov. 1906. The principal points noted were that the diffraction-pattern was unsymmetrical in character, the width of the bands instead of being the same throughout, as in the case of normal incidence, increasing continuously from one side of the pattern to the other: again, that the number of bands on one side of the pattern was limited. These results were shown to be simple consequences of the formula giving the positions of the minima of illumination in the pattern, i.e.

$$\sin i - \sin\theta = \pm\frac{n\lambda}{a}.$$

Later observations, in which the unsymmetrical character of the distribution of intensity in the pattern was noted, were described in the second paper quoted above. These observations were inexplicable on the ordinary (non-analytical) theory of diffraction. It was found that the broader bands on one side of the pattern were considerably feebler in intensity than the corresponding bands on the other side; whereas the ordinary theory required that they should be of equal

*A preliminary note on this subject was published in *Nature* (*London*), dated the 18th of November 1909.

†Communicated by the author.

intensity. 'The differences of intensity became very large as grazing incidence was approached. It was also observed that for any particular angle of incidence, the illumination in the pattern died away as the plane which sets the limit to the number of bands (on the side of the pattern at which these were broader) was approached. According to the ordinary theory, on the other hand, the intensity should have remained finite right up to this plane (which is the plane of the reflecting surface or aperture) and fallen discontinuously to zero at this point. It was this last, rather anomalous consequence of the ordinary theory which first drew my attention to the fact that the illumination in the diffraction-pattern was actually unsymmetrical in character.

The only explanation for these effects that I could find was that they were due to the varying obliquities at different points of the pattern; in other words, that the effects were due to the obliquity-factor of diffraction, of which no account is taken in the expression for the intensity derived from the ordinary theory. This explanation was developed in the paper quoted above, and was shown to be capable of accounting for the unsymmetrical character of the intensity-distribution.

One method of verifying the theory suggested would probably be to show that no other explanation of the observed effects is feasible. Any suggestion about defects in the experimental arrangements was found to be inadmissible. The focussing of the bands was or could be rendered perfect. The reflecting surface used was the face of a first-class prism of the kind used for accurate spectrometric work, and was absolutely fresh and clean: in fact, no perceptible effect was found to be produced by any moderate amount of dust or grease, the reason apparently being that the incidence was very oblique. The effect of scattered light, if any, in the field was in the opposite direction, and it could almost entirely be excluded by special precautions (to be described below in connection with the photometric work). In fact, all these factors were found to be quite incapable of accounting for the observed effects.

Even assuming that there was at work in the experiments some unknown factor (other than an obliquity-factor), it is difficult to see how it could have altered the relative intensity of the bands on the two sides of the pattern to a considerable degree without either (1) shifting the positions of the minima of illumination from those given by the ordinary theory of diffraction; or (2) without causing a finite sensible intensity to exist at the minima, which according to the ordinary theory are absolute zeroes of illumination. Both of these points were carefully tested by direct observation and by photography. With a sufficiently homogeneous but powerful source of light, there is no difficulty in observing perfectly black bands alternating with bright bands of great intensity. To secure a satisfactory negative showing clear minima, it is necessary first carefully to abolish halation by backing the photographic plates before use. If care is taken to avoid chemical fogging of the plates, it is possible to get photographs of the diffraction-pattern showing great density at the maxima with practically clear

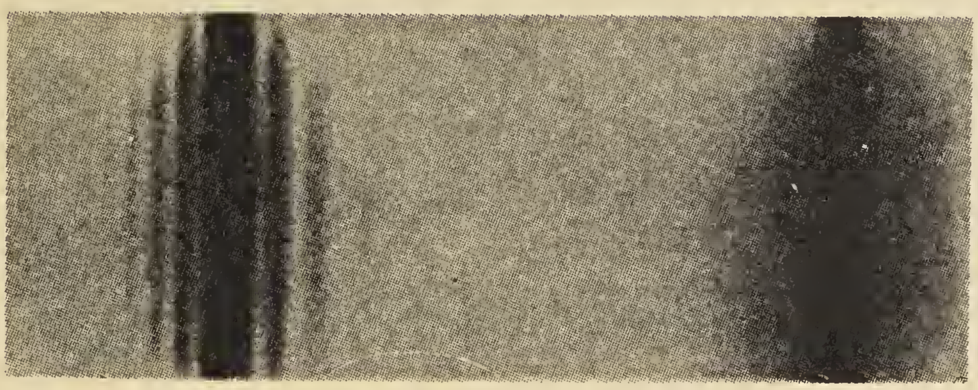


**Figure 1.** Enlarged photograph showing diffraction pattern.

**Plate I**

glass at the minima. An enlargement from such a negative is shown as figure 1 in plate I. In the diffraction-pattern photographed the theoretical ratios of illumination, calculated by a method to be explained below, should have been as follows:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| The 1st narrow | bright | band | 1·81 times | as intense | as the | 1st broad | band. | |
| 2nd | " | " | 2·92 | " | " | 2nd | " | " |
| 3rd | " | " | 5·68 | " | " | 3rd | " | " |
| 4th | " | " | about 20 | " | " | 4th | " | " |

which is too faint to be visible in the negative.

From an inspection of the original negative (or even of the enlargement, due regard being paid to the reduction of contrasts), it could be seen that the calculated values were of the right order.

The photographs were measured under a travelling microscope. The instrument was one which had been constructed by Messrs Hilger for spectrum-photograph measurements. The following tables show how nearly the observed positions of the *dark* bands agree with the calculated values.

**Table 1.** Position on the plate.

| | Observed | Calculated |
|---|---|---|
| The direct image | 7·5833 | 7·5833 |
| 1st broad dark band | 7·2550 | 7·2551 |
| 1st narrow „ „ | 7·1671 | 7·1667 |
| 2nd „ „ „ | 7·1335 | 7·1335 |
| 3rd „ „ „ | 7·1035 | 7·1041 |

**Table 2.** Positions on the plate.

| | Observed | Calculated |
|---|---|---|
| The direct image | 8·3626 | 8·3626 |
| 1st narrow dark band | 8·6579 | 8·6579 |
| 2nd „ „ „ | 8·6997 | 8·6997 |
| 3rd „ „ „ | 8·7339 | 8·7347 |
| 4th „ „ „ | 8·7656 | 8·7653 |

**Table 3.** Positions on the plate.

| | Observed | Calculated |
|---|---|---|
| The direct image | 8·4837 | 8·4837 |
| 2nd broad dark band | 8·8330 | 8·8332 |
| 1st „ „ „ | 8·8861 | 8·8857 |
| 1st narrow „ „ | 8·9610 | 8·9602 |
| 2nd „ „ „ | 8·9898 | 8·9903 |
| 3rd „ „ „ | 9·0166 | 9·0175 |

**Table 4.** Positions on the plate.

| | Observed | Calculated |
|---|---|---|
| The direct image | 7·0966 | 7·0966 |
| 3rd broad dark band | 8·2190 | 8·2191 |
| 2nd „ „ „ | 8·3186 | 8·3187 |
| 1st „ „ „ | 8·4010 | 8·4025 |
| 1st narrow „ „ | 8·5424 | 8·5427 |
| 2nd „ „ „ | 8·6028 | 8·6038 |
| 3rd „ „ „ | 8·6622 | 8·6607 |
| 4th „ „ „ | 8·7164 | 8·7141 |

The measurements do not reveal any decided departure from theory, of the positions of the minima of illumination.

The considerations advanced above seem to me to be sufficient proof of the general correctness of the explanation suggested by me. I realized, however, that actual measurements of the obliquity-effects would form a more convincing proof, and at the same time would have intrinsic experimental interest and serve also to test the correctness of the mathematical work.

There are *two* classes of obliquity-effects, which in practice are capable of experimental determination. The variation of the obliquity-factor over the diffraction-pattern profoundly modifies the distribution of illumination. In the first place the illuminations in corresponding bands on either side of the central band, instead of being equal, become widely different. The ratio of the

illumination in two corresponding bands is capable of photometrical determination and furnishes a method of measuring the obliquity-factor. Then, again, the curve of distribution of illumination in any particular *band* of the pattern, i.e. from one minimum to the next, is modified, and the position of the point of *maximum* illumination is shifted. This shift furnishes a second method. Both these lines of investigation have been worked on. In the present paper only the first method will be dealt with, the second being reserved for a separate communication.

The ratio of the illuminations at corresponding points (on opposite sides of the central band) is, as shown in the paper quoted above (*Philos. Mag.* Jan. 1909, page 214), equal to $\cos^2\theta_1/\cos^2\theta_2$, where $\theta_1$ and $\theta_2$ are the angles made by the diffracted rays with the normal to the plane of the reflecting surface or transmitting aperture. Putting

$$\theta_1 = \left(\frac{\pi}{2} - \delta_1\right) \quad \text{and} \quad \theta_2 = \left(\frac{\pi}{2} - \delta_2\right),$$

the ratio of illumination may be put equal to $\delta_1^2/\delta_2^2$, provided $\delta_1$ and $\delta_2$ are fairly small.

The ratio was determined experimentally by the method of revolving sectors. Plate II, figure 2 shows the photometric arrangement adopted. Two disks of

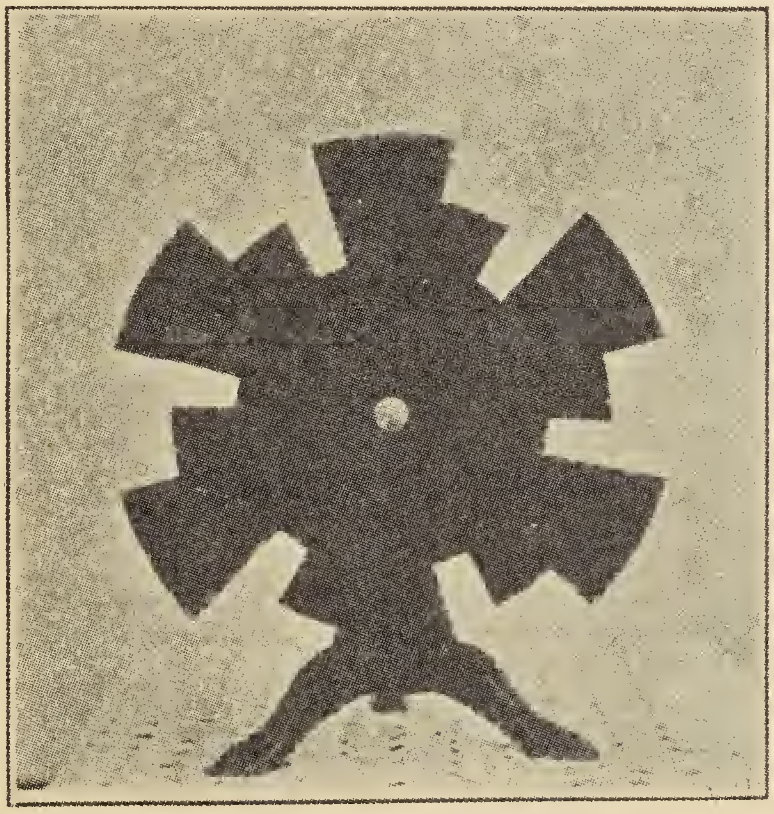


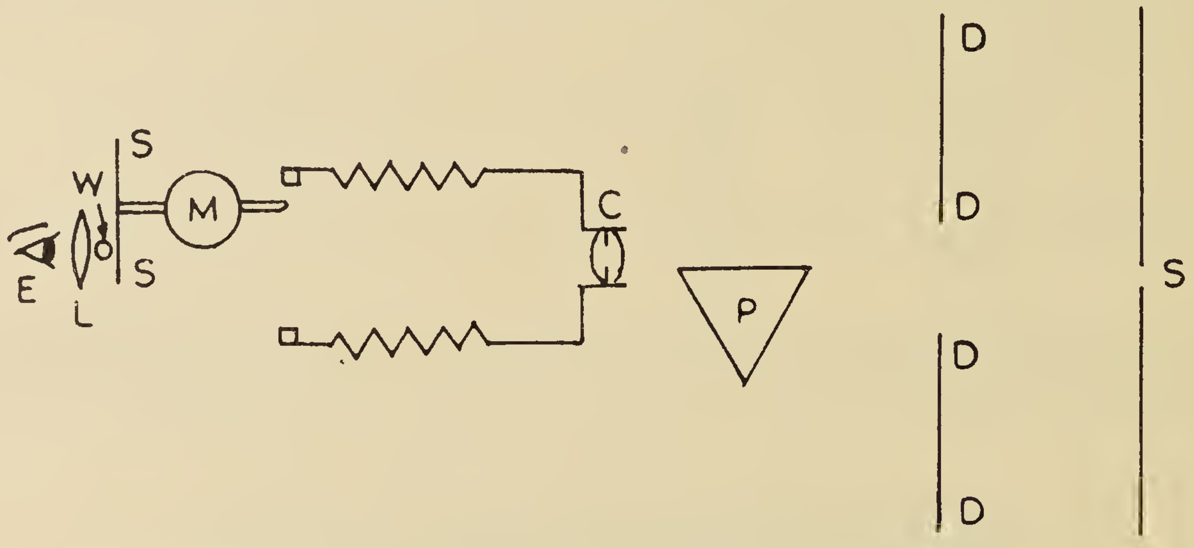
**Figure 2.** Photograph of black photometric disk.

**Plate II**

blackened cardboard about 24 and 21 cm in diameter respectively, are mounted concentrically, just touching each other on the shaft of a small electric motor. The outermost annulus of the first disk having a breadth of 6 cm is divided into 12 equal sectors of 30° each and alternate sectors are removed. The 2nd disk is similarly treated for a breadth of 3 cm from its circumference inward. It is

arranged so that the two disks can be clamped on the shaft in any desired position relative to each other. On working the motor, the two annuli (having a breadth of 3 cm each) are seen to transmit unequal intensities of light. The photometric disk lies and rotates in a vertical plane. The diffraction-pattern is focussed on the photometric disk, so that the central bright band (which, as also the other feebler ones on either side of it, is vertical) lies on the edge between the two annuli at right angles to the horizontal diameter of the disk. The broad bands on one side of the pattern lie on the brighter annulus the narrow ones on the other side in the feebler annulus.

The general disposition of the apparatus when at work is shown in the diagram below. The light from the slit S passes directly and partly also after reflection at




the surface of the prism P to the lens of a camera. The image of the slit and the diffraction-pattern are focussed by the lens on to the plane of the photometric disk, the adjustments being as described above. DD is a screen placed in front of the prism to shut off superfluous light. To further reduce the amount of scattered light, all the faces of the prism except the one used are completely blackened with a coat of varnish. The pattern is observed with the aid of a lens L placed behind the photometric disk. The central bright band of the pattern which would otherwise dazzle and confuse the eye, is cut off by a vertical wire W placed in front of the eyepiece.

The relative intensity of the first bands on either side of the central band is observed. Their brightnesses are adjusted to equality by sliding the disks one over the other, and thereby altering the angle of the smaller sectors. To secure accuracy, it was found essential to have the bands well-illuminated by using a brilliant source (in practice sunlight) to light up the slit. The adjustment to equality of brightness was found to be greatly facilitated if the upper or lower edge of the bands stood out sharply against a perfectly dark background.

When the adjustment is made, the lengths of the chords of the six larger and the six smaller sectors are read off with a scale. A divided glass scale is then placed in the focal plane of the camera so that the diffraction-pattern falls upon it, and the positions of (*a*) the direct image of the slit, (*b*) the maximum intensity in the central band, (*c*) and (*d*) the maximum intensities in the bands compared, are read off and

noted. The theoretical ratio of the illuminations can be computed from these measurements and compared with the value determined from the measurements of the sectors. Table 5 illustrates the method.

**Table 5**

| Chords of small sectors | Chords of large sectors | Scale-readings |
|---|---|---|
| 1·60 | 4·10 | $(a) = 7{\cdot}55$ |
| 1·69 | 4·10 | $(b) = 4{\cdot}85$ |
| 1·67 | 4·10 | $(c) = 5{\cdot}15$ |
| 1·55 | 4·15 | $(d) = 4{\cdot}57$ |
| 1·62 | 4·10 | $(a+b)/2 = 6{\cdot}20$ |
| 1·63 | 4·14 | $(a+b)/2 - c = 1{\cdot}05$ |
| Mean 1·63 | Mean 4·12 | $(a+b)/2 - d = 1{\cdot}63$ |

$$\text{Observed illumination ratio} = \frac{4{\cdot}12}{1{\cdot}63} = 2{\cdot}53.$$

$$\text{Calculated illumination ratio} = \left(\frac{1{\cdot}63}{1{\cdot}05}\right)^2 = 2{\cdot}41.$$

The results of measurements made on this plan over a pretty wide range of incidences are shown in table 6.

**Table 6**

| Ratio of illumination according to the ordinary theory | Ratio of illumination actually determined | Ratio of illumination calculated from obliquity |
|---|---|---|
| 1·00 | 1·48 | 1·44 |
| 1·00 | 1·72 | 1·81 |
| 1·00 | 2·44 | 2·22 |
| 1·00 | 2·53 | 2·41 |
| 1·00 | 3·01 | 3·18 |
| 1·00 | 3·89 | 4·00 |
| 1·00 | 4·88 | 5·10 |
| Total | 19·95 | 20·16 |

The totals at the foot of the table have no significance except that they serve to show by their agreement that the differences between the calculated and observed values in individual experiments lie pretty evenly on either side. The measurements entirely confirm the theory.

*Summary*: General considerations lead to the conclusion that the unsymmetrical character of the distribution of illumination in diffraction-patterns observed at oblique incidences, must be an obliquity effect. This is entirely verified by photometric work on the diffraction-pattern. The measurements show that the obliquity-factor follows the cosine-law, which may be stated thus: 'In the hemispherical wavelets emitted by each element of a transmitting aperture or reflecting surface upon which waves are incident at any angle, the amplitude of the light vector is, at any point in the plane of incidence, proportionate to the cosine of the angle made by the line joining that point and the element, with the normal to the plane of the element.'

This investigation was commenced at the Physical Laboratory of the Indian Association for the Cultivation of Science, Calcutta. My removal from that station necessitated its being completed elsewhere.

*Philos. Mag.* **30** 701–702 (1915)

# On intermittent vision

C V RAMAN, M.A.*

One of the most curious and interesting of the phenomena met with in the borderland between the physics and the physiology of vision, is the occasional appearance of 'intermittency' seen by an observer who watches a rapidly revolving object, e.g. a disk with alternate white and black sectors revolving in its own plane. The phenomenon has been investigated by Mr Mallock, who, in his paper on the subject†, puts forward the somewhat startling hypothesis that a slight mechanical shock to the head or body of the observer produces a periodic but rapidly extinguished paralysis of the perception of sight, and that the nerves on which seeing depends cannot bear more than a certain amount of mechanical acceleration without loss of sensibility. Mr Mallock's conclusions have been criticised in a recent paper‡ by Prof. Silvanus P Thompson, who, as the result of his experiments, comes to the conclusion that Mr Mallock's hypothesis is unnecessary, and adds that the explanation of the phenomenon appears to be that 'when the moving images of the white sectors on the retina are suddenly shifted by a minute displacement, they fall upon some of the rods and cones which are relatively unfatigued, and which for the instant are therefore of greater sensitiveness.'

I have recently had occasion to examine this subject, and I find that Prof. S P Thompson's suggestion that retinal fatigue is the cause of the effect is apparently also untenable, as it is inconsistent with the observed phenomena. There is no difficulty in testing the hypothesis that the fatigue of the rods and cones in the retina is the cause of the effect. If the images of the *white* sectors on the retina are *suddenly* moved, by some means, to positions of greater sensitiveness, namely, to the portions which the *dark* sectors had just passed over, we should, according to Prof. S P Thompson, expect the white sectors to flash out bright on a dark field. By observing the revolving disk in a mirror which is suddenly tilted by a small amount, the necessary conditions may be secured experimentally with a stationary retina, but as a matter of fact the expected effect fails to manifest itself. Paradoxically enough, it is by a sudden apparent displacement of the white sectors in the *opposite* direction, i.e. which brings them to positions on the retina

*Communicated by the author.

†*Proc. R. Soc. London* **A89**, 407.

‡*Proc. R. Soc. London* **A90** 448.

already fatigued by light, that we secure the desired result, namely, a sudden brightening and appearance of white sectors on a dark field. As the two ends of the diameter of the disk move in opposite directions, these two cases may be simultaneously observed and compared.

It is thus seen that the explanation in terms of retinal fatigue fails to account for the facts, and it seems unnecessary to postulate either it or else the nerve 'paralysis' suggested by Mr Mallock as the principal factor at work. The following explanation seems more in accordance with the facts: So long as the retina is absolutely at rest, and the white and dark sectors follow one another at intervals short compared with the period of persistence of vision, the disk appears uniformly illuminated. But if the retina is set in motion even for a small fraction of a second, say by a slight mechanical shock, or by the eye involuntarily following the motion of the sectors, and if the direction of this motion is such that the white sectors remain on any given portion of the retina for a *longer* interval than they otherwise would, the impression of light over the areas occupied by the dark sectors has time enough to die away appreciably, and we thus get the illusion of stationary white sectors on a dark ground. A movement of the retina in the *opposite* direction should, however, produce little or no perceptible effect, provided the rotation of the disk is sufficiently rapid. This is exactly what is found in experiment.

Calcutta
10 May 1915

*Nature* (*London*) **102** 205 (1918)

# The colours of the striae in mica

On examining even the most regularly split and transparent pieces of mica by diffuse reflected light, a few fine hair-like and rather irregular lines may generally be seen running along the surface. We have found that these lines or striae show some very interesting effects when mica is examined in a Töpler 'Schlieren' apparatus. The sheet as a whole, being optically good, remains invisible, but the striae shine out as brilliant and vividly coloured lines of light, the colours being different for different striae, and changing in a remarkable manner as the inclination of the mica relatively to the direction of the light in the apparatus is altered. For instance, a stria at normal incidence may appear crimson and, as the mica is rotated about an axis in its own plane, become successively purple, green, yellowish-green, yellow, orange, scarlet-red, green, yellow and red.

The phenomenon is being investigated in detail by one of us (P N Ghosh), but as to its general nature there appears to be little doubt. The striae are lines at which the thickness of the mica changes in a discontinuous manner, and the luminosity is due to the radiation from the discontinuity acting as a laminar diffracting boundary. For any particular wavelength the radiation is zero if the retardation of the wavefront on either side of the discontinuity differs by an even multiple of half a wavelength, and is approximately a maximum if the difference is an odd multiple of half a wavelength. The detailed mathematical investigation would follow the general lines indicated by Lord Rayleigh in his theory of the Foucault 'knife-edge' test (*Philos. Mag.*, February, 1917).

*C V RAMAN*
*P N GHOSH*

210 Bowbazaar Street, Calcutta
5 September

*Phys. Rev.* **13** 259–260 (1919)

# On the diffraction-figures due to an elliptic aperture

C V RAMAN

Synopsis. Photographs are shown of the diffraction pattern obtained near the focus of a converging pencil of rays from an elliptic aperture. The transition from the Fresnel to the Fraunhofer class of diffraction figure is traced and attention drawn to the geometric law to which the pattern conforms, namely, that the brightest part of the diffraction pattern outside the elliptic cross-section of the beam lies within the geometric evolute of this cross-section and is bounded by it.

It is well known that the diffraction pattern at the focus of a convergent pencil of rays limited by an elliptic aperture is made up of a central spot of light surrounded by alternate dark and bright rings which are all elliptical in shape, the direction of the major axis of the rings being the same as that of the minor axis of the aperture and *vice versa.** The distribution of luminosity in the focal plane may be readily deduced from the known results for the case of the circular aperture by a simple transformation of coordinates. The theory of the phenomena observed in *ultra-focal* planes is, however, considerably more complicated, and so far as the present writer is aware, has never been fully worked out for the case of elliptic apertures. Recently, while working in collaboration with Mr R S Deoras, the writer made some observations on the diffraction of light by elliptic apertures for convergent and also for divergent pencils, and has noticed that the configuration of the fringes presents some rather striking geometrical features, particularly in the cases in which the eccentricity of the elliptic aperture is considerable. It is thought that a brief account of the observations, and the reproductions of some of the photographs of the diffraction-figures secured in the course of the work (figures 2–11) may be of interest to the readers of the *Phys. Rev.*

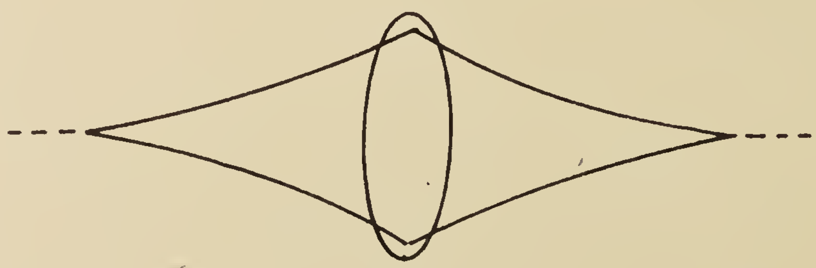
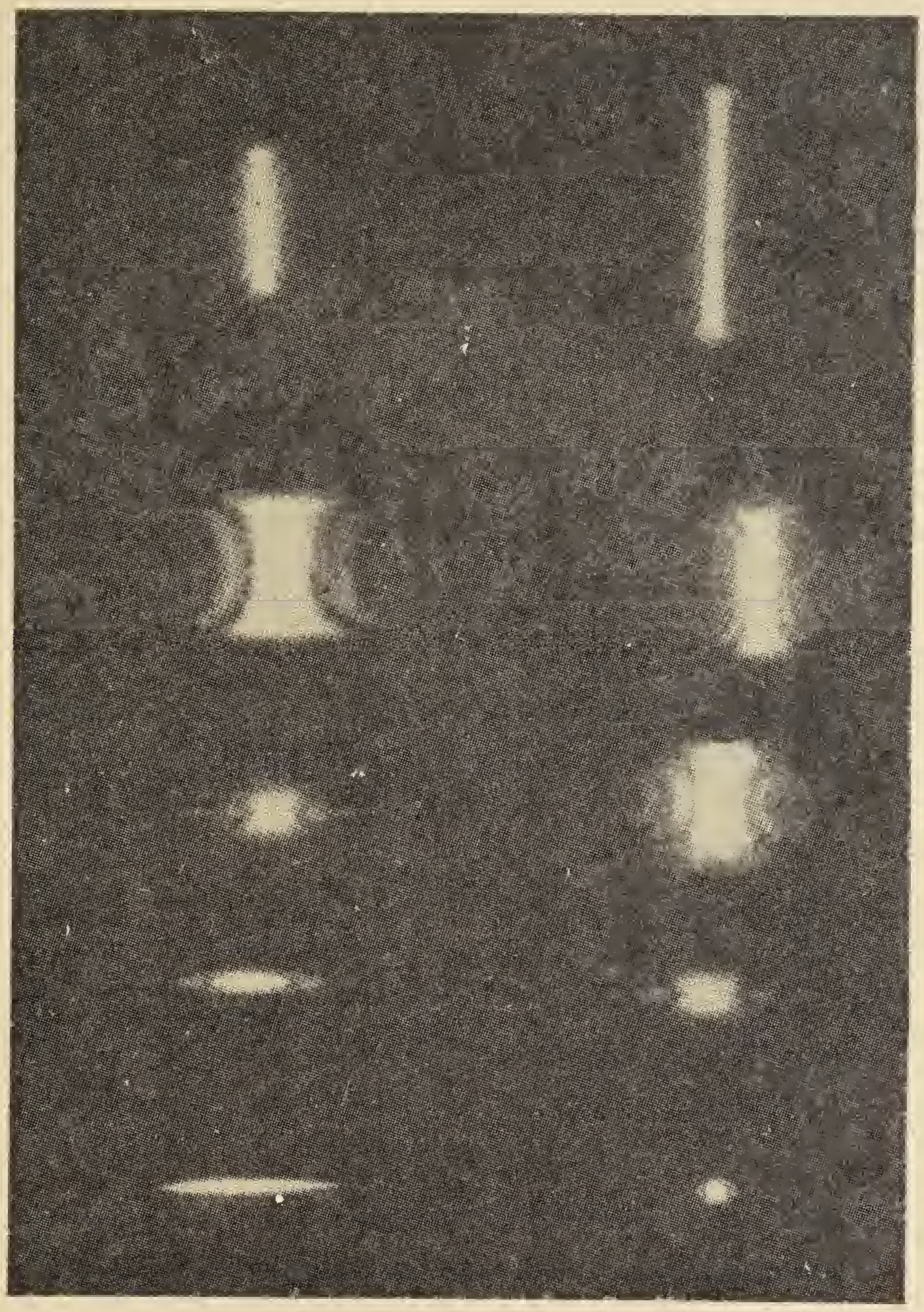


**Figure 1**

*Airy's Tract on the Undulatory Theory of Optics, Section 86.




**Figures 2–11**

Figures 2 to 9 represent the gradual transition from the Fresnel to the Fraunhofer class as the focus of the convergent pencil is approached. In all these cases, the aperture limiting the pencil was of considerable eccentricity and had its major axis vertical. The interesting feature to which the writer wishes to draw attention is the following simple geometrical law to which the diffraction-pattern is found to conform. In figure 1 the heavy lines represent the elliptic cross-section of the geometric pencil of rays by the plane of observation, and also the geometric evolute of this cross-section. [The ratio of the major to the minor axis of the ellipse is taken to be greater than $\sqrt{2}$, so that two of the cusps of the evolute lie outside the cross-section.] Observation shows that the brightest part of the diffraction-pattern outside the elliptic cross-section lies within the evolute and is, in fact, bounded by it. The complete figure of the evolute may be observed visually or even photographed with sufficiently long exposures. Beyond the two cusps of the evolute, a horizontal brush or extension may also be seen on each side, as shown in figure 1. The evolute boundary is clearly seen in figures 3, 4 and 6,

though in all these cases, the size of the photographic plate and the exposure were insufficient to record the whole of it.

Obviously, as the plane of observation approaches the focus of the convergent pencil, the cross-section of the pencil and the evolute both contract and ultimately reduce to a point. The horizontal brushes extending from the two cusps of the evolute, however, persist (vide figures 7 and 8) and break up into beads on both sides which, when the focal plane is further approached, ultimately join up and form the elliptic rings observed in the Fraunhofer pattern. Altogether, the case presents unusually interesting features in the transition from the Fresnel to the Fraunhofer class of diffraction-figure.

Figure 10 represents the elliptic rings at the focus, for the case in which the eccentricity is small. Figure 11 represents the rings observed in the focal plane when a *circular* reflecting surface held very obliquely forms the diffracting aperture. Some degree of asymmetry is observable in this figure.

The writer hopes to be able to present a fuller study and mathematical treatment in due course.

210 Bowbazaar Street, Calcutta
20 October 1918

*Philos. Mag.* **38** 568–572 (1919)

# The scattering of light in the refractive media of the eye

C V RAMAN*
(Palit Professor of Physics in the Calcutta University)

## 1. Introduction

In his treatise on Physiological Optics[†], Helmholtz has discussed the explanation of the interesting phenomena observed when a very small and intensely luminous source of light is viewed directly by the eye against a dark background. An enormous number of luminous streamers appear to emerge from the source, and stretch out from it in more or less exactly radial directions. Helmholtz was of opinion that these streamers were due to the diffraction of light at the irregular margin of the pupil of the eye; and this view is supported in his treatise by a description of the phenomena observed when the source of light is seen through a small hole in a metal plate placed in front of the eye so as partially or wholly to screen the margin of the pupil. Some doubt as to the adequacy of the explanation advanced by Helmholtz having been felt by me, a careful study of the effects was undertaken, the results of which are described in the present paper. The phenomena being of a subjective character, the assistance of a number of independent observers with normal vision was obtained in order to confirm my personal observations. This appeared all the more necessary, as the conclusions arrived at as to the origin of the phenomena differ from those of Helmholtz.

## 2. Description of the phenomena

The character of the luminous phenomena seen is widely different in the two cases in which the source emits *white* and *monochromatic* light respectively. For observing the phenomena with white light, the most suitable arrangement is to condense the light of an electric arc upon a pin-hole in a large dark screen, and to view the issuing light with the eye placed at a distance of about three or four yards from the screen. The luminous pin-hole appears surrounded in the first instance

*Communicated by the author.
[†]Page 180, 1896 (German) Edition.

by a circular patch full of luminous streaks starting out more or less radially from it, and occasionally crossing each other. These streamers are generally white, but appear here and there tipped with streaks of colour. The circular patch is surrounded by a relatively dark ring, outside which again the streamers reappear passing radially through a luminous coloured halo* surrounding the dark ring. The halo is, in fact, made up of short sections of the steamers which, here, are strongly coloured. Outside the halo, the streamers emerge again, but are much fainter, and they form a broad and somewhat ill-defined ring of luminosity extending to a considerable angular width from the source. The inner margin of this luminous ring is greenish-blue, and the outermost visible periphery is of an orange-red colour, but some fainter fluctuations of luminosity and colour may be observed within it.

For observations with monochromatic light, a Westinghouse 3000 c.p. silica mercury-vapour lamp with glass dome furnishes a suitable source. The lamp is suspended immediately behind a small aperture in a dark screen and is viewed from a distance of two or three yards. A green-ray filter may be put in front of the aperture, but is not essential. An alternative arrangement for obtaining a small and powerful source of homogeneous light is to load the carbon rods of an electric arc with plenty of common salt, and to condense the light from the luminous yellow mantle of the arc upon the pin-hole in the screen. With either arrangement, the appearance of radial streamers issuing from the source as seen in white light is *not* obtained. We see, instead, a circular area round the source filled with *granular* patches of light, and outside this, a relatively dark ring, followed by a well-defined circular halo, and some faint outer rings of luminosity.

A remarkable feature which is worthy of mention is the peculiar circulatory or irregular movement which is best seen in the radial streamers surrounding a white source of light, and less clearly in the granular patches surrounding a monochromatic source of light. These movements disappear gradually when the eye is held with a fixed gaze towards the source, but start again immediately whenever any movement of the eyeball or of the eyelids occurs.

## 3. Discussion of observations

The effects described above obviously present a striking resemblance to the phenomena observed when light is diffracted by a large number of irregularly placed apertures or particles of uniform size, e.g. lycopodium dust strewn on a glass plate. Recently, De Haas† has published an elaborate study of such

*Of angular radius a little less than 2 degrees of arc.

†*Proc. R. Soc. Amsterdam*, 1918, p. 1278. It may be remarked here that the phenomenon discussed in the present paper is different from the coloured haloes seen in certain pathological states of the eye and described by Tyndall in his lectures on Light, and by other writers.

diffraction phenomena, and has shown that the formation of the radial streamers in the coronas surrounding a source of white light, and their replacement by a granular field in monochromatic light, may be explained by considering the interference of light diffracted by individual particles which are assumed to be irregularly distributed over the aperture. The resemblance is not merely qualitative, as some careful visual estimates made by me seem to show that the relative intensity of the streamers in the area immediately surrounding the source and in the first circular halo is about the same as that observed when a source is viewed through a glass plate dusted with lycopodium. There thus seems little doubt that the phenomena described above have to be referred to the diffraction of light by a large number of particles of more or less uniform size included in the structure of the refractive media of the eye. The peculiar movements mentioned above would then naturally be ascribed to the movements of these particles. These may be imitated by observing a source of light through a plate of glass on which a little dilute milk has been flowed. The movements of the diffracted streamers of light can then be easily seen.

Observations show that the angular diameters of the circular patch containing the steamers and of the circular haloes surrounding it are entirely independent of the aperture of the pupil of the eye. This is readily proved by altering the intensity of the source of light under observation, with the result that the aperture of the pupil automatically adjusts itself. An ordinary candle-flame at three metres distance, and the light of an electric arc at the same distance or even nearer the eye, give identical measurements for the angular widths, though the aperture of the pupil must have been greatly different in the two cases. This is exactly what we might expect if the effects are due to particles contained in the structure of the eye, but it is very difficult to reconcile with the view of Helmholtz that the phenomena are due to diffraction at the margin of the pupil. The observed effects cannot be explained if we merely postulate any arbitrary irregularities in the circular shape of the margin of the pupil. It would be necessary also to assume a regular corrugation or periodicity in the margin of the pupil*, and even such an assumption, apart from its being purely hypothetical, fails to explain the observed effects. For, with any change in the aperture of the pupil, the distance between successive corrugations should also alter and influence the observed phenomena. This is inconsistent with the observed independence of the aperture and the observed effects. We are thus led to reject the view that the diffraction at the margin of the pupil determines the phenomena seen.

*A useful analogue is furnished by the milling on the circular edge of a coin. In a paper on diffraction which is in course of publication, S K Mitra has shown that a circular disk with corrugated edges, produces a coloured halo surrounding the usual Fresnel–Arago central bright spot, and the angular radius of this halo is equal to $n\lambda/2\pi a$, where $a$ is the radius of the disk and $n$ is the number of corrugations. A similar phenomenon is shown in convergent light by a circular aperture with a corrugated edge, but in a less striking manner. The number of corrugations remaining the same, the angular radius of the halo would vary inversely as the radius of the aperture.

It is useful in this connection to note that the intensity of the light diffracted by the pupil (supposed perfectly circular) in a direction making 30′ of arc with the source would be only 1/3,000,000 of that seen in the direction of the source. (This is calculated from the formula $I\alpha[J_1(z)/z]^2$, the radius of the pupil being taken to be 2 mm and $\lambda = 5600$ A.U.) Actually, the streamers surrounding the source can be seen in directions making an angle of 200′ and even more with its direction, and it seems safe to say that their intensity half a degree away from the source is a much greater fraction of its apparent intensity than 1/3,000,000.

A further test of the view that the effects are principally due to the structure of the eye and not to diffraction at the margin of the pupil, is furnished by experiments with very thin metal screens containing apertures placed in front of the pupil of the eye. Using a circular hole with smooth edges smaller than the pupil and placed in front of it, the intensity of the streamers surrounding the source of light is reduced, but does not vanish. When the screen is turned about an axis normal to its plane, the hole being continually kept in front of the pupil, the streamers of light seen in the field remain visible and fixed in position, showing that they are due to the structures of the eye through which the light passes, and not to the margin of the pupil, or the edges of the hole. Another and probably more convincing demonstration is obtained by using a *square* aperture smaller than the pupil of the eye and placed in front of it. (This may easily be contrived with the aid of four Gillette blades forming the four sides of a very small aperture.) In this case, the effect due to diffraction at the boundaries of the aperture is very clear and marked, but can be shown (on rotating the aperture in its own plane) to be entirely distinct and separate from the phenomena now under discussion.

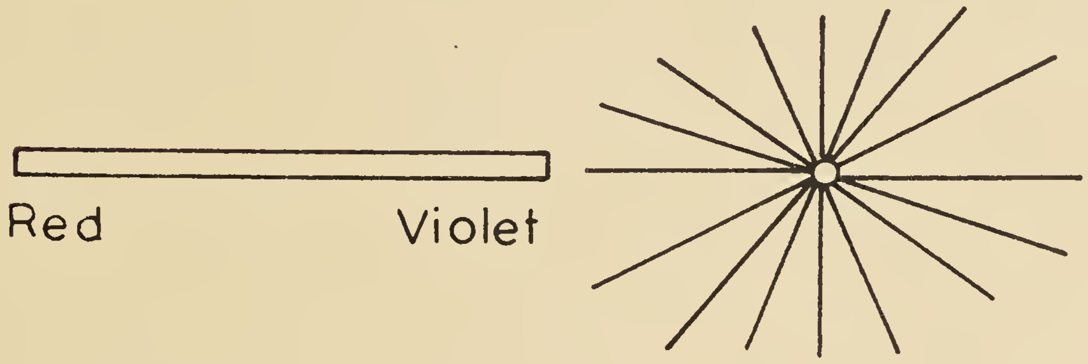
The angular diameters and intensities of the haloes are such as to suggest that we are dealing not with *one* but with *two* sets of structures contained in the refractive media of the eye, averaging in size about 13 $\mu$ and 7 $\mu$ respectively. The structures of the latter (smaller) size are indicated by the outermost halo, which appears to be composite in character and due to the superposed effect of the two sets of particles. These structures in the living eye are presumably to be localised in the cornea and in the vitreous humour, as histological evidence of the existence of cellular structures in these bodies is available. Upon this question, however, the author does not venture to express any opinion.

Calcutta
22 April 1919

*Nature (London)* **108** 12 (1921)

# The "radiant" spectrum

The title refers to an interesting optical effect observed and described many years ago by Sir David Brewster (*Philos. Mag.*, September, 1867), which appears, however, never to have been satisfactorily explained. When a small brilliant source of light is viewed through a prism held in front of the eye, a remarkable appearance is noticed, represented roughly in the accompanying diagram (figure 1). In the continuation of the spectrum of the source, but considerably




beyond its violet end, is seen a patch of light consisting of streamers radiating from a centre, as shown. A brief statement on the cause of this effect, as determined in an investigation made by me, may be of interest to readers of *Nature (London)*.

The phenomenon is due to the diffraction of light in its passage through the eye by the corneal corpuscles. Were there no prior dispersion of the light by the prism, the diffraction-halo would appear to consist of streamers surrounding the source and radiating from it directly. The effect of the dispersion on the diffraction-halo is to shift its achromatic centre towards the side of the shorter wavelengths—in fact, to a point lying considerably beyond the violet of the spectrum of the source, exactly as observed. The streamers in the halo really consist of elongated diffraction-spectra, and the effect of the prism is to reorient them, so that they now appear to diverge from the altered position of the achromatic centre. This explanation of Brewster's phenomenon is strikingly confirmed by the fact that very similar effects may be observed in the diffraction-halo due to a glass plate dusted with lycopodium, when held in front of the eye along with a 60° glass prism.

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22 Oxford Road, Putney, S.W.15
12 August

*Nature* (*London*) **109** 175–176 (1922)

# The radiant spectrum

Dr Hartridge's objections to my explanation of this phenomenon (*Nature* (*London*), September 1, p. 12, and December 8, 1921, p. 467) seem to be based on an imperfect appreciation of Brewster's observations on the subject. Brewster brings out two facts clearly in his paper: First, when a very small and intense source of white light is viewed directly by the eye it appears surrounded by a system of radiating streamers which appear to diverge directly from it; secondly, when a prism of small dispersive power is interposed in front of the eye the streamers are deviated and now appear to diverge from a point lying beyond the violet end of the spectrum into which the source itself is drawn out. It is clearly illogical to suggest, as Dr Hartridge does, that the prism is responsible for the radiant phenomenon in view of the fact that, in its essential features, the effect is observed even before the introduction of the prism.

Using a sufficiently intense source of light and a prism of small angle with optically good and clean faces, and making the observations in a dark room, it should be easy for anyone to satisfy himself by simple tests of the kind referred to by Dr Hartridge that he is in error, and that Brewster's phenomenon really arises from the scattering of light in the eye, the prism merely acting as a dispersive apparatus modifying the colour and disposition of the streamers in the halo surrounding the source. Judging from the statements made in his letter, Dr Hartridge would appear to have been particularly unfortunate in his choice of experimental conditions. Any noticeable imperfection in the optical surfaces of the prism would, of course, give rise to scattering, masking the true phenomenon due to the eye itself. This is indeed clearly suggested in Brewster's own paper.

A further and absolutely crucial test is also available. In my paper on the scattering of light in the refractive media of the eye (*Philos. Mag.*, November, 1919, p. 568), I have described the character of the diffraction-halo arising from this cause in considerable detail. With a source of white light the halo shows a radiating fibrous structure and clearly marked alternations of colour and intensity in its outer parts. A monochromatic source, on the other hand, exhibits a halo with a granular structure and a succession of bright and dark rings. These features are explained in my paper as due to the diffraction of light by corpuscles of more or less uniform size included within the structure of the eye. On this view we should expect one half of the first diffraction ring outside the central portion of the halo to be partially achromatised on the introduction of the prism and to appear as a detached semi-circular arc lying beyond the violet end of the spectrum and the displaced position of the achromatic centre. No mere

imperfections or irregularities in optical surfaces could, on the other hand, give rise to such a phenomenon. Actual trial confirms the expectation from theory and puts its correctness on an unassailable basis.

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210 Bowbazaar Street, Calcutta
4 January

*Philos. Mag.* **43** 357–358 (1922)

# On the phenomenon of the "radiant spectrum" observed by Sir David Brewster

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(Palit Professor of Physics in the Calcutta University)

In a paper on 'The scattering of light in the refractive media of the eye,' published in the *Philos. Mag.* for November 1919 (p. 568), I discussed the explanation of the luminous effects observed when a small brilliant source of light is viewed directly by the eye against a dark background, and especially of the marked difference between the cases in which the source emits white light and highly monochromatic light respectively. In both cases the source appears to be surrounded by a diffraction-halo; but the structure of the halo is markedly different in appearance. In the former case, the source appears to shoot out streamers of light radiating from it in all directions, these streamers shown marked colour, and in fact appearing as elongated spectra in the outer parts of the halo. With the monochromatic light-source, on the other hand, the radiant structure of the halo is not observed, and we have instead surrounding the light-source a halo showing dark and bright rings and exhibiting a finely mottled or granular appearance. It was pointed out in the paper that these effects are precisely what might be expected on the hypothesis that the halo seen surrounding the source is due to the diffraction of light by a large number of particles of constant size—presumably the corneal corpuscles—present in the refractive media of the eye. The radiant structure of the halo in white light and its granular structure in highly monochromatic light is, on this view, due to the field of light diffracted by individual particles varying arbitrarily in intensity from point to point as the result of the mutual interference of the effects of the large number of such particles. A closely analogous structure of the luminous field may be observed in diffraction-haloes obtained in other ways, e.g. with the aid of a glass plate dusted with lycopodium powder through which a small distant source of light is viewed.

The facts mentioned above provide a very simple explanation of a remarkable observation made long ago by Sir David Brewster, and communicated to the *Proc. R. Soc. Edinburgh* (6, p. 147†; see also *Philos. Mag.* September 1867), which has not up to now been satisfactorily accounted for, and to which my attention

*Communicated by the author. Read before the *R. Soc. Edinburgh*, Nov. 7, 1921.
†See also a brief note by Tait, *Proc. R. Soc. Edinburgh*, **6** 167.

has been recently drawn by Dr C G Knott while I was on a visit to Edinburgh. Brewster noticed that when a spectrum of a small brilliant source of white light is formed, either by a prism or by diffraction, and viewed directly by the eye, a patch of light is seen lying in the continuation of the spectrum well beyond its violet end and exhibiting streamers radiating from its centre. That this is a diffraction-effect is shown by the fact that a similar and even more striking effect may be observed in the diffraction-halo due to a glass plate dusted with lycopodium held together with a 60° glass prism before the eye, when a small distant source of white light is viewed through the combination. The prism disperses the image of the source into a spectrum. It also disperses the diffraction-halo, and since the diffraction-rings are of different size for the different wavelengths and are shifted to different extents owing to the dispersive power of the prism, the achromatic centre of the halo is shifted laterally to a considerable extent, its new position generally lying at a point much removed beyond the violet end of the spectrum of the source itself. The elongated spectra which form the radiating streamers are rotated through various angles by the dispersion of the prism, being drawn out laterally on one side and shut up or drawn together on the other side, and they then appear to diverge from the shifted position of the achromatic centre of the halo, which, as remarked above, now lies well beyond the violet end of the spectrum of the source. The analogy between this effect and Brewster's phenomenon is so striking that there can be no doubt that the latter is essentially of the same nature, the diffraction in this case being due to the structures within the eye itself.

*Philos. Mag.* **41** 338–347 (1921)

# On the colours of mixed plates—Part I

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[Plates I and II]

## 1. Introduction

The colours exhibited by a mixed plate or film consisting of two interspersed transparent media were investigated by Thomas Young[†], and were ascribed by him to the interference of the coherent streams of light passing through the media and emerging from the film having suffered different retardations. Later, it was pointed out by Brewster[‡] that the colours were due to laminar diffraction; but his treatment was not very complete, and apparently did not convince later writers such as Verdet and Mascart, who dealt with this case in their treatises as one of simple interference, and ignored the part played by diffraction. More recently Profs. Charles Fabry[§] and R W Wood[‖] have attempted to give an explanation of the colours of mixed plates from the standpoint of elementary diffraction theory. On investigating the subject, however, we have found that there are several features in the observed phenomena which the treatments proposed fail to explain. For instance, when a mixed plate consisting of a uniform film of liquid enclosing a large number of air-bubbles of widely varying radii is held in front of the eye, and a distant source of white light is viewed through it, we should expect, according to the theory given by Wood[¶], that the halo seen surrounding the source should be throughout of a more or less uniform colour complementary to the tint of the regularly transmitted light, and fluctuate as a whole when the thickness of the film is varied. As will be seen in the following section, this is very

*Communicated by the authors.
[†]*Philos. Trans. R. Soc. London* **1802**, 390 and *Elements of natural philosophy* vols 1 and 2.
[‡]*Philos. Trans. R. Soc. London* **1638**, 73.
[§]*J. Phys.* **8**, 595 (1899).
[‖]*Philos. Mag.* April 1904, and *Physical optics*, 2nd edition, p. 252.
[¶]*Loc. cit.*

far indeed from being in agreement with what is actually observed. Further, the experimental examination of the subject which we have carried out has brought to light a number of interesting features which appear hitherto to have been overlooked. It is proposed, in part I of this paper, to give a general description of the observed effects. The subsequent instalments of the paper will contain a description of further observations and experiments on the subject and the detailed discussion of the theory of the phenomena.

## 2. The diffraction-haloes due to a mixed plate

The most instructive case is that referred to above—namely, that of a mixed plate which is held *normally* in front of the eye, a distant point-source of light being observed through it. The plate should have a thickness as uniform as possible, and to minimise the effects of slight unavoidable variations of thickness it should be brought up very close to the eye. In order to observe the phenomenon at its best, it is useful to work with a fairly powerful source of illumination, which should be completely enclosed except for a small aperture through which the light issues and falls upon the plate held at a sufficient distance from the source. The observations should be made in a darkened room so that the fainter extensions of the diffraction-halo surrounding the source can be easily seen. A tungsten-filament lamp of, say, 400 c.p. is a suitable source for observations in white light. For observations in monochromatic light a quartz-mercury vapour lamp with green ray filter is the most convenient source to use, though good results may also be obtained with a monochromator illuminated by the electric arc, or with a bead of salt held on a platinum wire in the hottest part of the flame of a Meker burner.

A mixed plate of uniform thickness may easily be obtained by spreading a few drops of saliva or of egg-albumen between two plates of glass 5 in. × $2\frac{1}{2}$ in. in size and $\frac{1}{4}$ in. thick, and working up the material into a film of uniform consistency by circular sliding movements of the plates over each other. Examined under the microscope, a film of this description shows a thin layer of liquid enclosing a larger number of air-bubbles widely varying in size, irregularly arranged and of shape often departing considerably from circularity (see figure 1 in plate I), but showing no bias towards elongation in any particular direction*. The air-bubbles can be distinguished from the liquid by the slightly diminished intensity of the transmitted light, and also by the presence of very minute drops of liquid on the surface of the plates enclosed within the bubbles. (These can be seen in the micro-photograph, figure 1 in plate I. The laminar boundaries appear as black lines in transmitted light under the microscope.

*Cases in which the bubbles have distorted forms elongated specially in one direction will be considered below in a separate section.

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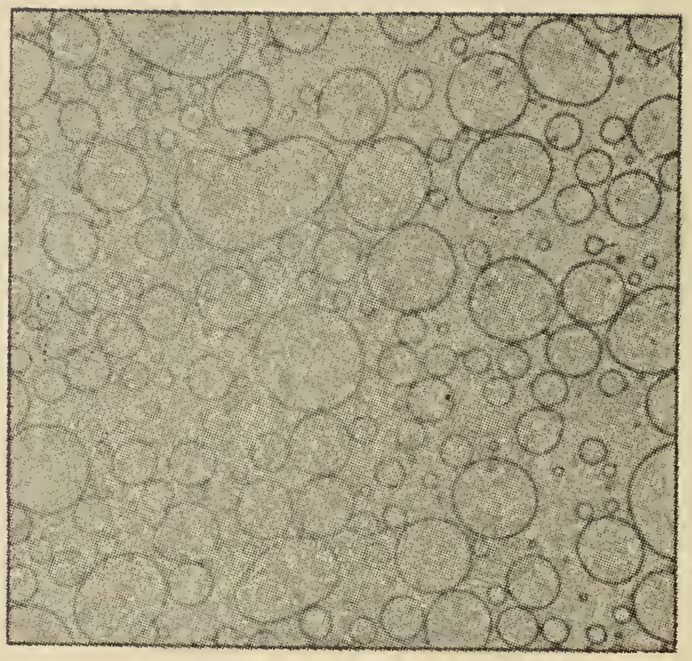

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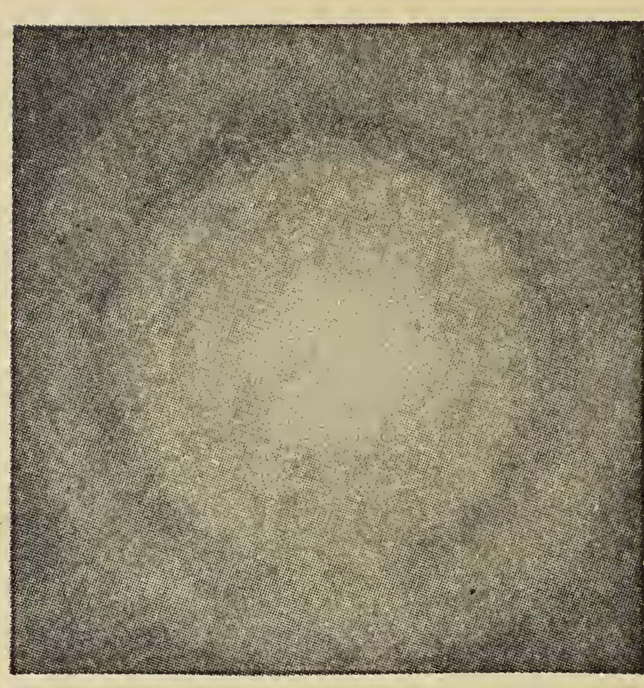

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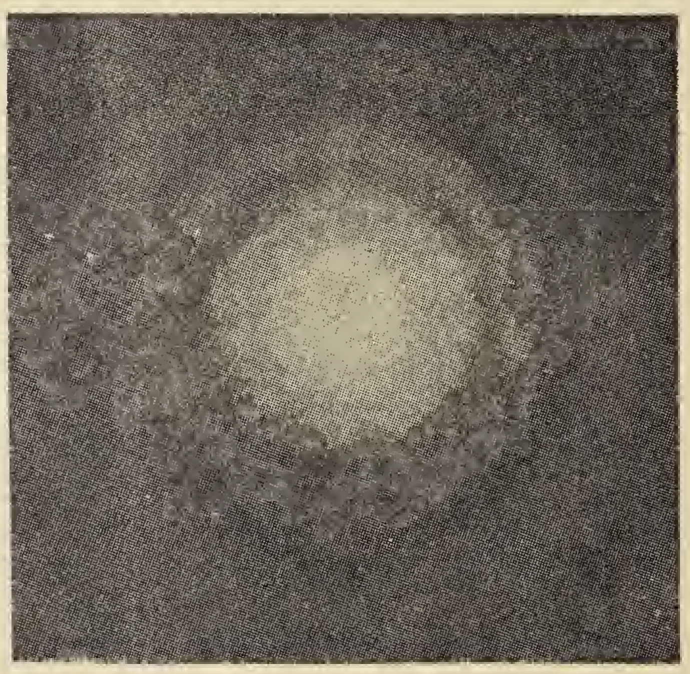

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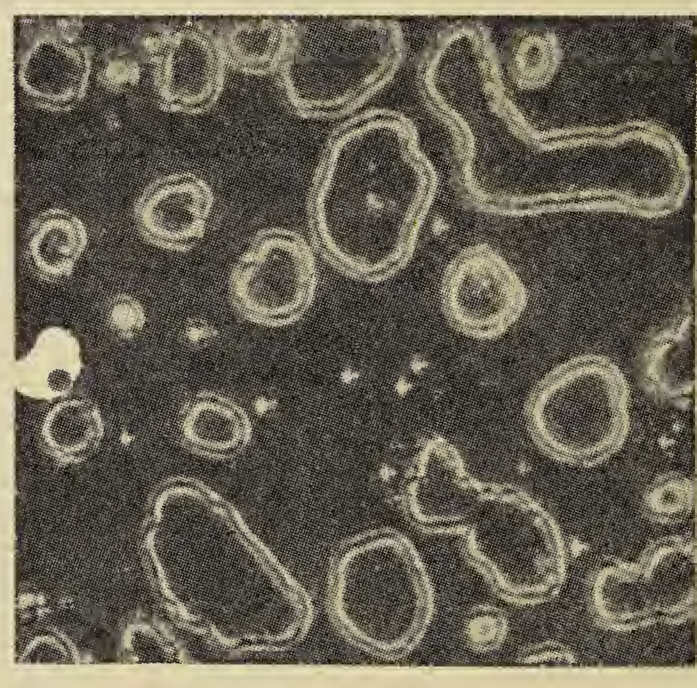

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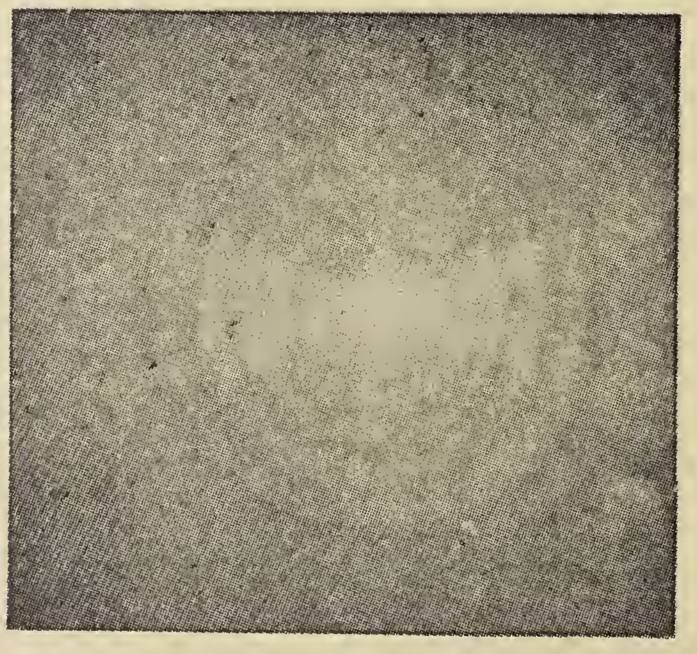

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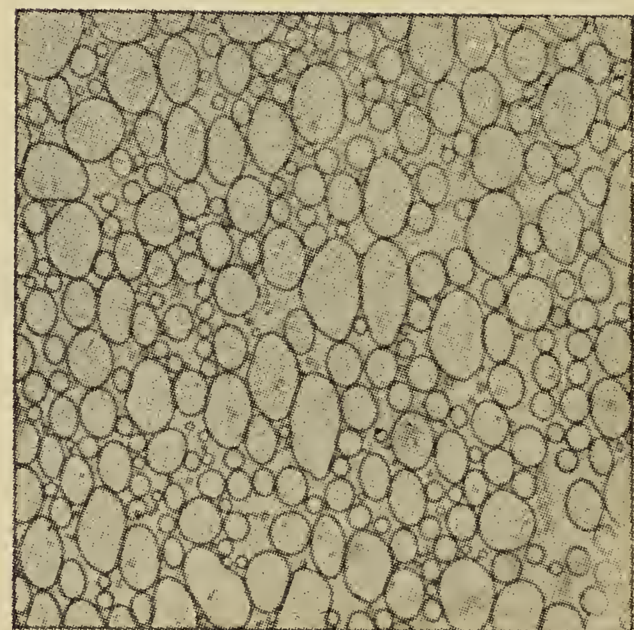


**Plate I (figures 1–6)**

Using the method of observation described above and a *freshly prepared* plate, some remarkable effects are observed which will now be described.

## Observations in monochromatic light

Surrounding the source is seen a brilliant halo, which, if the source is sufficiently small and distant, shows especially near the centre a finely mottled or granular structure. The halo consists of a succession of dark and bright circular rings of which the number and position depend on the thickness of the film. With thick films fifteen or twenty rings may easily be obtained, with thin films a proportionately smaller number. The most remarkable features of the ring-system are: *firstly*, that the successive rings are closer together near the centre of the halo and wider apart towards the margin of the halo, where they are very broad and faint; *secondly*, the dark rings in the halo are all more or less perfectly black, *except the two or three rings nearest the centre of the halo* which are not so dark as the rest, and fluctuate in sharpness and intensity with their exact position in the halo. Figures 2 and 3 in plate I show the ring-system above described for a moderately thin film. The photographs of the halo here reproduced were secured by using a short-focus wide-angle lens (Zeiss-Tessar F/3·4 lens of focal length 5 cm), the mixed plate being placed as close to the lens as possible. Figure 2 is a light print showing the faint and broad outer rings, and figure 3 a deep print showing more clearly the part of the halo near the centre.

When the mixed plate is moved in its own plane so as to bring a thinner part of the film in front of the eye, the rings seen in the diffraction halo move *inwards*, closing up at the centre so that the number visible in the field decreases and the rings appear wider apart. As each of the dark rings in the inner part of the halo contracts and moves inwards, it undergoes a periodic fluctuation of intensity, becoming alternately broad and diffuse, and then sharp and black, just as it is about to close up at the centre. Further away from the centre, however, the rings merely contract and move inwards without any noticeable change in their appearance. In fact, what is seen distinctly suggests that near the centre of the halo there is a second and fainter ring-system superposed on the first, with the result that in this region the dark rings vary in sharpness and intensity according as, at a given point, the two sets of rings are in or out of step. A similar closing in of the rings is observed if an electric arc with monochromator is used as the source and the wave-length of the light incident on the plate is gradually increased.

## Observations in white light

In this case the halo has a fibrous radial structure, and shows a series of coloured rings surrounding the source. The *outermost* ring in the halo is very broad and

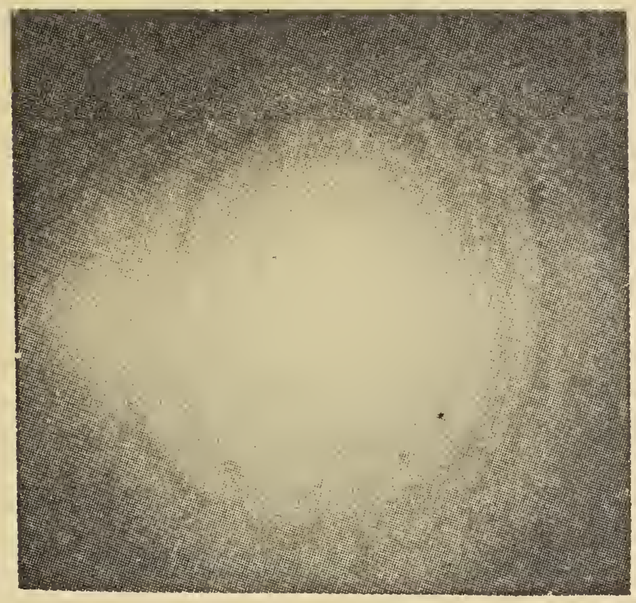
practically *colourless* or *achromatic*. The first few rings subsequent to this achromatic ring within the halo are strongly coloured even if a thick film be used, the succession of colours being not very dissimilar to that observed in passing from the achromatic centre to the coloured bands in the diffraction pattern of the Fraunhofer class due to a rectilinear slit. The colours near the centre of the halo are very weak and impure, unless the film is so thin that the total number of coloured rings in the halo is rather small. In the latter case the halo shows vivid colours, even near the centre.

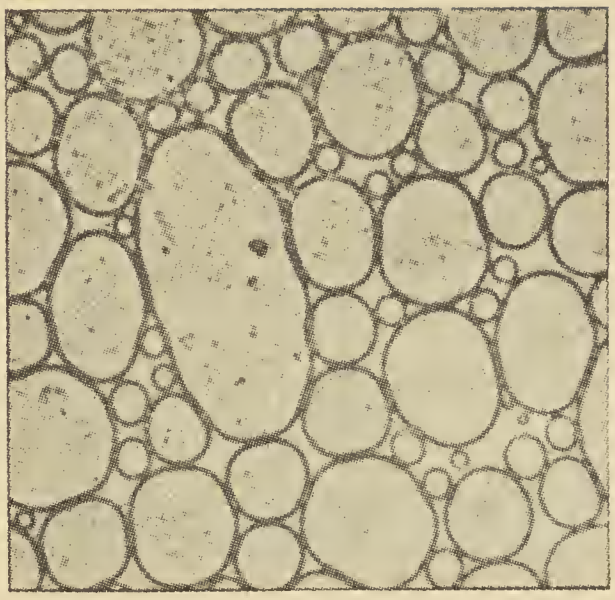
In all cases the source of light as seen through the mixed plates appears enfeebled, as is evidently to be expected in view of the fact that part of the incident energy appears in the scattered light forming the halo. When the observations are made in monochromatic light, and the plate is moved in front of the eye so as to pass from a thick to a thinner part of the film, the intensity of the source suffers periodic fluctuations, being greatest when the innermost dark ring in the halo is just about to close in at the centre and least when one of the bright rings just surrounds the source. When similar observations are made in white light, the source appears to fluctuate both in intensity and colour as the plate is moved so as to alter the thickness of the film in front of the eye. The part of the halo immediately surrounding the source and the source itself appear to be of complementary colours.

## 3. Haloes due to obliquely-held plates

If the plate instead of being held normally to the light is gradually tilted, some very striking results are obtained, especially when the observations are made in monochromatic light. As the obliquity of the plate to the incident light is increased, new rings appear at the centre of the halo and move outwards: when a dark ring first appears it is very clear and well-defined, but in moving out it becomes faint and broad, and then again sharper and blacker, and so on. These fluctuations, however, occur only when the dark ring is near the centre, and become imperceptible when it has moved to the position of the third or fourth ring from the centre, beyond which the dark rings are all more or less perfectly black. At the same time, the form of the rings undergoes alteration, becoming elliptical near the centre and of an oval form in the outer parts of the halo. With increasing obliquity these ovals assume unsymmetrical shapes, the curvatures at the two ends of the minor axis (which lies in the plane of incidence) being unequal. This effect is shown in figure 7 in plate II. Indeed, on the flatter side the ovals may actually straighten out and even reverse their curvature—that is, become concave outwards; while on the other side, which corresponds to directions more nearly parallel to the plate, the rings remain convex outwards. At the same time, the different parts of the rings in the halo appear very unequal in their illumination, the light becoming more or less completely concentrated in the

(7)





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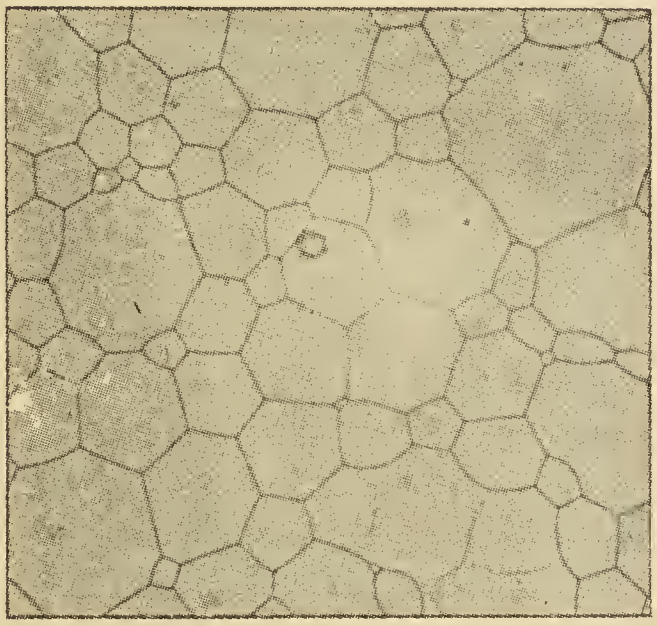

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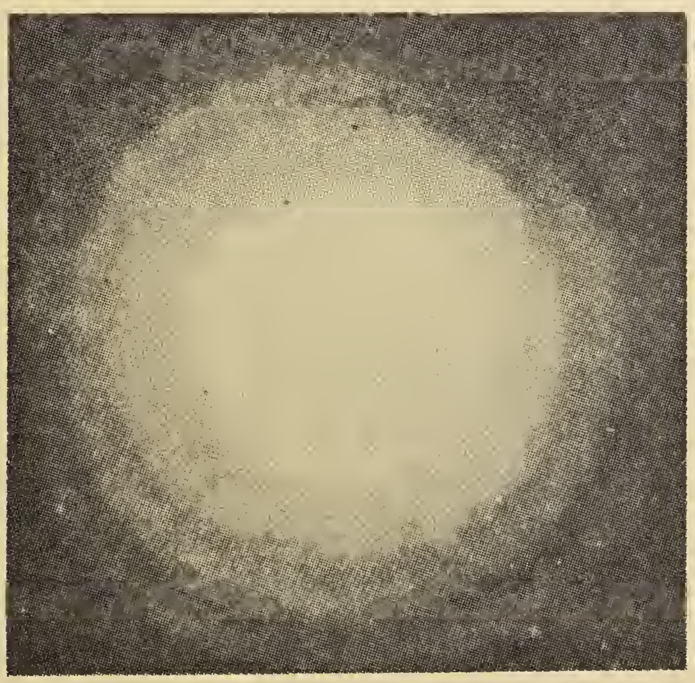

(12)

**Plate II (figures 7–12)**

plane of incidence and the flatter side of the rings appearing brighter than the more strongly curved side (figure 7 in plate II).

## 4. Films containing distorted boundaries

Mixed plates freshly prepared by rubbing up saliva or white of egg between glass plates generally show, when examined under the microscope, that the air-bubbles tend to take up a circular form, or at any rate do not show a bias towards elongation in any particular direction. But if the glass plates enclosing a mixed film of white of egg and air are pressed together and continuously moved over each other in any one direction, the bubbles in the film become distorted, assuming elliptic or oval shapes of which the major axis is in a direction *perpendicular* to that of movement of the plates. Figure 6 in plate I is a micro-photograph of a film obtained in this manner. When the diffraction-halo due to such a plate (normally held) is observed, it is found that the rings in the halo retain their relative position and circular shape, the only conspicuous effect of the altered form of the boundaries being to increase the luminosity of the halo in directions transverse to the preponderating direction of the boundaries. This is illustrated in figure 5. This observation, taken along with those described in the preceding section, clearly shows that the essential feature of the halo, namely, the succession of dark and bright rings of widths increasing from the centre outwards, does not depend for its formation on the position, size, or shape of the air-bubbles in the film, and is determined only by the thickness of the film containing the bubbles.

When glycerine, turpentine, fats, or oils are used for forming the mixed plates, the tendency towards elongation of the air-liquid boundaries in the film becomes excessive, in these cases occurring parallel to the direction of movement at any instant of the enclosing glass plates. With such liquids it requires some dexterity to prepare a film showing the complete circular haloes. Generally only a diametrical streak is obtained which shows different colours at different parts of its length, and turns round and round as the glass plates enclosing the film are slid over each other with a circular movement in front of the eye.

## 5. Effect of partial or complete drying of an albumen film

When a mixed plate is prepared with white of egg between parallel glass plates and is allowed to stand for an hour or two, some very remarkable changes occur in the structure of the film and in the optical effects produced by it. The bubbles of air in the film which at first lie about indiscriminately (figure 1 in plate I) soon draw together, coming into contact over a considerable portion of their edges

(figure 8 in plate II), and this process continues gradually till the edges everywhere touch each other. (With very thin films, especially those formed between curved surfaces, this process may be indefinitely retarded.) Ultimately, the edges which have joined up straighten out, and when after a day or two the film has completely dried up, it is found on examining the film under a microscope that the albumen is confined to a number of very fine ridges holding the two glass plates together, the form of these ridges being that of a number of irregular hexagons, pentagons, or quadrilaterals forming a network (figure 9 in plate II) film. The diffraction-halo seen round a distant source of white light when such a completely dried plate is held normally in front of the eye is entirely different in character from that due to a mixed plate freshly prepared; in fact, the relative position of the achromatic and coloured portions and the spacing of the rings are completely reversed with the dried film. The halo in this case is much fainter. It shows a broad central area which is achromatic, followed *outside* by rings of gradually *decreasing* width which are strongly coloured. The radial fibrous structure of the halo is exceedingly well marked (figure 11 in plate II). In fact, the diffraction-halo seen round the source when viewed through the completely dry film is remarkably similar in appearance to the well known diffusion-rings observed around the focus of a thick concave mirror with a dusted surface. The resemblance extends also to the case in which the plate is held obliquely in front of the eye. As the plate is gradually tilted, the halo runs out on one side, fresh fringes appearing on that side of the source; and ultimately, when the plate is held at a moderate obliquity, the halo consists (at least in its brightest part) of a system of circular arcs which are unequally spaced, the arc which passes through the source being *achromatic* and those on either side of it being strongly coloured in white light (see figure 12 in plate II). [The oblique streak seen in the figure running across the circular arcs was due to an accidental circumstance]. In fact, the halo in its brightest part is very similar to the diffusion rings due to a dusted mirror when the latter is tilted. It should be mentioned, however, that in the outer fainter parts of the halo (which do not appear in the photograph reproduced) certain more complex effects are observed. The detailed description of these may be deferred for the present.

When the film is only partially dry (as, for instance, in figure 8 in plate II), both sets of rings, that is those due to a freshly prepared film and those shown by a completely dry film, appear simultaneously, and owing to their superposition, the phenomena appear somewhat confused, especially near the centre of the halo. The effect of partial drying of the film is also clearly noticeable in the diffraction-halo due to an obliquely held plate. The elliptic or oval rings appear traversed by a system of circular arcs running transverse to the plane of incidence. These are seen (somewhat faintly) in the photograph reproduced in figure 10 in plate II. They become more and more prominent as the drying of the film progresses, and ultimately remain alone in the field (figure 12, plate II) when the elliptic rings have disappeared.

## 6. Non-uniform films

Standing in close relation with the phenomena described above due to mixed plates of uniform thickness, are the effects observed with mixed plates of variable thickness. The difference between the two cases is principally as regards the method of observation. In the former case the film is placed close to the eye, and the diffraction-halo surrounding a distant source is observed. With mixed films of variable thickness, on the other hand, the most suitable method of observation is to focus the eye on the film itself, the latter being held at a suitable distance from the observer. As in the former case, it is necessary, if the effects are to be studied critically, to use a light-source of small dimensions and to place the film at a sufficient distance from it. A film of variable thickness may be readily formed between the surfaces of two lenses similar to those used for observation of Newton's rings. The thickness of the film at its centre need not necessarily be zero; and in fact, if egg-albumen is used, there is considerable difficulty felt in forcing the lenses into actual contact at the centre. We shall confine our attention here to the phenomena observed with *freshly prepared* films.

### Observations in white light: Normal incidence

When the mixed plate is held at a sufficient distance in the line of sight between the eye and a distant source of light, vividly coloured rings localized on the film are seen (provided its thickness is not too great), these rings being the lines of equal thickness on the film. In this case the light reaching the eye is that diffracted through small angles by the air-liquid boundaries which the film contains; and, indeed, it is these boundaries which appear luminous to the eye and *not* the whole continuous film*. The source of light itself appears coloured, and is of a complementary tint to the part of the film through which it is seen. When the eye is moved a *little* out of the direct line between the plate and source of light, some remarkable changes occur in the appearance of the film. The colours become feeble and impure, and on moving the eye further out of the direct line, the colours reappear again vividly, the rings having simultaneously expanded and moved outwards. With still further movement of the eye the phenomenon repeats itself, but with much less marked fluctuations in the vividness of the colours; and if the film were initially so thick as to show a coloured centre, fresh rings also appear and move outwards from the centre, until finally an achromatic centre develops

*Figure 4 in plate I is a photograph (much enlarged) of these luminous boundaries in a dark field as observed by the method of the "Foucault test". Each of the laminar boundaries appears as a pair of brilliantly coloured lines running parallel to each other and separated by a perfectly black line coinciding with the exact outline of the boundary. The theory of this effect will be more fully considered in part II of this paper.

and expands so as to cover the whole area of the film when viewed sufficiently obliquely. When the source of light is fairly powerful (as, for instance, when the film is held normally in the track of a parallel beam of light from an optical lantern), this expansion of the rings with increasing obliquity of observation may be followed up till the direction in which the film is seen makes an angle up to 90° with the direction of the incident light. It is a noteworthy fact, that a film too thick to show colours when observed nearly in the direction of the transmitted light will show the coloured rings vividly when viewed at a moderate obliquity. Ultimately all films, whether thick or thin, appear practically achromatic in a sufficiently oblique direction. The thinner the film, the smaller the angle of diffraction necessary for this. The thinnest films, which may be obtained by forcing the glasses together till they nearly come into contact, scatter much less light than the thicker films. Hence the film which is achromatic when viewed obliquely shows a darker area near its centre.

## Observations in monochromatic light: Normal incidence

In this case, if the mixed plate is held directly in the line of sight between the eye and the source of light at a sufficiently great distance from both of them, a series of *perfectly black* rings (alternating with bright rings) may be seen on the film, even if this be fairly thick. Bringing the film nearer the source of light or increasing the dimensions of the latter has a very deleterious effect on the perfect blackness and sharpness of the rings. Placing the eye a little out of the direct line also results in the rings becoming blurred in appearance; and, indeed, on merely bringing the film nearer the observer in the line of sight so as to increase the angle it subtends at the eye, the rings may appear broken up and blurred in parts. (The effect is the more striking the thicker the film and the larger its area). If a particular dark ring be watched as the eye is gradually moved out of the line through film and source, it will be noticed that the ring becomes blurred and broadened, then again sharp and perfectly black, but with its position shifted outwards in the film; this process further repeats itself, but with rapidly diminishing fluctuations in the intensity and sharpness of the ring. Viewed at a moderately large obliquity, the dark rings are always perfectly black, and their sharpness is much less dependent upon the use of a light-source of restricted area. The rings expand with increasing obliquity of observation and move out of the film, until finally no rings are visible at all. A darker patch at the centre, where the film is thinnest, can be seen, as in the case of white light.

*Obliquely incident light*—When the film is tilted relatively to the direction of the incident rays, the rings seen on the film contract and move inwards. As in the case of normal incidence, perfectly black rings may be seen on the film, if it be viewed in monochromatic light very nearly in the direction of the transmitted rays, and the appearance of these rings alters with the obliquity of observation in

much the same way. It should be remarked, however, that in the present case the effects observed vary not only with the angle between the transmitted pencil and the direction of observation, but also with the particular plane in which the latter direction lies. The maximum permissible angle of observation varies with this plane. In the plane of incidence it is $\pi/2 - \alpha$ and $\pi/2 + \alpha$ respectively on the two sides of the transmitted pencil, where $\alpha$ is the angle of incidence. If $\alpha$ is considerable, the rings continue to be visible when the film is viewed nearly along the surface of the plate on one side of the transmitted pencil, while on the other side the rings move out and disappear from the film at a moderate obliquity, so that in white light the film appears achromatic over a wide range of angles of observation.

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# On the colours of mixed plates—Part II

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## 1. Inadequacy of the elementary diffraction theory

The optical phenomena exhibited by mixed plates have been described in detail, with illustrations, in the first part of the paper. We now proceed to consider their explanation. It is obvious that the phenomena must be classed as laminar diffraction effects. But, as has already been remarked in the introduction to the first part of the paper, many of the features observed differ from what one might expect on the usual elementary theory of diffraction phenomena. Conspicuous amongst these is the special character of the diffraction-halo seen surrounding a distant light-source viewed through a mixed plate of uniform thickness. Many of the observed features of this halo are not explicable on the elementary diffraction theory: namely, the succession of dark and bright rings of widths rapidly increasing from the centre outwards, the perfect blackness of the dark rings in the outer part of the halo, and the obviously composite structure of the inner part of the halo. In seeking for an explanation of these effects, a clue is furnished by the observation already recorded, that when a mixed plate is observed by the light diffracted by it, *the whole surface of the film does not appear luminous, but only the laminar boundaries or lines of separation of the two media forming the film.* The optical effects of mixed plates are thus, in fact, the optical effects due to the scattering or radiation of light from laminar diffracting boundaries. It is necessary to study the manner in which an individual laminar boundary scatters or diffracts light incident on it; to compare this with the indications of theory, and from the observed effects to infer the aggregate result of the scattering by a large number of such boundaries irregularly situated on the film. These points we now proceed to discuss.

*Communicated by the authors.

## 2. Examination of mixed plates by the method of the Foucault test

The most convenient way of examining the scattering of light by a laminar boundary in directions nearly *coincident with that of the incident waves* is by the method of observation known as the Foucault knife-edge test or the Töpler Schlieren method. The theory of this method was developed by the late Lord Rayleigh on the basis of the usual elementary treatment of diffraction phenomena*, and it was shown by him that a discontinuous laminar boundary should appear as a luminous line when examined by the method of the Foucault test. A beautiful illustration of Rayleigh's theory is furnished on examining a clear piece of mica by the Foucault test, when it will be found that the striae or boundaries in the mica between regions having slightly different thicknesses shine out as vividly-coloured lines of light in a dark field†. To study the phenomena of mixed plates by a similar method, the following arrangement is suitable. Light from a small circular aperture illuminated by an incandescent filament lamp falls upon a good achromatic lens, and is brought to a focus at a distance from it. Two pieces of good plate-glass, pressed together with a drop or two of water mixed with air between them, are placed in front of the achromatic lens. The film thus enclosed between the glasses forms the 'mixed plate,' which is observed through a telescope placed with its objective just behind the focus of the achromatic lens. The appearance of the film, as seen with this arrangement, depends on the form of the aperture or stop regulating the admission of light from the focal plane of the lens into the object-glass of the observing-telescope: (A) With an aperture placed centrally in the focal plane so as to admit the light coming to the geometrical focus but cutting off the diffracted light, the water-air boundaries in the mixed plate appear as coloured lines in a bright white field. (B) When a central stop is placed symmetrically so as to cut off the light coming geometrically to a focus, and an annular aperture surrounding the stop admits only diffracted light into the observing telescope, the whole field appears dark, except the water-air boundaries, which are seen apparently doubled, shining out as two brightly-coloured lines of light running parallel to each other and separated by a fine perfectly dark line coinciding with the exact outline of the boundary. The colours seen depend only on the thickness of the film, and are independent of the size or shape of the boundaries. They are complementary to those seen in case A. A magnified photograph of the film under these conditions, showing the apparent doubling of the boundaries, was reproduced in figure 4 of the plate accompanying the first

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*"On Methods for Detecting Small Optical Retardations and the Theory of the Foucault Test." *Philos. Mag.* Feb. 1917.

†"On the Colours of the Striae in Mica," C V Raman and P N Ghosh. *Nature* (*London*) October 1918 See also P N Ghosh, *Proc. R. Soc.* **A96** 257 (1919).

part of the paper. (C) When, instead of a symmetrical annular aperture, we have only a small aperture placed eccentrically in the focal plane admitting diffracted light into the observing telescope, then the full outlines of the water-air boundaries are not seen, but only two small portions of each closed boundary, such that the normals to the boundary at the two points visible are parallel to the radius vector joining the focus with the aperture placed in the focal plane. With such an eccentrically-placed aperture the phenomenon of the doubling of the boundaries noticed under (B) does not occur, and we merely get a single luminous coloured line in a dark field running along the portions of the boundary visible.

The phenomena described above are closely analogous to those exhibited by the striae in mica*. The observations show clearly that the laminar boundaries in a mixed plate act as centres or sources of diffracted radiation. Each element of a laminar boundary may be regarded as sending out *two* streams of radiation—one on the more retarded, and one on the less retarded side of the wave-front. In directions nearly coincident with that of regular transmission of the incident waves, these two streams are of practically equal intensity and of opposite phases. In such directions the phenomena observed are in agreement with the indications of Lord Rayleigh's theory, according to which the colour of the laminar boundary, as seen in the Foucault test, should be complementary to the colour of the central fringe in the laminar diffraction pattern produced by it. For very small angles of diffraction, therefore, the elementary diffraction theory gives results which are substantially valid. As we shall see presently, this ceases to be true when we consider larger angles of diffraction.

## 3. The unsymmetrical scattering of light by laminar boundaries: Normal incidence

Very simple observation suffices to show that the scattering of light through larger angles by laminar boundaries exhibits features not indicated by the elementary theory. For this purpose, a thin film of liquid mixed with air enclosed between two glass plates is placed normally in the track of a strong pencil of light from a lantern, and viewed obliquely by the eye with or without the aid of a magnifier. It will be noticed at once that the edges of separation of liquid and air diffract light in a strikingly unsymmetrical manner. Any given edge can easily be observed diffracting light at all angles up to 90° when viewed on the side passing through the liquid; but viewed on the side passing through air, it can hardly be seen at all except in directions making less than about 10° or 15° with the direction of the incident beam, so small is the intensity of the diffracted light in this region. If a closed curved boundary enclosing air be viewed at a slight

*P N Ghosh, *loc. cit.*

obliquity to the direction of the incident light, the two limited portions of the boundary visible appear differently coloured, one being much fainter than the other. The fainter portion which is seen through air vanishes altogether when viewed at greater obliquity, while the part of the boundary seen through the liquid, i.e. on the more retarded side of the wave-front, remains visible throughout, its colour changing periodically and becoming richer as the obliquity of observation is increased, and finally appearing achromatic when viewed in a direction nearly parallel to the plate. An edge too thick to show colour when observed in a direction nearly normal to the plate, will appear vividly coloured when viewed obliquely on one side of this direction, and be practically invisible from the other side. It should also be remarked that in oblique directions the portions of any one curved boundary that can be seen at a time become greatly reduced—in fact, nearly contract to single points. The normal to the boundary at these points lies in the plane of observation. Each element of a curved boundary is therefore effective in scattering light principally in a plane normal to its own direction*.

For a closer examination of the manner in which the colour of the light scattered by a laminar boundary varies with the direction of observation, the mixed plate may be placed on the table of a spectrometer, and viewed through a low-power microscope which replaces the telescope ordinarily used in the instrument. The laminar boundaries under observation should be illuminated by a somewhat narrow pencil normally incident on the plate, and in order to screen this from entering directly into the field of view of the microscope, a wire may be placed immediately in front of the objective. On turning the microscope about the axis of the spectrometer, the phenomena described in the preceding paragraph may be readily observed and studied. Viewed nearly in the direction of the incident light, the laminar boundaries appear of a uniform colour depending on the thickness of the film. On turning the microscope aside to a slightly oblique direction, each of the boundaries seen changes colour, that differently in its two parts which are seen respectively through the more and less refrangible media. For instance, a closed boundary which, seen in a nearly normal direction, appears throughout golden yellow, viewed at a slightly greater obliquity, appears red on the portion seen through the liquid and greenish blue on the portion seen through air, the latter appearing much fainter. The colours and intensities of the two parts of each boundary are interchanged when the microscope is turned over to the other side. Viewed at still greater obliquities, further fluctuations of colour occur, the sequence of these variations being quite asymmetric with respect to the two sides of the direction of the regularly transmitted pencil. A clear idea of these phenomena will be obtained from figure 1.

*This is generally true of all curved diffracting boundaries on which light is normally incident. See *Philos. Mag.* Jan. 1919, p. 127, and Sept. 1919, p. 219.

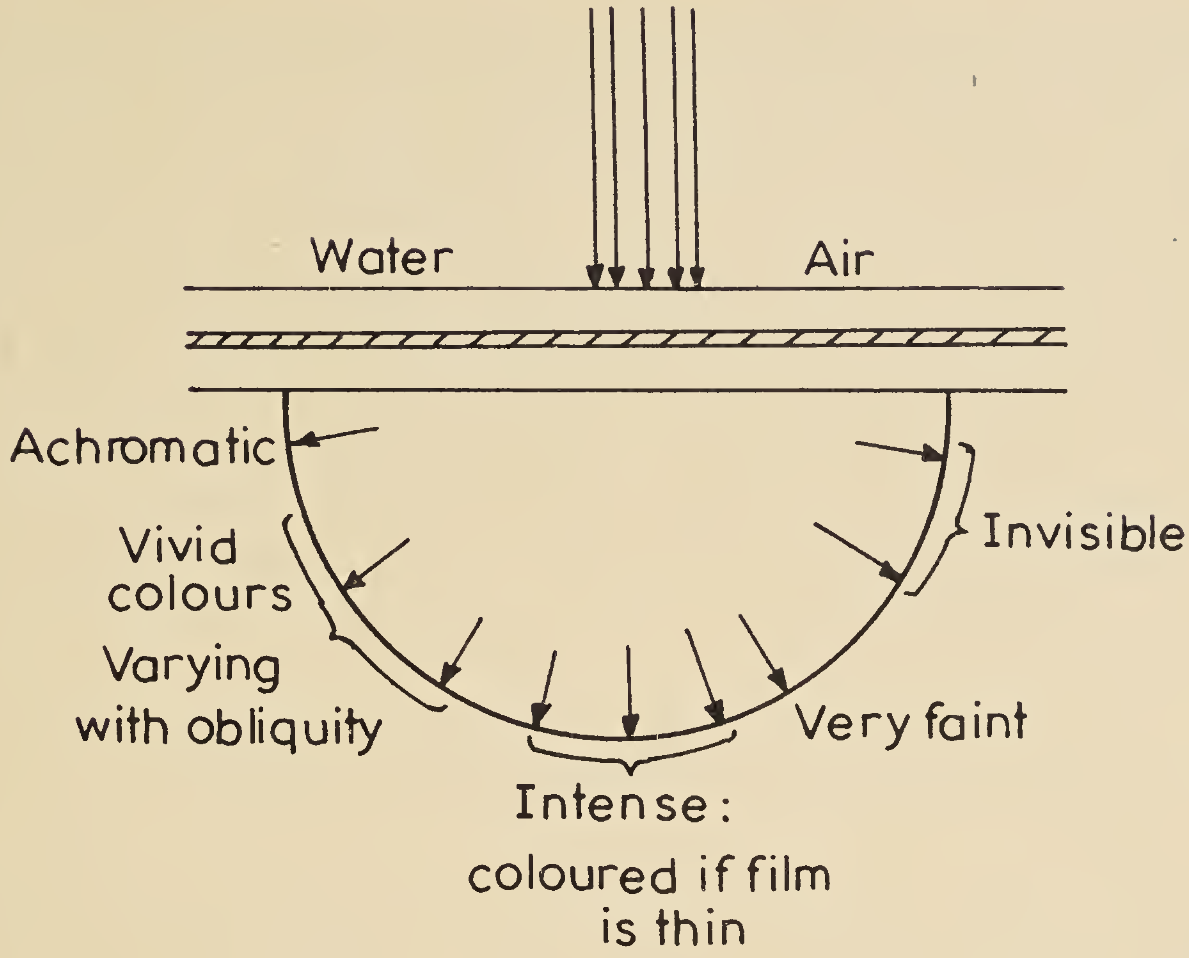


**Figure 1**

The fluctuations of colour with obliquity are most striking in the region on the left of the diagram where they commence, and are continued over to the right, where they are much less vivid, apart from the greatly decreased intensity of the diffracted light on this side. If, instead of white light, a pencil of monochromatic light be used to illuminate a laminar boundary, the light scattered by it shows a similar asymmetry, the intensity fluctuating as the obliquity is varied, and vanishing in a series of directions which are closer together or wider apart according to the thickness of the film.

## 4. Explanation of the diffraction-haloes

We are now in a position, on the basis of the observations described in the preceding section, to form a general idea of the manner in which the diffraction-haloes due to a mixed plate arise. To begin with, it may be assumed that the mixed plate is of uniform thickness, and that the light is incident normally upon it. Consider the light diffracted by the film in a direction making an angle $\theta$ with the incident pencil of rays. To the aggregate scattering in this direction the laminar boundaries in the film all contribute, the effective elements of each boundary being those normal to the plane containing the incident and scattered rays. Since, as we have seen, each boundary scatters light in an unsymmetrical manner with respect to the direction of the transmitted pencil, we may divide all the effective

elements into two groups, namely those which scatter light in the given direction respectively through the more and less refrangible media in the film. In summing up the effects of the elements in each group in the given direction $\theta$, the relative phases of the scattered rays which depend on the positions of the scattering elements have to be taken into account; and as the boundaries are irregularly distributed on the film, the phases of the elements in each group may be assumed to vary arbitrarily. Further, as the dimension of each boundary—that is, the diameter of each air-bubble in the film—varies between wide limits, there is also no fixed phase-relation between the corresponding elements in the two groups. Thus, the intensity of the scattered light in the direction $\theta$ is *statistically* equal to the sum of the intensities due to the two groups separately, and the intensity due to each group is, similarly, the sum of the intensities due to its discrete elements. But for individual values of $\theta$ there may be large deviations from the statistical average, and this gives rise to the granular structure of the halo in monochromatic light, and its radial fibrous structure in white light*. We have seen that the colour and intensity of the scattering by each elementary boundary varies with $\theta$ in a manner depending on the thickness of the film, and since this is constant, the *average* aggregate effect of each group varies with $\theta$ in the same way as for individual elements. Considering together all the possible directions of the scattered pencil, we see that each group will give rise to a diffraction-halo with circular rings surrounding the direction of the source. The diffraction-halo due to one group extends from $\theta = 0°$ up to $\theta = 90°$, while that due to the other group is of sensible intensity only for small values of $\theta$. Near the centre, therefore, the two sets of rings due to both groups of elements are superposed and the halo is composite; while in the outer part, only those elements which diffract light through the more refrangible medium have a sensible effect, and the halo is therefore simple. It is thus seen that the summation of the effects of the individual boundaries in the film leads to results in close agreement with the features of the halo already described.

The foregoing treatment also enables us at once to explain the increased intensity of the halo in certain directions in the case of films containing elongated boundaries. In so far as relates to the general configuration of the halo, the arguments of the preceding paragraph apply *mutatis mutandis* also in the case of such films, and it is clear why the distortion of the boundaries leaves the circular form and positions of the rings in the diffraction-halo unaffected; the only difference is as regards the relative intensity of the halo along different radii, which depends on the aggregate length of the scattering elements effective along the respective directions. If we divide up each boundary in the film into $n$ parts, such that the successive normals at the points of division make angles of $2\pi/n$ with

*Compare De Haas, "On the Scattering of Light by Small Particles." *Proc. R. Soc. Amsterdam*, 1918 p. 1278; also Lord Rayleigh, *Philos. Mag.* Dec. 1918.

each other, the aggregate scattering effect of each of the $n$ groups of parallel elements in the film in the planes respectively normal to them would be the same, provided the *average* length of an element in each of the $n$ groups were the same. The latter condition is satisfied, provided the boundaries in the film show no bias towards elongation in any particular direction. But if they do show such bias, the average length of an element is greatest in respect of the groups running parallel to the general direction of elongation, and least in the groups running transverse to such direction. From this, it follows that the intensity of the halo should be greatest in the plane perpendicular to the direction of elongation, and least in the plane parallel to it. This is exactly what is observed. The intensities should, in fact, be quantitatively proportional to the square of the average length of an element in each group.

## 5. Mathematical theory: Normal incidence

We have now to consider the explanation of the unsymmetrical scattering by the laminar boundaries in a mixed plate, and to express the results in quantitative form. No *rigorous* treatment of the problem of diffraction by plane transparent laminae bounded by edges appears as yet to have been put forward. In practice, the precise shape of the diffracting-edge should obviously have a considerable influence in determining the manner in which it scatters light in directions much removed from that of regular propagation of the incident waves. For instance, the striae in mica are often found, under the microscope, to possess an echelon-like structure*. The striae diffract light asymmetrically through large angles, but the effects observed with different striae differ in a manner which suggests that the results are influenced by the structure of the laminar edge as well as by its total thickness. In the case of mixed plates, owing to the action of surface tension, the laminar edges are not perpendicular to the surface of the film, but have the form of a meniscus (figure 2). We shall assume that the angle of contact of the liquid with the plate is zero, and that the meniscus is of semicircular form. The diagram is assumed to be drawn perpendicular to the plane of the film and also to the element of the scattering boundary under consideration.

It is obvious, especially in view of the curvature of the surface of separation, that the incident light would be scattered very differently towards the two sides of the boundary between the two media. On either side, scattered disturbances would emerge which have traversed different paths partly through one medium and partly through the other, and the problem is to find their relative intensities and the path-differences under which they interfere. As regards the light scattered

*P N Ghosh, "On some Phenomena of Laminar Diffraction observed with Mica." *Proc. Indian Assoc. Cultiv. Sci.* **6**, 51 part 1 (1920).

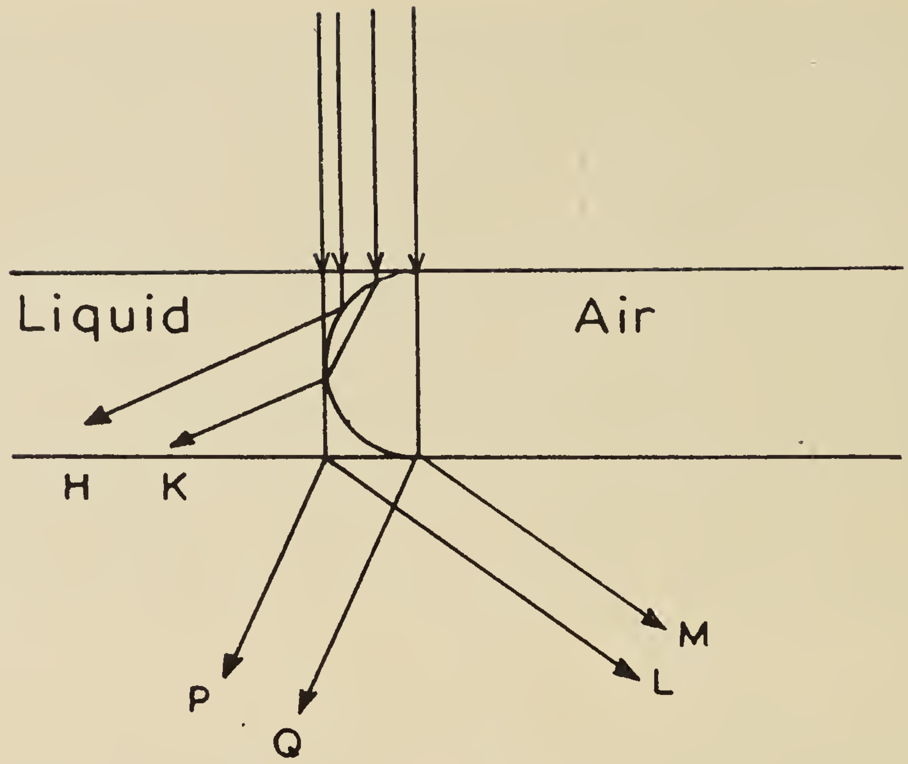


Figure 2

towards the side of the more refrangible medium, it is convenient to assume tentatively that the paths traversed are those given by the laws of geometrical optics. Part of the light incident on the curved meniscus would be reflected within the liquid, and if the angle of incidence were greater than the critical angle, the reflection would be total. Part would also be refracted into the rarer medium, and after a second refraction emerge again into the liquid. If these disturbances finally travel in parallel directions, such as those shown by H and K in the plane of the diagram, the path-difference between them may be readily evaluated and shown to be

$$t(1-\mu\sin i)(\mu\cos i-\sqrt{1-\mu^2\sin^2 i})+\delta. \tag{1}$$

In this formula, $t$ is the thickness of the film, $\mu$ the refractive index of the liquid, and $i$ the angle of incidence on the meniscus of the light which is twice refracted. $\delta$ is the correction necessary on account of the change of phase in total reflection. If $i$ be nearly equal to zero, both pencils emerge nearly parallel to the direction of the incident rays, $\delta$ has the value which obtains at nearly grazing incidence, and the expression for the path-difference reduces to

$$(\mu-1)t-\lambda/2. \tag{2}$$

The light scattered through small angles thus interferes under a difference of path which is the same as that of the regularly-transmitted pencils less half a wave-length. This agrees with what we should expect on the simple diffraction theory, the scattered light being of a colour complementary to that due to the interference of the regularly-transmitted pencils. For larger angles of scattering, the difference of path between the interfering pencils given by (1) steadily falls off in magnitude, and finally becomes zero when the angle of incidence on the meniscus is just equal

to the critical angle, as the pencils then become coincident and $\delta$ is also equal to zero. We should thus expect to observe a series of maxima and minima of intensity in the scattered light in different directions, which is exactly what is found in experiment. The deviation of the interfering pencils within the liquid film is given by $2(r - i)$ where $\mu \sin i = \sin r$, and when this is equal to $(\pi - 2\alpha)$ where $\alpha$ is the critical angle, the path-difference vanishes, and we should expect the scattered light to be achromatic. The angle of scattering $\theta$ on emergence from the film is given by the relation $\sin \theta = \mu \sin 2(r - i)$. It is worthy of note that, as the correction $\delta$ for the change of phase in total reflection depends on the plane of polarization of the incident light, the positions of the maxima and minima in the scattered light should be slightly different for light polarized in and at right angles to the plane of incidence, the difference being greatest when the path-difference given by (1) is rather small. This point will be noticed again hereafter.

We should, of course, also consider disturbances such as those indicated by $P$ and $Q$ in the diagram, which have traversed paths lying wholly in one medium or the other and are then diffracted in oblique directions from the edges of the regularly-transmitted wave-fronts. If $\theta$ be the angle of diffraction on emergence from the film, the path-difference under which such disturbances interfere is

$$(\mu - 1)t - t/2 \cdot \sin \theta - \lambda/2, \tag{3}$$

the deduction of $\lambda/2$ being made on account of the phase-reversal of the diffracted ray. When the angle of scattering is not large, the path-differences given by equations (3) and (1) are identical, as may be readily shown on expanding (1) and neglecting the second and higher powers of $i$. Indeed, in this case it follows from the well-known principle of minimum or stationary path, that the actual course followed within the film by either of the interfering disturbances is a matter of indifference, provided the deviations from the geometrical path are not large. On the other hand, when the angle of scattering is large, the intensity of the light scattered from the curved interface between the two media towards the more refrangible medium would be far larger than that diffracted from the edges of the wave-fronts. Hence, both for small and large angles of scattering we would be justified in regarding the expression (1) for the path-difference as substantially valid.

Passing on now to consider the light scattered towards the less refrangible medium, it is clear in this case that no sensible portion of it is contributed by the curved interface between the media. The scattered light which emerges consists entirely of disturbances (such as those indicated by L and M in the diagram) diffracted from the edges of the wave-fronts. These interfere under a path-difference

$$(\mu - 1)t + t/2 \cdot \sin \theta + \lambda/2, \tag{4}$$

which now *increases* with the increasing obliquity of the diffracted light; in directions nearly normal to the film the scattered light is, as before, of a colour complementary to that due to the interference of the regularly-transmitted

pencils, and its intensity is equal or comparable with that of the light similarly diffracted towards the side of the more refrangible medium. At larger angles of diffraction, however, the intensity falls off with great rapidity, and is far less than on the side of the more refrangible medium where the light scattered from the curved interface plays an important part.

We have not, so far, discussed the relative intensity of the interfering pencils scattered in any given direction. The mathematical treatment of this question may be deferred till a later stage. For the present, it may suffice to remark that experimental observation as already detailed shows the interferences to be remarkably perfect, and hence the interfering pencils must be of comparable or equal intensity throughout the region in which we have maxima and minima in the scattered light. There is no difficulty in understanding, at least in a general way, why this is the case. In directions nearly normal to the film, light is diffracted chiefly from the wave-front regularly transmitted through the film, and the contributions to the scattered radiation from the part of the wave-front lying on either side of each boundary should obviously be equal. In more oblique directions the scattering occurs chiefly at the curved interface between the two media, and a calculation on the principles of geometrical optics shows the intensities of the pencils emerging respectively after two refractions and after total reflection to be comparable throughout, the intensity of the former being at first greater, then equal and finally less than that of the latter, as the angle of scattering increases. Thus there is reason to expect that throughout the range in which the scattered light can be observed, the interferences should be strongly marked.

## 6. Concluding remarks

Numerical computation of the position of the dark and bright rings in the halo from formula (1) of the preceding section gives results in general agreement with experiment. The width of the successive rings increases rapidly as we proceed outwards from the centre of the halo. For instance, in the particular case of a film for which the path-difference for normal transmission through the two media is five wavelengths, the angular radii of the five dark rings in the halo as given by the formula are $\theta = 0°$, $\theta = 9° 20'$, $\theta = 20° 27'$, $\theta = 37° 12'$, and $\theta = 73°$, respectively. These quantities are of the same order as those actually observed.

A detailed quantitative comparison between experiment and theory, together with a discussion of the theory of the diffraction-haloes due to obliquely-held plates and of the effects observed with non-uniform plates, as also of the special phenomena observed with dry films of albumen, will be given in the concluding instalments of the paper.

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# On the colours of mixed plates—Part III

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## 1. Introduction

A detailed description of the phenomena exhibited by mixed plates, including several features of interest not previously noticed, was given in the first paper of the series†. In the second paper‡ it was shown by a discussion of these and other effects that, the theory of the colours of mixed plates usually put forward is inadequate, and a new treatment was suggested in order to explain the observed phenomena. We now proceed to summarize the theory, to develop its consequences in detail, and to show that the results obtained are in quantitative agreement with experiment.

## 2. Statement of theory proposed

The explanation of the various phenomena exhibited by mixed plates is closely related to that of the effects observed in one fundamental case—that is, the character of the diffraction halo seen surrounding a distant source of light when viewed through a mixed plate of uniform thickness held normally close to the eye. The remarkable feature is that though the bubbles of air enclosed in the film forming the mixed plate vary arbitrarily in size and shape and are irregularly arranged in the film, nevertheless the diffraction halo seen in this case exhibits a regular structure consisting of a series of circular rings which are closer together in the centre of the halo and wider apart in its margin, the number of such rings depending only on the thickness of the film and its composition. The theory proposed is that these effects are connected with the special character of the

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laminar edges in the film. Owing to the action of surface tension, the edges of the bubbles are not cylindrical, but are drawn inwards as a meniscus, the form of which depends on the angle of contact between the liquid and plate. If the angle of contact be assumed to be zero, the cross-section of the meniscus should be semi-circular. The investigation on this basis (given in the second paper of the series) indicates that the intensity of the light scattered in its passage through the film at these laminar edges should exhibit maxima and minima in various directions besides being strongly asymmetrical in its distribution—that is, differing very greatly in directions lying on either side of the regularly transmitted light. For very small angles of diffraction—that is, in directions very nearly coincident with that of regular propagation—the results of the investigation agree with the usual elementary theory; but in other directions differ entirely from it. Considering the aggregate effect of all the elementary laminar boundaries assumed to be irregularly arranged in the film, it is shown that the scattered light would give rise to two sets of bright and dark circular rings in the halo, the first set extending up to very large angles of diffraction, and the second set being of inappreciable intensity except near the centre of the halo. The position of the dark rings in the first set is given by the angle of diffraction $\theta$, where

$$\left.\begin{aligned} t(1-\mu\sin i)(\mu\cos i-\sqrt{1-\mu^2\sin^2 i})-\delta &= (2n-1)\frac{\lambda}{2}, \\ \mu\sin i=\sin r, \quad \text{and} \quad \sin\theta &= \mu\sin 2(r-i). \end{aligned}\right\} \tag{1}$$

In this formula, $t$ is the thickness of the film, $\mu$ the refractive index, $\lambda$ the wavelength and $\delta$ is the phase-change occurring in total reflection, and $n$ is any integer (counting from zero).

The position of the dark rings in the second set is given by the formula

$$(\mu-1)t+\tfrac{1}{2}t\sin\theta-\frac{\lambda}{2}=(2n+1)\frac{\lambda}{2}. \tag{2}$$

The two sets of rings would be superposed on each other near the centre of the halo.

## 3. Comparison of theory and experiment for uniform films: Case of normal incidence

It has already been indicated in the second paper of the series that there is a general qualitative agreement between the facts observed and the results of the theory outlined above. This becomes clearer on representing the expressions for the path-difference of the interfering rays given in formulae (1) and (2) graphically. Taking $\mu$ for egg-white to be 1·35, the expressions $t(1-\mu\sin i)$

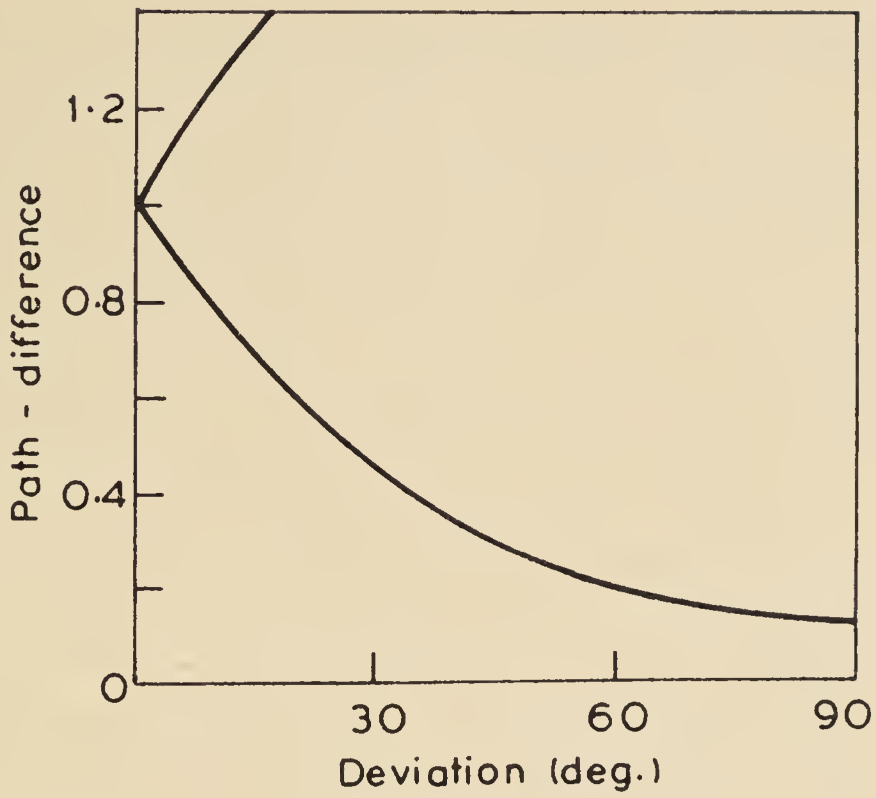


**Figure 1**

$(\mu \cos i - \sqrt{1 - \mu^2 \sin^2 i})$ and $t\{(\mu - 1) + \frac{1}{2}\sin\theta\}$ in formulae (1) and (2) respectively have been calculated as fractions of $(\mu - 1)t$ and plotted in figure 1 against the corresponding values of $\theta$, the angle of diffraction. It will be seen that in one case the path-difference diminishes continuously (at first rapidly and later more slowly) from $(\mu - 1)t$ up to a very small fraction of it for the largest angles of diffraction, while in the other case the path-difference actually increases with $\theta$. We should accordingly have in the former case a series of circular rings in the halo, closer together near its centre and wider apart near its margin. The value of $\delta$, the phase-change in total reflection, decreases gradually from $\lambda/2$ to zero as $\theta$ increases. The outermost ring in white light corresponding to a negligibly small path-difference is therefore achromatic, while those following within it should be strongly coloured*. Near the centre of the halo the colours would not be very pure (except in the case of relatively thin films) owing to the increasing path-difference under which the interferences take place, and especially owing to the superposition of the two sets of rings in this neighbourhood. These results agree with observation.

For a quantitative test of the correctness of the theory, it is necessary to use a monochromatic light source. A small aperture illuminated by a quartz mercury lamp with green ray filter is viewed from a distance through a mixed plate of fairly uniform thickness, which may be obtained by rubbing up egg-white between

*If the exact sequence of the colours in the outermost part of the halo is to be investigated, the variation of refractive index $\mu$ with the wavelength $\lambda$ should, strictly speaking, also be taken into account in making the calculations.

moderately thick flat plates of glass and regulating the pressure over different parts of the film in a suitable manner. The film is held normally close to the eye, and moved about till one of the dark rings in the halo just closes in on the centre. The part of the film in front of the eye then corresponds to a thickness for which $(\mu - 1)t$ is an integral number of wavelengths. The angular width of the dark rings in the halo may be determined by making them coincide in succession with a movable faint luminous reference mark placed in the same plane as the source. The actual value of $(\mu - 1)t$ may be determined in one or other of two ways. The first method is to observe the spectrum of white light transmitted through the portion of the film under consideration, and by noting the wavelengths of the light freely transmitted through the film without interference to determine the value of $(\mu - 1)t$ by calculation. The second method utilizes the fact that the number of wavelengths, say $n$, comprised in $(\mu - 1)t$ is the same as the total number of dark rings visible in the halo, provided the latter is not too large; for the outermost dark ring in the halo then corresponds to interference under a path-difference $\lambda/2$, and the innermost dark ring to a path-difference $n\lambda - (\lambda/2)$. The results of both methods agree. Table 1 shows the results of a few determinations of the angular diameter of the dark rings in the halo.

**Table 1**

| $(\mu - 1)t$ | | Angular diameter of dark rings | | | | | Remarks |
|---|---|---|---|---|---|---|---|
| $7\lambda$ | obs. | 0° 0′ | 6° 45′ | 14° 12′ | 22° 42′ | 32° 40′ | [Outer rings |
| | calc. | 0 | 6 15 | 13 40 | 22 16 | 33 10 | not measured] |
| $6\lambda$ | obs. | 0 | 7 36 | 16 10 | 28 40 | 45 0 | Ditto |
| | calc. | 0 | 6 42 | 15 40 | 27 30 | 44 0 | |
| $5\lambda$ | obs. | 0 | 10 30 | 19 6 | 35 24 | | Ditto |
| | calc. | 0 | 9 27 | 20 27 | 36 50 | | |

In the cases under consideration, where $(\mu - 1)t$ is an integral number of wavelengths, formulae (1) and (2) agree in making the first dark ring coincide with the centre of the halo. Elsewhere the intensity of the halo given by formula (2) is too small appreciably to influence the *positions* of any of the observable dark rings, though it may result in partial blurring of one or two rings nearest the centre. The theoretical values of the angular diameters of the dark rings may therefore be taken as given by formula (1) exclusively, and have accordingly been calculated and shown in table 1. The agreement between theory and observation is seen to be satisfactory.

## 4. Non-uniform films: Normal incidence

When a mixed plate is held at some distance from the eye between it and a source of light of small dimensions, the film becomes visible by the light diffracted in passage through it. Strictly speaking, it is not the whole film that appears luminous, but only the laminar diffracting boundaries in it or such portions of them as are effective in scattering light in directions reaching the eye of the observer. Since, as we have seen, the light scattered by each elementary laminary boundary exhibits maxima and minima of intensity in various directions depending only on the thickness of the film, the appearance of the mixed plate should vary with the direction in which it is observed. This is actually the case, and the effects observed are most striking with a non-uniform film of graduated thickness formed between two lenses of the kind used for observation of Newton's rings. A film of this kind is very suitable for quantitative work in testing the accuracy of formulae (1) and (2) given above, as the value of $t$ varies from point to point of the film in a perfectly regular and determinable manner. Taking first the case in which the film is observed very nearly in the direction of the incident light, so that the angles of diffraction are small, formulae (1) and (2) agree in showing that, for the regions at which $(\mu - 1)t$ is an integral number of wavelengths, the diffracted light is of minimum intensity—that is, the film as seen by the light scattered by it shows a succession of black rings at these points. On altering the angle of observation, the appearance of the film alters, and we have now to consider separately the two sets of laminar boundaries which diffract light to the observer's eye according to formulae (1) and (2) respectively. From the graphs exhibited in figure 1 it is evident that these should behave differently. Therefore, the rings seen on the film should bifurcate, one set moving outwards, that is from the thinner to the thicker part of the film, and the other set should move inwards. For certain angles of diffraction the two sets would be completely out of step, and this would result in a blurred appearance of the film. At other positions they would fall into step, and the dark and bright rings on the film would again be conspicuous.

The foregoing indications of theory are in agreement with what is actually observed. Owing to the finite area of the film, the angle of observation is different for different portions of its surface; and as the eye is gradually moved outwards away from the line of direct vision of the source, one or two patches of imperfect visibility of the rings appear on the film. On further increasing the angle of observation, these disappear and the rings again become sharp and clear, and continue to move *outwards* on the film in accordance with formula (1), the second set of rings which move inwards according to formula (2) becoming of negligible intensity. With very oblique observation the dark rings indeed move completely out of the thinner portions of the film, and it is possible to determine the thickness of any part of the film, provided it is not too large, by merely counting the number of rings that pass across it as the angle of observation is gradually increased up to

90°. As the path-differences under which the interferences observed in the film occur decrease with increasing angle of diffraction, it is clear why even a thick film when viewed obliquely shows vivid colours.

Table 2 shows some observations of the angles of diffraction corresponding to various thicknesses of the film at which a dark ring appears on it, and for comparison the theoretical values calculated from formula (1). It is seen that here again the agreement is satisfactory.

**Table 2**

| $(\mu - 1)t$ | | Angles of diffraction at which a dark ring appears on film | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $10\lambda$ | obs. | 0° 0′ | 3° 40′ | 8° 50′ | 14° 30′ | 20° 6′ | 27° 10′ | 35° 0′ | 46° 20′ | 62° 0′ |
| | calc. | 0 0 | 3 48 | 9 10 | 14 15 | 20 20 | 26 50 | 35 54 | 47 20 | 66 1 |
| $9\lambda$ | obs. | 0 0 | 5 0 | 10 42 | 16 30 | 24 30 | 32 6 | 42 18 | | |
| | calc. | 0 0 | 4 44 | 10 40 | 15 49 | 23 16 | 31 28 | 43 10 | | |
| $7\lambda$ | obs. | 0 0 | 6 12 | 13 36 | 22 12 | 32 54 | 50 18 | | | |
| | calc. | 0 0 | 6 15 | 13 40 | 22 16 | 33 10 | 50 9 | | | |
| $5\lambda$ | obs. | 0 0 | 9 50 | 21 30 | 36 42 | | | | | |
| | calc. | 0 0 | 9 27 | 20 27 | 36 50 | | | | | |
| $4\lambda$ | obs. | 0 0 | 12 0 | 27 42 | 52 0 | | | | | |
| | calc. | 0 0 | 12 6 | 28 5 | 59 40 | | | | | |
| $3\lambda$ | obs. | 0 0 | 18 0 | 44 27 | | | | | | |
| | calc. | 0 0 | 16 31 | 47 20 | | | | | | |

In calculating the positions of the dark rings given by formula (1), the phase-change $\delta$ occurring in total reflection was taken to be given by the formula

$$\cos\frac{2\pi}{\lambda}\delta = \frac{1 + \mu^2 - 2\mu^2\cos^2(r - i)}{\mu^2 - 1}.$$

This corresponds to light polarized in the plane of incidence. For light polarized in a perpendicular plane,

$$\cos\frac{2\pi}{\lambda}\delta = \frac{(\mu^2 + 1) - (\mu^4 + 1)\cos^2(r - i)}{(\mu^2 - 1) - (\mu^4 + 1)\cos^2(r - i)}.$$

These two formulae should give slightly different values for the positions of the rings. That such a difference actually exists is verified on observation of the rings of non-uniform plates obliquely through a nicol, when it will be found that the rings shift slightly on rotating the nicol. The effect is, however, perceptible only for fairly large angles of diffraction. For instance, numerical calculation shows that the difference between the two values of $\delta$ corresponds to an alteration of the

path-difference of the interfering rays of only $0{\cdot}02\,\lambda$ for an angle of diffraction $9^\circ 30'$. For a deviation of $62^\circ$ it increases to $0{\cdot}14\,\lambda$, and this gives an easily measurable difference in the position of the rings, especially with films which are fairly thin. This indication of theory has been confirmed by quantitative measurement.

The angles of diffraction at which the rings on the film appear blurred may be readily calculated. As these angles are generally small, we may use for this purpose an approximate form of formula (1) giving the path-difference directly in terms of the angle of diffraction:

$$(\mu - 1)t - \tfrac{1}{2}t\sin\theta = n\lambda. \tag{3}$$

Formula (2) may be written as

$$(\mu - 1)t + \tfrac{1}{2}t\sin\theta = n\lambda. \tag{4}$$

The diffraction rings whose positions are given by (3) and (4) would be completely out of step if $t\sin\theta = \lambda/2, 3\lambda/2$, etc. Table 3 shows for comparison the calculated and observed values of the angles of the first blurring in a few cases exhibiting satisfactory agreement. The second blurring, which occurs at a larger angle of diffraction, is hardly so conspicuous as the first.

**Table 3**

| $(\mu - 1)t$ | Observed angle | Calculated angle |
|---|---|---|
| $7\lambda$ | $1^\circ 30'$ | $1^\circ 20'$ |
| $3\lambda$ | 3 40 | 3 20 |
| $2\lambda$ | 5 15 | 5 2 |

## 5. Uniform films: Oblique incidence

The unsymmetrical haloes with elliptic or oval rings seen on observing a light-source through an obliquely held mixed plate have already been described in the first paper of the series. We now proceed to consider their explanation. It is clear that in this case, the elementary laminary boundaries do not all diffract light in an identical manner. The meniscus forming the boundary is differently situated with reference to the incident light at different portions of the periphery of an air-bubble in the film, and the discussion of the manner in which it would diffract the light incident on it is obviously in general a three-dimensional problem. In order to obtain an idea of the principal features of the case, it is sufficient to consider three elements of the boundary of each bubble: (1) an element running perpendicular to the plane of incidence and having the meniscus convex towards the incident rays; (2) an element running perpendicular to the plane of incidence

but with the meniscus concave to the incident rays; and (3) an element running parallel to the plane of incidence. Of these, (1) and (2) would diffract light in directions lying in the plane of incidence, and (3) would diffract light in directions lying along the surface of a cone which has the element as its axis and the incident ray as generator. (For small angles of diffraction, this cone practically coincides with a plane drawn perpendicular to the element.) By investigating these three cases, we get the positions of the maxima and minima of the diffracted light along the directions referred to, and thus obtain an idea of the general configuration of the haloes surrounding the source. Cases (1) and (2) may be dealt with as two-dimensional problems.

*Case* (1)—The section of the meniscus by the plane of incidence and the course of the rays emerging in parallel directions after having traversed different

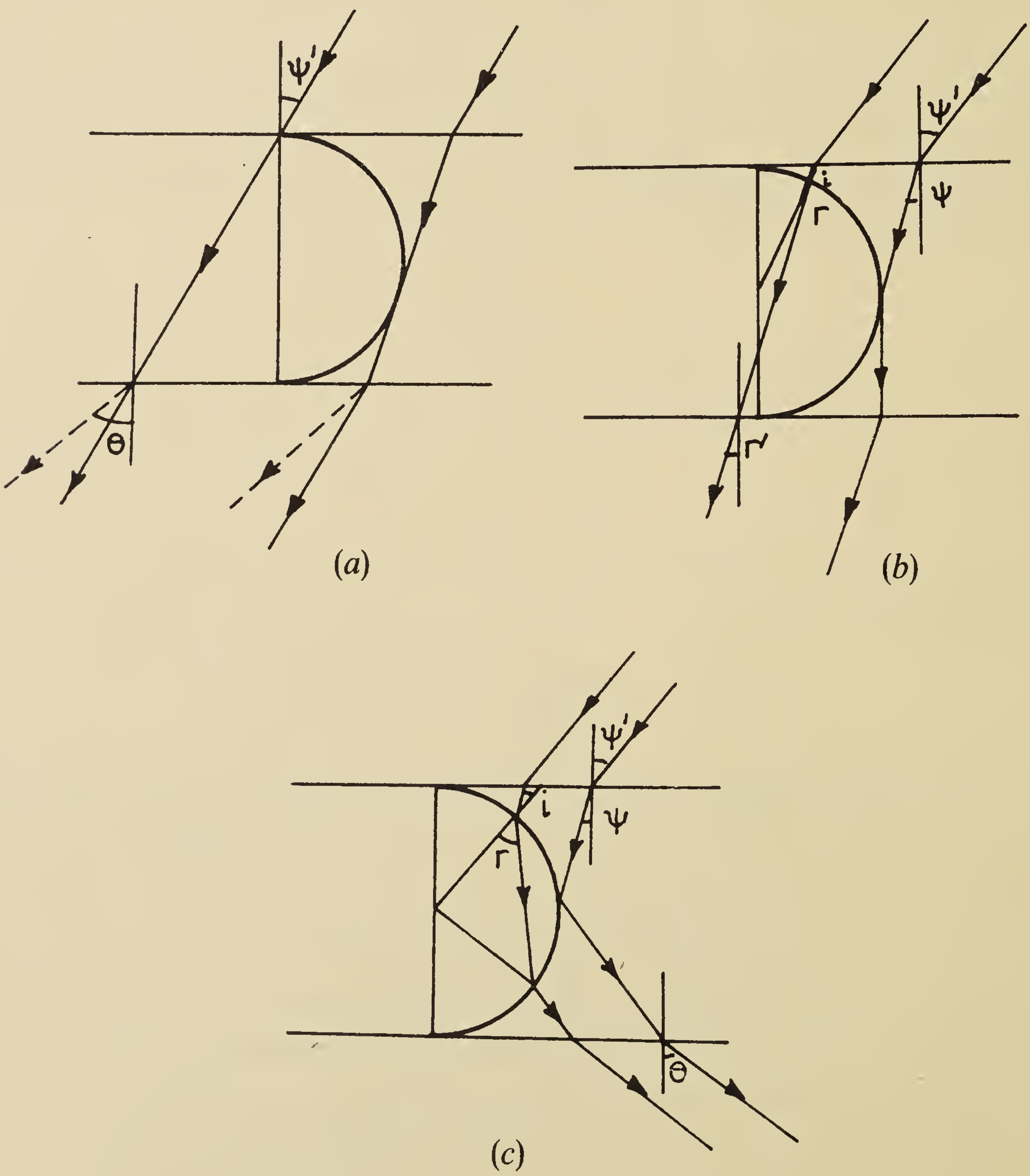


**Figure 2**

paths indicated by the ordinary laws of geometrical optics is shown in figure 2(*a*), (*b*), and (*c*), which corresponds to gradually increasing deviations of the rays passing through the film.

Figure 2(*a*) shows the course of two rays, one of which passes wholly through the air-bubble and the other wholly through the liquid just grazing the meniscus, both emerging without deviation in their original direction. The path-difference between the two rays is

$$t(\mu\cos\psi - \cos\psi'),$$

where $\psi$ and $\psi'$ are the angles of incidence and refraction at the surface of the plate.

In figure 2(*b*) we have one ray undergoing a single refraction at the surface of the meniscus and emerging into the air, and the other ray passing wholly through the liquid and totally reflected at the surface of the meniscus. The path-difference in this case is

$$-\frac{t}{2}(\cos r + \cos r') - \mu t \sin\frac{\psi - i'}{2} + \tfrac{1}{2}\mu t(\cos i + \cos i') - \delta$$

and

$$\sin\theta = \sin r' = \sin(\psi - \overline{r - i}).$$

In the formula, $i$ and $r$ are the angles of incidence and refraction at the surface of the meniscus, and $i'$ and $r'$ are the angles of incidence and refraction at the rear surface of the plate. $\delta$, as before, is the phase-change at total reflection.

In figure 2(*c*) we have one ray undergoing two refractions at the surface of the meniscus, and the other ray passing wholly through the liquid and totally reflected at the meniscus. The path-difference in this case is

$$\mu t(\cos i - \sin\overline{r - i}) - t\cos r - \delta$$

and

$$\sin\theta = \mu\sin(\overline{2r - i - \psi}).$$

In figure 3(*a*), (*b*), (*c*) we have the corresponding cases of the meniscus concave to the incident rays.

The expressions for the path-difference and the direction of the emergent ray are respectively:

$$t(\mu\cos\psi - \cos\psi'). \qquad \text{Figure 3}(a)$$

$$\left.\begin{aligned} &-\tfrac{1}{2}t(\cos r + \cos\psi') - \mu t\sin\frac{i' - \psi}{2} \\ &\qquad + \tfrac{1}{2}\mu t(\cos i + \cos\psi) - \delta \\ \text{and} \quad &\sin\theta = \sin r' = \mu\sin(\psi' + r - i). \end{aligned}\right\} \qquad \text{Figure 3}(b)$$

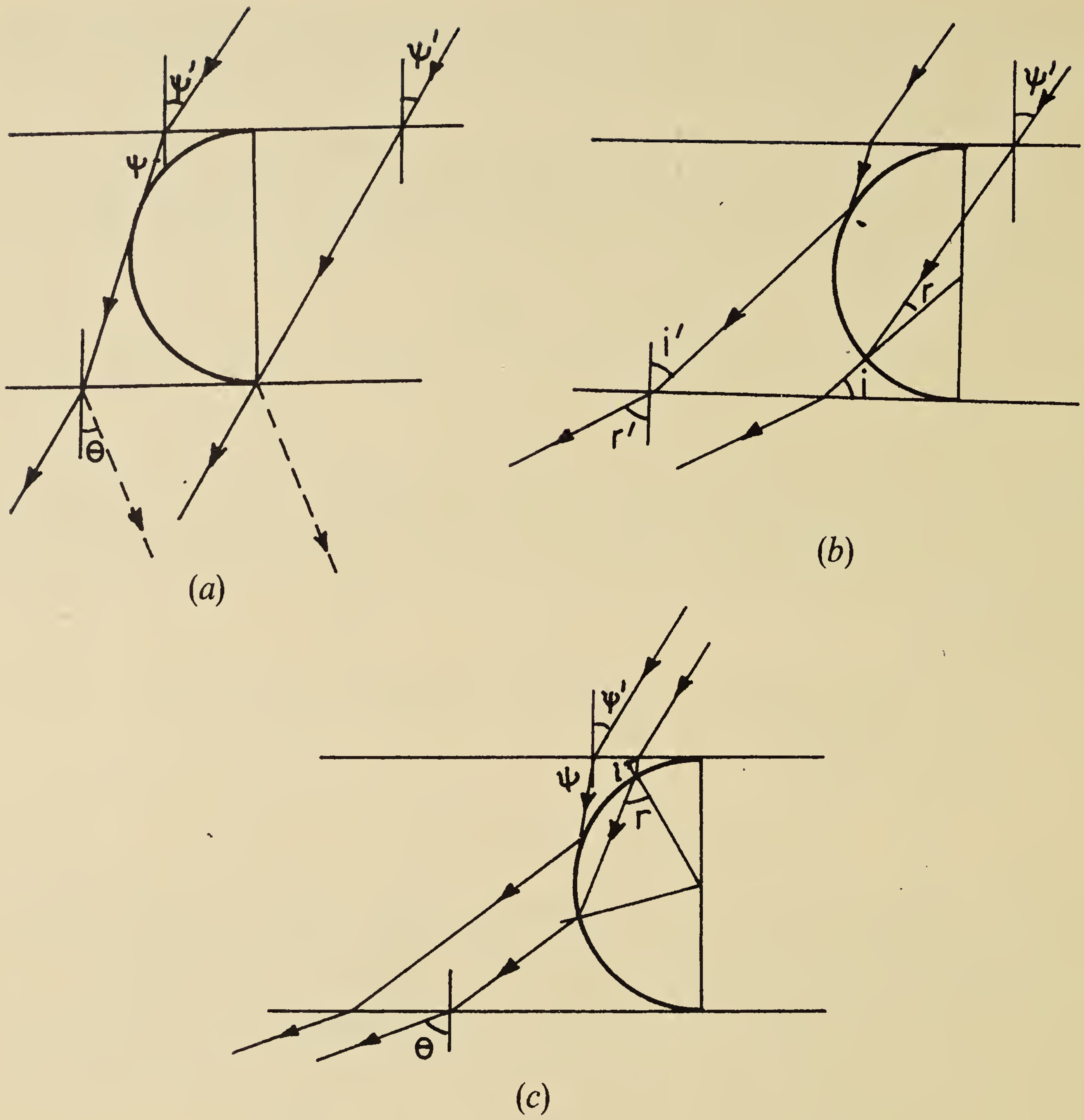


**Figure 3**

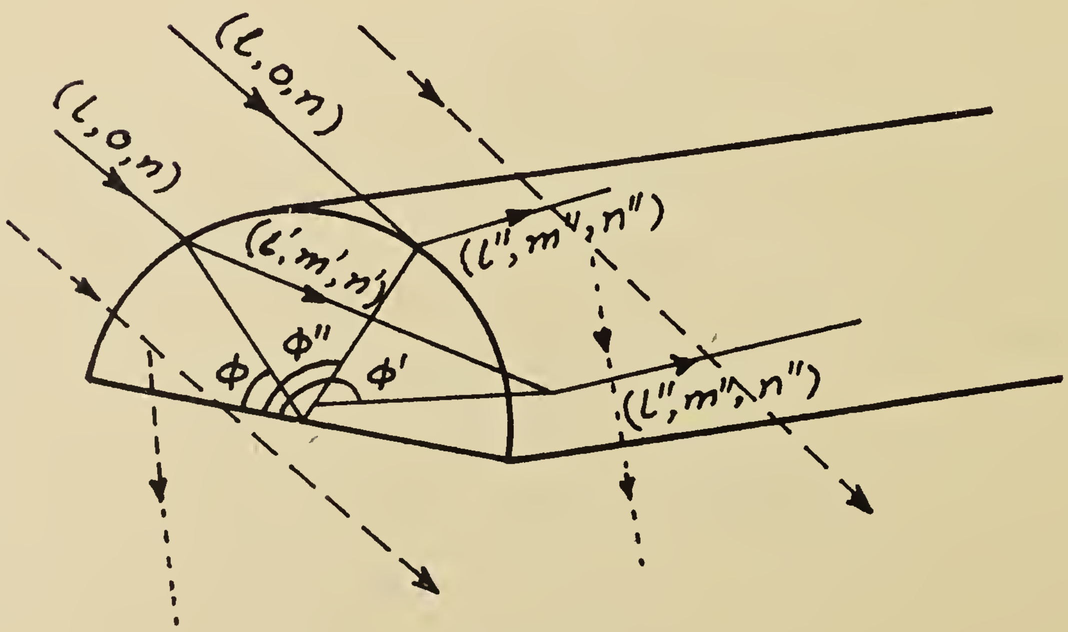


**Figure 4**

$$\left.\begin{aligned} &\mu t(\cos i - \sin \overline{r - i}) - t \cos r - \delta \\ \text{and} \qquad &\sin \theta = \mu \sin(\overline{2r - i + \psi}). \end{aligned}\right\} \qquad \text{Figure } 3(c)$$

In figure 4 we have a representation of the three-dimensional case of the laminar edge running parallel to the plane of incidence.

Taking the direction of the edge to be the $z$-axis and the meniscus to be a section of a semicircular cylinder, $l$, $o$, $n$ to be the direction-cosines of the ray incident on the meniscus, $l'$, $m'$, $n'$ to be the direction-cosines of the ray after one refraction, $l''$, $m''$, $n''$ its direction-cosines after two refractions on emergence into the liquid, the path-difference between this and another ray emerging parallel to it which has passed wholly through the liquid and is totally reflected at the surface of the meniscus may be readily calculated and shown to be

$$\tfrac{1}{2}t\left\{\frac{\cos\phi - \cos\phi'}{l'} - 2\mu l(\cos\phi - \cos\phi'') - \mu^2 n^2 \frac{\cos\phi - \cos\phi'}{l'}\right\} - \delta,$$

where the angles $\phi$, $\phi'$, $\phi''$ are connected by the relations

$$\phi + \phi' = 2\phi'',$$

$$\tan\frac{\phi' - \phi}{2} = \frac{k}{\mu l \sin\phi},$$

where

$$k^2 = 1 - \mu^2(1 - l^2 \cos^2\phi),$$

$$l' = k\cos\phi + \mu l \sin^2\phi.$$

The deviation $D$ of the scattered light is given by the relation

$$\cos D = l_1 l_2 + \mu^2 n^2,$$

where $l_1$, $o$, $n_1$ and $l_2$, $m_2$, $n_2$ are the direction-cosines of the incident and the final emergent ray respectively, and

$$l_2^2 = 1 - \mu^2(1 - l^2\cos^2 2\phi).$$

With the help of the foregoing expressions, the phase-difference of the interfering rays and their deviations may be readily calculated. The results show that the path-differences increase with the obliquity at which the plate is held and for the same deviation depend on the plane in which the emergent rays lie, and are different also on the two sides of the regularly-transmitted pencil in the plane of incidence. Figure 5 shows the decrease of the path-difference of the interfering rays with increasing deviation in the three cases indicated in figures 2, 3, and 4, the angle of incidence being 27°·5. $\delta$ has not been taken into account in drawing the graphs. It will be noticed that the path-difference in figure 2 falls off throughout much more rapidly than in figure 3 or figure 4, and, as regards the two latter, their graphs intersect at a deviation of about 35°.

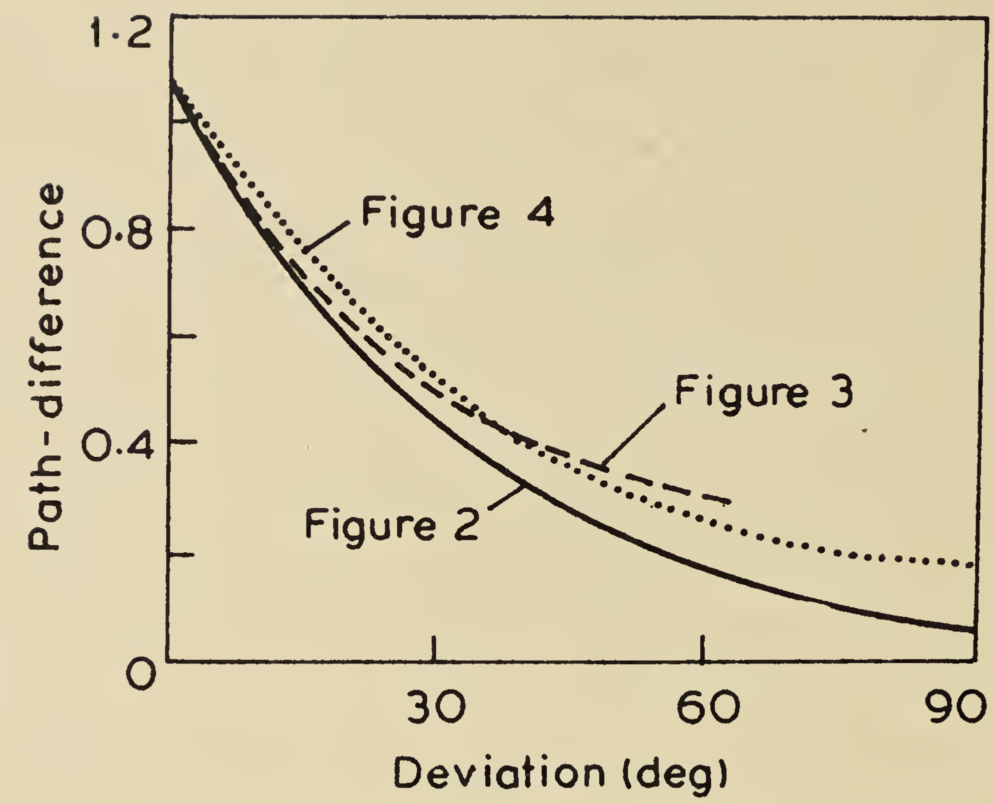


**Figure 5**

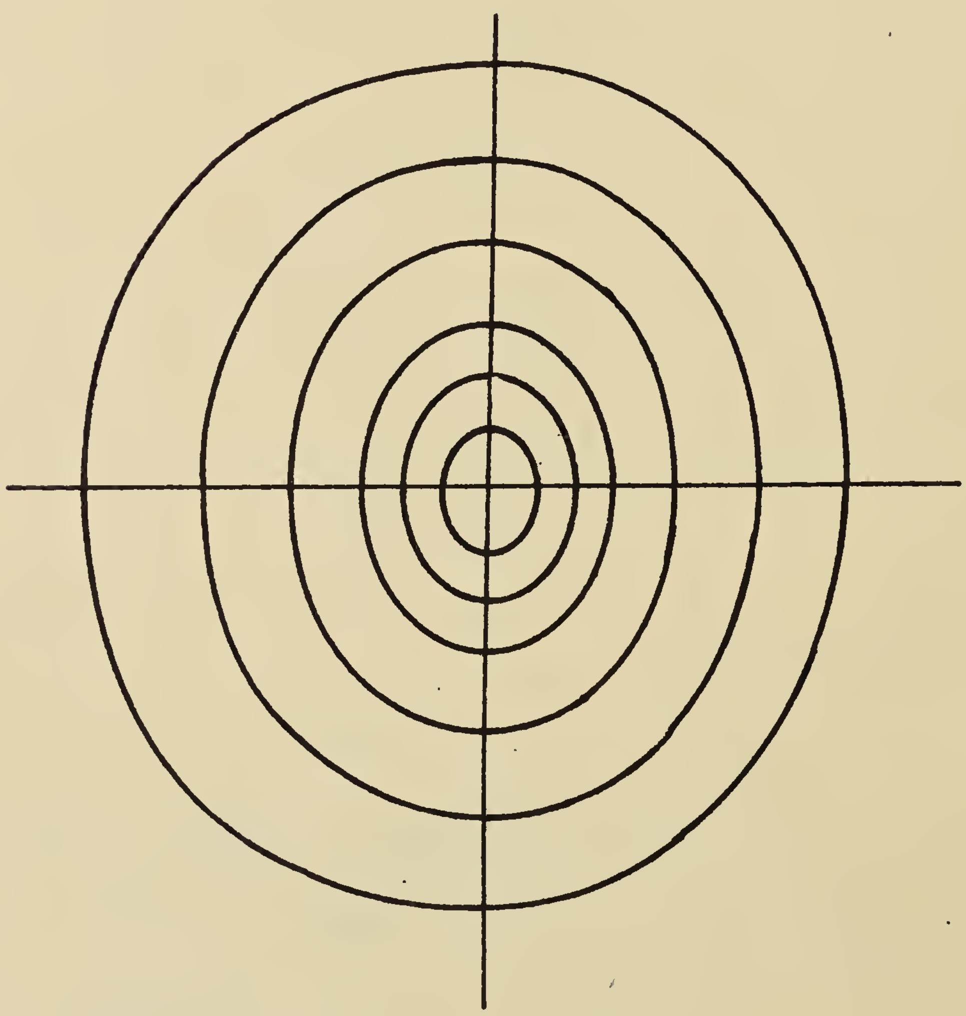

**Figure 6**

From these graphs the position of four points on each of the dark rings in the halo may be readily found, and hence the general shape of the rings may be ascertained. This has been done in figure 6 for the first few rings for a case in which $(\mu - 1)t = 10\lambda$. The oval asymmetrical shape of the rings and other characteristic

features of the halo shown in this graph are fully in agreement with those observed in experiment for this case. The total number of rings seen in the halo increases when the plate is held obliquely, as the path-difference between the undeviated rays passing wholly through the two media is greater than for normal incidence.

As in the case of normal incidence, we have also to consider a second set of rings in the halo due to light diffracted from the edges of the wave-fronts towards the less refrangible medium after traversing paths lying wholly in one or the other of the two media. The course of such diffracted rays is indicated by dotted lines for the three cases dealt with above in figure 2(*a*), figure 3(*a*), and figure 4. The path-difference of such rays may be readily evaluated, and the positions of the rings in the halo produced by them may be found.

The expressions for the path-differences are given by

$$\left.\begin{aligned}&\mu t \sec\psi + t\left\{\tan\psi' + \tfrac{1}{2}\tan\left(45+\frac{\psi}{2}\right) - \tan\psi\right\}\sin\theta\\ &\quad -\tfrac{1}{2}t\cdot\tan\left(45+\frac{\psi}{2}\right)\sin\psi' - t\sec\psi'.\end{aligned}\right\} \qquad \text{Figure 2}(a)$$

$$\left.\begin{aligned}&\mu t \sec\psi + t\left\{\tan\psi' + \tfrac{1}{2}\tan\left(45+\frac{\psi}{2}\right) - \tan\psi\right\}\sin\psi'\\ &\quad -\tfrac{1}{2}t\cdot\tan\left(45+\frac{\psi}{2}\right)\sin\theta - t\sec\psi'.\end{aligned}\right\} \qquad \text{Figure 3}(a)$$

$$\mu t\cos\psi - t\cos\psi' + \tfrac{1}{2}t\sin D. \qquad \text{Figure 4}$$

When $D = 0$ all the three expressions become identical and equal to

$$(\mu t\cos\psi - t\cos\psi').$$

The general result is the same as before—namely, that owing to the superposition of the two sets the first few rings in the halo vary in visibility with their position. It should be noticed that the rays diffracted towards the liquid in figure 2 will be superposed on those diffracted towards the air in figure 3, and *vice versa*. Hence the visibility of the rings will not fluctuate in identically the same way on both sides of the direction of the source and some asymmetry may be expected in respect of this as well. These indications of theory are supported by observation.

## 6. Non-uniform films: Oblique incidence

It remains finally to explain the phenomena observed in this case. The general nature of the effects noticed in experiment has already been sufficiently indicated

in the first paper of the series, and is connected with the case discussed in Section 5 in the same way as the contents of Sections 3 and 4 for the case of normal incidence are related to each other. The increase in the number of rings seen on the film on turning it to an oblique position, the blurring of the rings at certain angles varying with the thickness of the film and the plane of observation, the changes of the appearance of the film at different angles of diffraction and in different planes, are precisely what we should expect on the basis of the theory worked out in the present paper, in view of the asymmetrical character of the haloes for an obliquely-held uniform film exhibited in figure 6. To test the matter even further, a few observations have been made of the angles of diffraction for an obliquely-held film at which a dark ring appears on it. The agreement between theory and observation is here again seen to be satisfactory.

**Table 4.** Angle of incidence, 27° 30′

Direction of observation in the plane of incidence towards the plate

| $(\mu-1)t$ | | Angle of observation | | | | |
|---|---|---|---|---|---|---|
| $4\lambda$ | obs. | 7° 42′ | 23° 18′ | 46° 0′ | | |
| | calc. | 8 50 | 25 40 | 55 0 | | |
| $7\lambda$ | obs. | 4 30 | 10 54 | 17 40 | 28° 10′ | 43° 0′ |
| | calc. | 4 10 | 11 10 | 20 0 | 30 20 | 46 0 |

Direction of observation in the plane of incidence away from the plate

| $(\mu-1)t$ | | Angle of observation | | |
|---|---|---|---|---|
| $4\lambda$ | obs. | 10° 30′ | 27° 6′ | |
| | calc. | 10 9 | 31 10 | |
| $6\lambda$ | obs. | 7 24 | 17 10 | 31° 0′ |
| | calc. | 7 15 | 15 50 | 30 15 |

Direction of observation perpendicular to the plane of incidence

| $(\mu-1)t$ | | Angle of observation | |
|---|---|---|---|
| $5\lambda$ | obs. | 8° 30′ | 23° 10′ |
| | calc. | 8 30 | 21 10 |

Angles of blurring

| $(\mu-1)t$ | | Blurring angle | | |
|---|---|---|---|---|
| | | Figure 2 | Figure 3 | Figure 4 |
| $6\lambda$ | obs. | 1° 30′ | 1° 40′ | |
| | calc. | 1 18 | 1 20 | |
| $5\lambda$ | obs. | — | — | 3° 0′ |
| | calc. | — | — | 2 20 |

## Summary and conclusion

In the present paper, the consequences of the theory of the colours of mixed plates put forward by us have been fully worked out, and quantitative data in its support are furnished. It is shown that the results of theory and experiment are in complete accord. The following are the principal points brought out and explained in the paper: (1) The character of the diffraction-halo observed round a light-source viewed through a mixed plate of uniform thickness; (2) the changes in the appearance of a non-uniform film with the angle of observation and especially the blurring of the rings when viewed in certain directions, and the observed influence of the plane of polarization of the light on the position of the rings; (3) the remarkable asymmetrical character of the haloes observed with obliquely-held plates; and (4) the corresponding phenomena with non-uniform films.

In the final and concluding instalment of the paper, the special effects exhibited by partially or completely dry films of albumen and some further studies of the phenomena of mixed plates will be discussed.

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# On Quetelet's rings and other allied phenomena

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[Plate I]

CONTENTS



## 1. Introduction

When a distant point-source of light is viewed by reflection from a plane mirror silvered on the back, the scattered light surrounding the reflected image of the source exhibits a system of coloured rings, the brilliancy of which is greatly enhanced by purposely dimming the front surface of the mirror, as, for instance, by breathing upon it. These rings (generally referred to in the literature as Quetelet's rings) belong to the same class of diffraction phenomena as the well known "diffusion" rings surrounding the focus of a thick concave mirror discovered by Newton. The generally accepted theory of their formation was put forward originally by Stokes† and was subsequently elaborated by Lommel‡ and Exner§. It is proposed in this paper to draw attention to several novel features

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†*Mathematical and Physical Papers*, **3**, 155–196.
‡*Ann. Phys.* (*Leipzig*) **8**, 193 (1879).
§*Sitzungsberichte* of the Vienna Academy, **90**, 827 (1884), and *Ann. Phys.* (*Leipzig*) **9**, 239 (1880).

which the authors have observed in the course of an experimental study of the phenomena under various conditions. In considering the explanation of these features, it will be shown also that the hitherto accepted theory of Quetelet's rings and other related phenomena requires revision in certain respects.

## 2. Observations of Quetelet's rings at oblique incidences

So far as the authors are aware, no observations have been published of Quetelet's rings with plates on which light is incident at large obliquities. Stokes in his paper* describes an unsuccessful attempt in this direction—"As the angle of incidence increases, the bands become finer and finer, and after they have become too fine to be distinguished by the naked eye, they may still be seen through a small telescope, provided the source of light be sufficiently small.... I saw traces of the bands when the angle of incidence was about 25° 40′, but they were not at all well-formed beyond an angle of about 10° 40′, after which they began to be confounded with rays which shot in all directions from the image of the luminous point." He also suggests that with a thinner mirror the observations might probably have been pushed further.

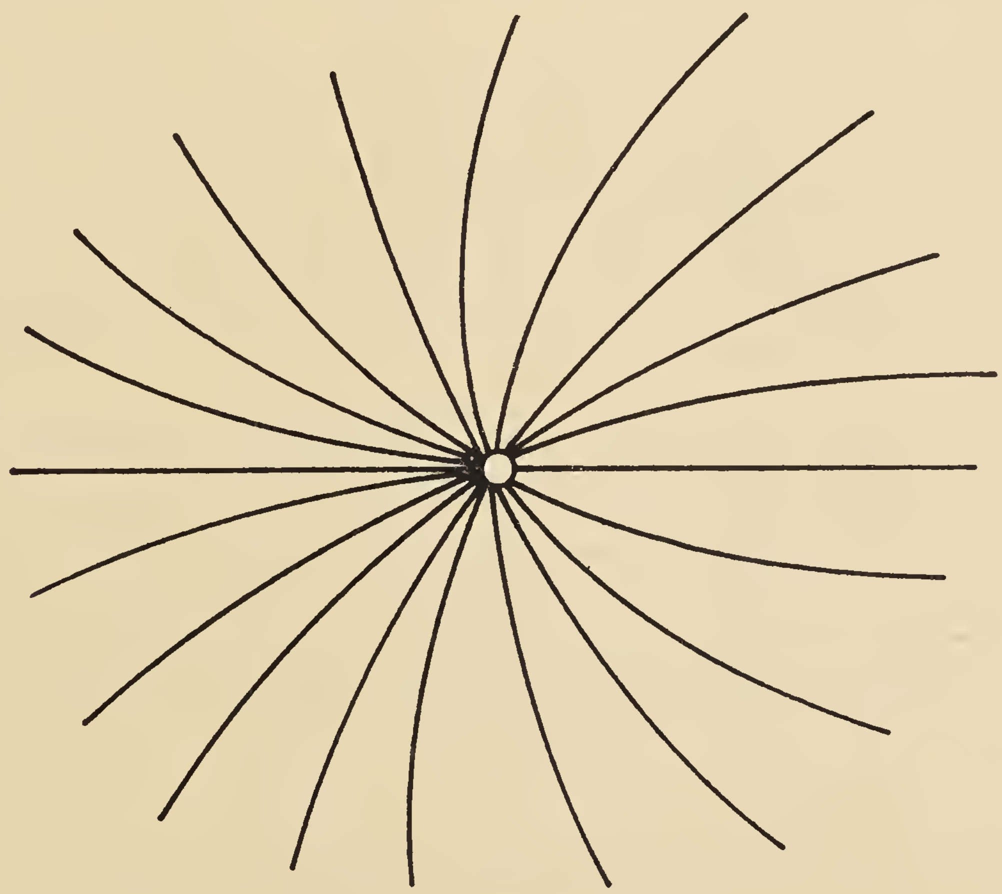

**Figure 1**

As remarked by Stokes, the difficulty in observing the rings given by a plate at any considerable obliquity arises from the structure of the luminous field in which

*Loc. cit.* p. 184.

they should appear. The structure (which is radial* and fibrous in white light, and granular in monochromatic light) arises from the irregular distribution of the diffracting particles or heterogeneity of the film which dims the surface of the mirror and is accordingly fundamental to the experiment. Its effect may be minimized by increasing the aperture of the observing telescope or diminishing the thickness of the mirror, but considerations of convenience and the difficulty in obtaining very thin mirrors with optically plane and parallel faces limit the possibilities in either direction.

The present authors have found two methods of experimenting by which it is possible to push the observations of Quetelet's rings to angles of incidence right up to 90°. The *first* method is to use a front-silvered glass plate (ordinary $\frac{1}{4}$ in. mirror glass does very well), and to coat the surface of a second glass plate with a scattering film and place it face to face with the silvered surface on the first plate, thin slips of paper or mica being placed at the corners between the two plates to adjust the distance between them. In this way, very satisfactory optical surfaces separated by any desired space may be obtained. The authors have found that a light coat of ammonium chloride on the surface of the plate (deposited by volatilization) gives a highly homogeneous and uniform scattering film of which the thickness can be accurately controlled. This gives much more satisfactory results than the film of milk which is usually recommended, the granulation of the field being practically absent.

The *second* method is to use a clear sheet of mica which can be obtained readily and split to any desired thickness, and to coat one of its surfaces with a thin film of ammonium chloride by volatilization. The back of the mica may be silvered if so desired, but this is not essential if a bright source of light be available. As is well known, the surfaces of a good sheet of mica are parallel to an extreme degree of accuracy, and if the sheet be suitably held, it need not appreciably deviate from planeness.

By both methods, it is possible to obtain such small separations between the mirror and the scattering film that it is entirely unnecessary to use an observing telescope, even at very oblique incidences. Gorgeously-coloured rings are seen when the arrangement is held close to the eye, and a small source of light is viewed by reflection from it. Using the methods of observation here described, several interesting results have been obtained which will now be detailed.

## 3. The configuration of the rings

There is a noteworthy difference between the configuration of the rings as seen with the air-film and with the sheet of mica. As is well known, the diameters of

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*With an obliquely-held plate, the streamers instead of running out quite radially from the reflected image of the source, curve unsymmetrically as shown in figure 1.

Quetelet's rings are given by the formula

$$2\mu t(\cos r - \cos\theta) = \pm n\lambda, \tag{1}$$

where $t$ is the thickness of the plate, $\lambda$ the wavelength of the light, $r$ and $\theta$ are the angles which the *regularly transmitted* and *diffracted* rays *inside* the plate make with the normal to it surface, and $\mu$ is the refractive index of the substance of the plate. The achromatic ring for which $r = \theta$ passes through the reflected image of the source. The spacing of the rings depends on the variation only of $\theta$, and is thus practically the same as for Haidinger's rings or the "interference-curves of equal inclination" as they are called, observed by reflection at the surfaces of the plate of an extended source of monochromatic light. The positions of the latter are given by the formula.

$$2\mu t\cos r = \pm n\lambda, \tag{2}$$

$r$ in this case being also the direction *within* the plate of the regularly emergent rays. The corresponding angle of emergence $i$ from the plate is given by the relation

$$\sin i = \mu \sin r. \tag{3}$$

The angular width d$i$ of a ring is given by the relation

$$\mathrm{d}i = \frac{\mu\lambda}{t}\cdot\frac{\cos r}{\sin 2i}. \tag{4}$$

From (4) it is seen that when $\mu \neq 1$, the rings are broad *both* for normal and grazing emergence, and the width of the rings is a minimum for some intermediate direction, whereas if $\mu = 1$, the width decreases continually from normal to grazing emergence. These features in the spacing of the rings are also easily observable with the Quetelet's rings. With the air-film, $\mu = 1$, and the rings decrease continually in width from normal to grazing emergence as indicated by the theory. With the mica however, $\mu \neq 1$, and the rings seen with white light are seen first to become narrower and narrower as the incidence is increased till they reach a minimum width, and then expand till they become very wide again at grazing emergence. Stokes* in his paper predicted from theory that this should occur, but, as already mentioned, he was unable to observe any rings at such great obliquities. It should be mentioned that the feature here dealt with, namely the widening of the rings beyond a certain point, depends only on the angle of *emergence* of the scattered light, and not upon the angle of incidence of the light from the source. Hence, even at normal incidence it is possible to observe the effect. For this special case, it is necessary to use a very powerful monochromatic light-source (i.e., a quartz-mercury vapour lamp with a green filter) and to work in a dark room, so that the light scattered even through 90° can be observed.

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**Loc. cit.* Arts. 23–26 and 29.

A few measurements are here given of the width of Quetelet's rings observed with mica to show the widening of the rings towards the direction of grazing emergence, and to exhibit the agreement of the observations with the theory.

**Table 1.** Quetelet's rings in mica. $\mu = 1{\cdot}6$ approximately.

| Angle of incidence on plate | Observed width of achromatic ring in arbitrary units | Calculated width |
|---|---|---|
| 10° | 38 | 39 |
| 20° | 22 | 21 |
| 30° | 15 | 14·8 |
| 40° | 13 | 12·5 |
| 50° | 12 | 12 |
| 60° | 14·5 | 13·1 |
| 70° | 18 | 17 |

## 4. Quetelet's rings by multiple reflection

In observing the system of rings obtained with the air-film held *obliquely*, a brilliant source of white light being used, it was noticed that besides the usual

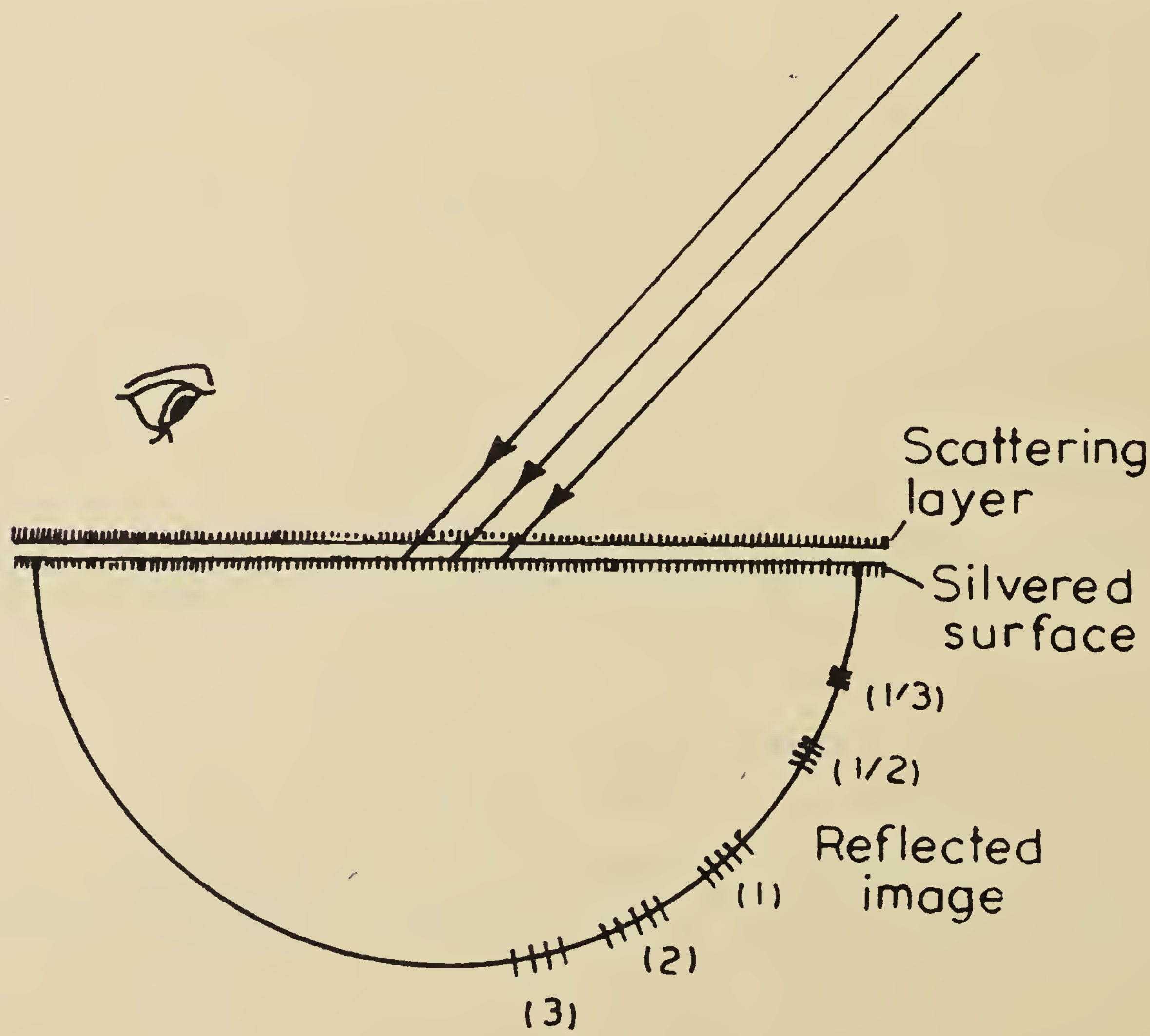



**Figure 2**

system of which the achromatic ring passes through the reflected image of the source, three or four other systems of coloured bands concentric with it were also visible in the field, each with its own achromatic band and the usual colour-sequence, but of different widths. The appearance in the plane of incidence is diagramatically illustrated in figure 2.

The position of the reflected image and the usual system of coloured bands on either side of it is indicated by (1). The symbols ($\frac{1}{2}$) and ($\frac{1}{3}$) indicate two other systems, the fringe-widths of which are smaller roughly in the ratio of the fractions which indicate them. (2) and (3) are two more systems, of which the latter is, in general, somewhat less easy to observe. These are of greater fringe-width than (1).

The explanation of the formation and character of these new systems of Quetelet's rings is interesting. In the usual (primary) system of Quetelet's rings, each of the interfering pencils suffers a single reflexion at the silvered surface and passes through the plate twice. The systems now described arise in a different way, one of the interfering pencils passing twice through the plate and suffering a single reflection, and the other passing four or six times through the plate, suffering multiple reflexion. To illustrate this, the paths for cases (1), ($\frac{1}{2}$), and (2) are shown diagrammatically in figures 3(*a*), (*b*), and (*c*) respectively.

It will be seen from the manner of formation that in the cases ($\frac{1}{2}$) and ($\frac{1}{3}$), the pencil undergoing multiple reflection deviates from the geometrical path at entry, whereas in (2) and (3) this occurs at emergence. In all the cases, the decrease in the intensity of one of the interfering pencils due to multiple reflection is to some extent compensated by the increase in the obliquity of the incidence at which the reflection occurs; but this is much more so in the cases ($\frac{1}{2}$) and ($\frac{1}{3}$) than in the cases (2) and (3), and hence the former systems are much more easily seen. It should be particularly noticed that the scattering film used should be very thin and transparent, in order that these systems of higher order may be well seen.

From the figures showing paths, we may readily write down the formulae

$$2t(\cos r - 2\cos\theta) = \pm n\lambda \qquad 5(\tfrac{1}{2})$$

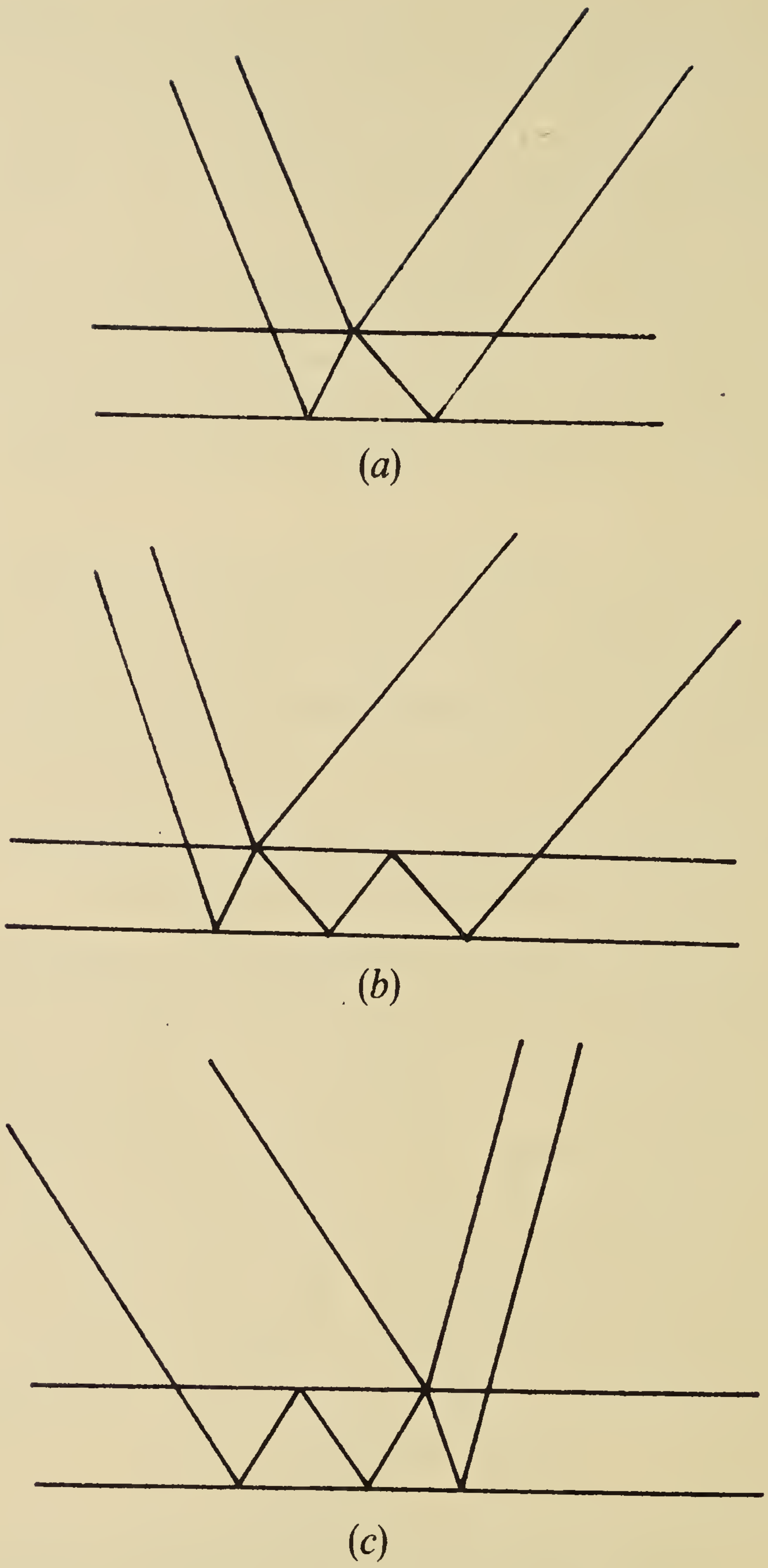
$$2t(\cos r - 3\cos\theta) = \pm n\lambda \qquad 5(\tfrac{1}{3})$$

$$2t(\cos r - \cos\theta) = \pm n\lambda \qquad 5(1)$$

$$2t(2\cos r - \cos\theta) = \pm n\lambda \qquad 5(2)$$

$$2t(3\cos r - \cos\theta) = \pm n\lambda \qquad 5(3)$$

The position of the achromatic centres for the different systems are thus given by the relations $2\cos\theta = \cos r$; $3\cos\theta = \cos r$; $\cos\theta = \cos r$; $\cos\theta = 2\cos r$; $\cos\theta = 3\cos r$. It will be seen that the two latter systems of rings cannot be formed unless $r > \cos^{-1}(\frac{1}{2})$ and $r > \cos^{-1}(\frac{1}{3})$ respectively. The formulae also readily enable the width of rings in the different systems to be calculated, and the results are in agreement with observation.



(a)

(b)

(c)

Figure 3

## 5. Quetelet's rings in crystalline plates

No reference has hitherto been made to the interesting effects that arise in the rings observed with the mica owing to its doubly-refractive property. As is well known, mica is a bi-axial crystal, the planes of cleavage being practically perpendicular to the bisectrix of the angle between the optic axes. In muscovite mica the apparent (external) angle between the axes is about 70°. That special

effects must arise from the doubly refractive property is clear in view of the analogy with the Haidinger's rings observed by reflected monochromatic light which have been recently very fully studied by Chinmayanandam*. It appears that in the latter case we have not *one*, but two sets of interference-curves corresponding respectively to light polarized in and perpendicular to the principal plane, which are approximately ellipses in shape. The curves of blurring or minimum visibility along which the maxima and minima respectively of the two sets are superposed, have the same form as the "isochromatic lines" in polarized light of a plate of twice the thickness†. From the photographs of the Haidinger's rings in mica secured by Chinmayanandam, it is seen that along these curves of minimum visibility the visible ring-system formed by the superposition of two sets of rings appears dislocated—that is, the minima on the side of a curve of minimum visibility run into the maxima on the other side, and *vice versa.*

Examination of the Quetelet's rings in *white light* exhibited by a sheet of mica having one surface dimmed by a film of ammonium chloride, showed somewhat similar phenomena. For this purpose it is desirable to use a fairly bright source of light, and work in a darkened room so that the complete rings may be observed. When the incidence on the mica is normal, nothing special is noticed; but when the mica is tilted somewhat, the ring-system expands, and it will be noticed at once that the coloured circles appear dislocated along certain curved arcs, the bright rings on one side running into the dark rings on the other, and *vice versa.* When the plane containing the optic axes of the mica is parallel or perpendicular to the plane of incidence, the dislocations occur in a symmetrical manner with reference to the diameter of the rings passing through the reflected image of the source; but in other cases the dislocations occur in regions unsymmetrically situated with reference to this line.

It should be understood that the form and position of the lines of minimum visibility in Quetelet's rings are by no means necessarily the same as for Haidinger's rings in the same plate. Though the *spacing* of the rings is practically identical in the two cases, the exact positions of the rings (for either component of polarization) may differ slightly, the formulae giving the absolute positions of the rings being different (see equations (1) and (2) above). This may result in a very considerable shift of the curve along which the maxima and minima respectively of the two sets of rings coincide. The fact that one of the rays on emergence is scattered may also introduce complications in regard to its polarization that do not arise in the theory of Haidinger's rings. The further examination, both theoretical and experimental, of this subject must be reserved for a separate paper.

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* *Proc. R. Soc.* **A95** (Jan. 1919).

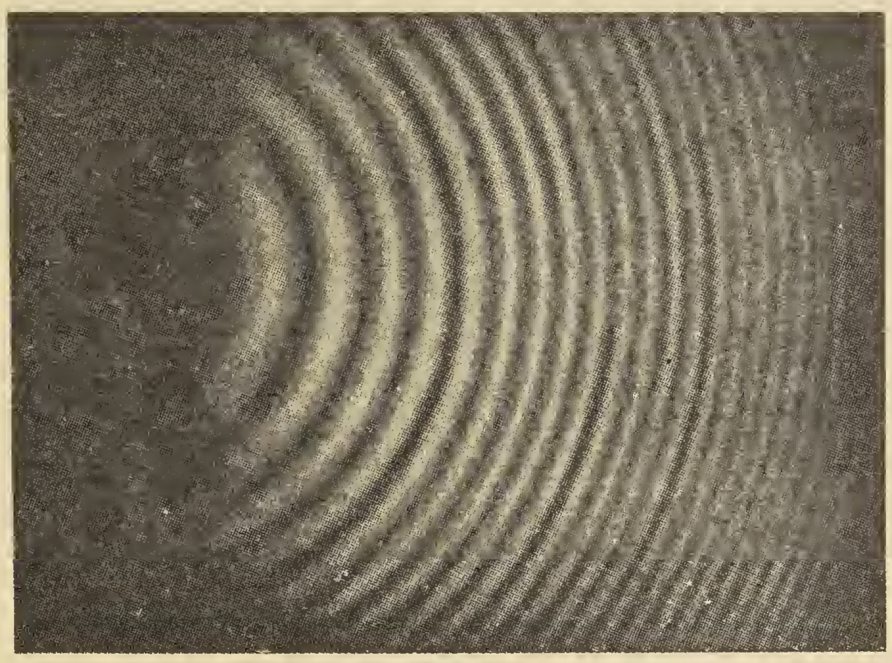
† Chinmayanandam, *loc. cit.*

## 6. Influence of the structure of the scattering film

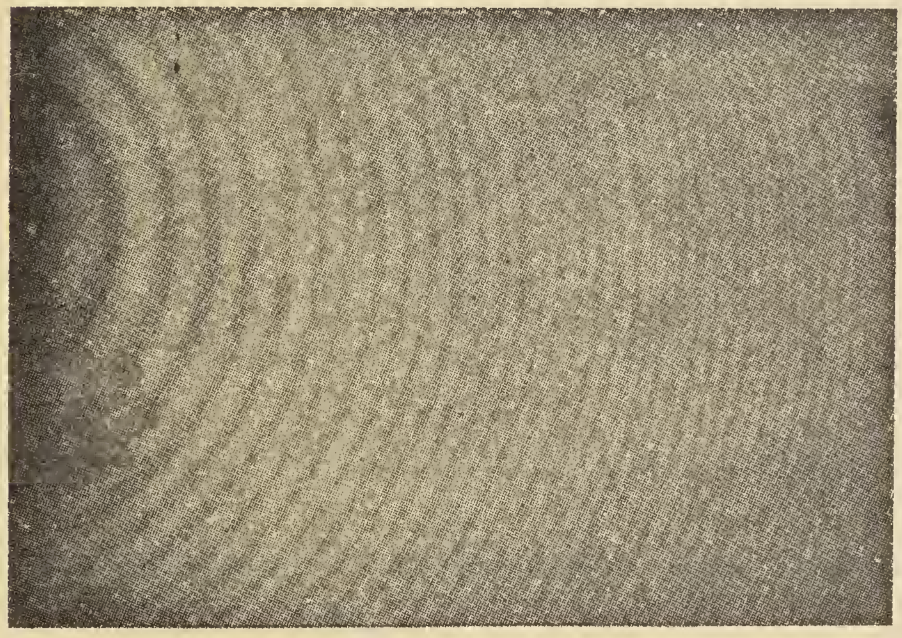
It has already been remarked that the luminous field in which Quetelet's rings appear shows a fibrous or granular structure depending on the irregular distribution on the surface of the plate of the sources of diffracted light, and that this structure may be controlled by altering the aperture of the observing instrument. Apart from this, there is another important respect in which the nature and distribution of the diffracting particles influences the observed phenomena. It seems generally to be supposed that in Newton's Diffusion Rings or in Quetelet's rings as observed in focus in monochromatic light the interference minima are perfectly black; in other words, that the visibility of the rings is the maximum possible. This is, however, very far from being generally true. Observation shows that the visibility is different under different conditions. The most interesting results under this head are obtained when the surface of the mirror is dimmed by a more or less continuous film. The effects are then seen to depend upon the structure of the film.

Probably the easiest way of exhibiting the effect mentioned above is to observe the diffusion rings at the focus of a concave mirror in the usual way, the surface of the mirror being dimmed by breathing on it. The structure of the deposit may be controlled by breathing either less or more heavily and for different intervals. It will be noticed that the lightly-breathed upon mirror shows rings which are rather evanescent no doubt, but which are very vivid and beautifully coloured so long as they are seen. The heavier deposit gives rings which are much duller in appearance. The distribution of the scattered light around the focus is also different in the two cases. When white light (with or without a red glass to approximately monochromatize the light) is used, the effect may be seen with the rings focussed on a screen in the usual way. The effects are, however, far more striking when the monochromatic light of the quartz-mercury vapour lamp with a green filter is used. Instead of receiving the rings on a screen, it is preferable to view them directly by the eye, a fairly large totally-reflecting prism being placed near the focus to make any desired part of the field visible to the observer who takes his stand on one side. The aperture through which the light issues should be as small as possible, the light of the mercury arc being concentrated upon it with a condenser. When the mirror is lightly breathed upon, a very large number of beautifully sharp and clear rings may then be seen. With a heavy deposit the rings are very poorly visible, and may even entirely disappear in the higher orders. Some improvement, however, occurs as the deposit evaporates and is about to vanish completely.

With a tarnish of milk or grease on the mirror similar effects may be noticed, the appearance of the rings deteriorating with the heaviness of the deposit. Figures 4 and 5 in plate I exhibit the difference in the appearances observed with a very thin and a heavier coat of dried milk-and-water respectively. It will be noticed that though the heavier deposit does not distort the rings in any way, it



(4)



(5)

**Figures 4 and 5.** Showing effect of thickness of scattering film on the visibility of the rings. **4.** Thin film, **5.** Thick film.

**Plate I**

diminishes the observable contrast between the maxima and minima. Perhaps the most interesting results are those observed when a film of ammonium chloride is deposited by volatilization on the surface of the mirror. The deposit as initially formed is highly fine-grained in texture and scatters light through very large angles. By breathing lightly on it, the structure of the film may be altered permanently, the ammonium chloride being drawn by the moisture before it evaporates into small microscopic globular masses. Still heavier breathing further makes the structure coarser, semi-crystalline aggregates being formed. These changes are accompanied by corresponding alterations in the distribution of light in the field and in the visibility of the rings.

That these changes of visibility of the rings depend on the magnitude of the phase-changes which occur in the passage of the light through the more or less irregular structure of the film, is made clear by the following experiment. A front-

silvered glass plate and another plate of glass one of whose surfaces is smoothly ground by fine emery, are placed face to face with little slips of paper at the corners to keep them at a fixed distance apart. With this combination, Quetelet's rings cannot be observed at all in the field of light diffused by the glass. When, however, a little water is allowed to enter the space between the plates, the rings can be faintly seen, only the first few rings, however, being visible even in monochromatic light, and the contrast between the maxima and minima is very small. When, however, instead of water, a little benzol is let in between the plates, many more rings are visible, and their appearance is very greatly improved.

It should also be remarked that when observations of Quetelet's rings are made with the air-film, there is a marked and progressive diminution in their visibility as we pass from the case of normal incidence to that in which the film is very obliquely held. This diminution is not due to the structure of the field, as the influence of the latter may be sufficiently reduced by using relatively thin films, and yet the effect continues to persist. This deterioration of the visibility of the rings with increasing obliquity of incidence is also influenced in a very marked degree by the structure of the film. The thinnest and most uniform films show the effect to a much less degree than the relatively coarser films.

## 7. Theory of the phenomena

The effects described in the preceding section are not explicable on the theory put forward by Stokes, Lommel, and Exner. According to these authors, the interferences observed are supposed to be those of the two sets of waves diffracted at the dimmed surface of the mirror, in one case at entry and in another case at emergence. The two diffractions are supposed to occur independently, no given portion of the wave-front being affected *both* at entry and emergence. Stokes shows in his paper that on these assumptions, the two diffracted waves which *each* particle gives rise to should be of equal intensity and are in permanent phase-relation so that the interference-minima should be perfectly black. That this is not always the case shows that the theory is imperfect. The assumption that no given portion of the wave-front is affected *both* at entry and emergence, though plausible when we are dealing with small opaque particles very sparsely distributed on the surface of the mirror, is inappropriate in other cases, as, for instance, when the particles are densely distributed, and is not at all permissible when the diffraction is due to a more or less continuous film of heterogeneous structure on the surface of the mirror. In such circumstances *every* portion of the wave-front is affected *both* at entry and emergence, and the theory developed by Stokes ceases to be applicable.

In the cases in which Quetelet's rings are formed by breathing upon the mirror or by tarnishing its surface with a film of milk or grease or a deposit of ammonium chloride, or otherwise roughening its surface, we should regard the phenomenon as

due to laminar diffraction, and not as a case of diffraction by opaque particles. The film imposes arbitrary variations of phase at different points of the wave-front passing through it, both at entry into and emergence from the mirror, and the problem is to determine the nature of the interferences that arise.

A practicable method of dealing with this case is one analogous to that adopted by Chinmayanandam* in discussing the specular reflection from rough surfaces. Consider the configuration of the wave-front immediately on emergence from the scattering film. It will not be a perfectly plane wave-front, because different portions of it will have been unequally retarded by the random distribution of the particles in the film. We may assume that the deviations of the wave-front from perfect planeness follow the well known law of errors. Certain parts of the wave-front would have traversed a greater thickness of the material of the film, and some less. Let $x$ denote the difference between the average thickness traversed by the whole wave-front and the thickness traversed by a specified element of the wave-front. Then the total area of the portions of the wave-front in advance of or behind the mean for which this difference lies between $x$ and $x + \mathrm{d}x$ may be taken proportional to $\exp(-Ax^2)\mathrm{d}x$. The resultant vibration due to these portions of the wave-front is, therefore, in the direction of regular propagation

$$y = \exp(-Ax^2)\cos\left\{\omega t - \frac{2\pi}{\lambda}\mu x\right\}\mathrm{d}x,$$

if the equation of the vibration for the mean wave-front be $y = \cos\omega t$. The average vibration due to a unit area of the complete wave-front will then be

$$y = \frac{\int_{-\infty}^{+\infty}\exp(-Ax^2)\cos\left\{\omega t - \frac{2\pi}{\lambda}\mu x\right\}\mathrm{d}x}{\int_{-\infty}^{+\infty}\exp(-Ax^2)\,\mathrm{d}x}.$$

This will make the average intensity of the transmitted light

$$I = \exp(-2\pi^2/A)\left(\frac{\mu}{\lambda}\right)^2 = \exp[-B(\mu/\lambda)^2], \tag{6}$$

where the constant $B$ is determined by the structure of the film. The remainder of the energy is scattered.

We have thus analysed the disturbance when it has entered the film into two portions: (*a*) a regularly-transmitted portion of reduced intensity, and (*b*) a portion scattered in various directions in a manner depending on the structure of the film†. This complex disturbance is reflected from the silvered surface, and in

***Phys. Rev.* **13** 96 (Feb. 1919).

†In regard to the distribution of energy in different directions in the light scattered in passing through a heterogeneous medium, see N K Sethi, *Proc. Indian Assoc. Cultiv. Sci.* **6** 128 (1920).

passing out through the film is again dealt with in the same way by it. Finally, when the light emerges from the mirror, we have four types of disturbance: (*a*) a regularly-transmitted wave, (*b*) a scattered-transmitted disturbance, (*c*) a transmitted-scattered disturbance, and (*d*) a twice-scattered disturbance. Of these, (*a*) gives the geometrical reflected image of the source; (*b*) and (*c*) are in permanent phase-relation, and, being of equal intensity, give interferences of maximum visibility; (*d*) appears as an overlying general illumination which diminishes the visibility of the interferences due to (*b*) and (*c*). The importance of (*d*) relatively to (*b*) and (*c*) depends on the transmitting power of the film as defined by equation (6), and also on the manner in which the energy of the scattering light is distributed in different directions. A relatively opaque film or a film that concentrates most of the scattered light in a small solid angle, would give interferences of smaller visibility than a very transparent film or a film that scatters through very wide angles. It can easily be shown that in oblique transmission through the film its transparency as determined by the phase-differences in passage through it must decrease. Hence the visibility of the interferences must decrease with increasing obliquity of incidence. This is exactly what is observed. Further, the improvement up to a certain stage in the visibility of the interferences produced by interposing a liquid between the rough surface and the mirror is also what is to be expected on the present theory as the operative phase-differences are diminished.

## 8. Summary

The following are the principal results of the investigation described in the paper:

(*a*) Two simple methods of experimenting are described in which a highly homogeneous scattering film of ammonium chloride on the surface of a plate enables Quetelet's rings to be satisfactorily observed at all incidences up to 90°.

(*b*) An important difference is observed in the configuration of the rings in the cases in which the refractive index of the material of the plate is equal to or greater than unity. In the former case the width of the rings diminished continually from normal to grazing incidence, whereas in the latter the rings first contract, reach a minimum width and widen out again.

(*c*) A new class of Quetelet's rings has been observed in which scattered light which has passed twice through the plate interferes with light which has traversed the plate four or six times, undergoing multiple reflection and scattering.

(*d*) Observations of Quetelet's rings with a sheet of mica of which one surface is dimmed by a film of ammonium chloride show interesting effects, due to the doubly refracting properties of the plate. The rings seen by white light show dislocations along curved arcs of minimum visibility analogous but not quite similar to those observed in Haidinger's rings by monochromatic light.

(*e*) The interferences observed in Quetelet's rings and other allied phenomena do not always have maximum visibility. The contrast between the maxima and minima varies with the magnitude of the phase-differences introduced by passage of the light through the scattering film.

(*f*) A new treatment of the theory of Quetelet's rings is developed, applicable to the case of a continuous scattering film, which enables the features mentioned in (*e*), as also the effect of obliquity on the visibility of the rings, to be satisfactorily explained.

22 Oxford Road, Putney, London, S.W.15
31 May 1921

*Nature* (*London*) **107** 714 (1921)

# The colours of breathed-on plates

The phenomena of breath-figures on glass are of considerable interest, and have been written upon in the columns of *Nature* (*London*) by the late Lord Rayleigh, Dr John Aitken, and others. One specially interesting aspect of the subject to which I have recently devoted some attention is the explanation of the beautiful optical effects exhibited by breathed-on plates of glass. If a clean, cold plate of glass is lightly breathed-on and then held in front of the eye, and if a small distant source of light is viewed through it, coloured haloes will be seen surrounding the source. The characteristic feature of the halo exhibited by a moderately heavy (but not too heavy) deposit is that the outermost ring in it is achromatic, with a reddish or brown inner margin, followed inside by a succession of rings of various colours. As the film of moisture evaporates, the halo contracts and the coloured rings move inwards, ultimately disappearing at the centre of the halo. The entire halo presents a radiating fibrous structure.

The explanation of these phenomena presents some difficulties. One is tempted to suppose (as, indeed, Donlé and Exner have already) either that the minute droplets of water condensed on the plate which diffract the light are of approximately equal size or that they are arranged at more or less constant distances from each other. A microscopic examination of the condensed film shows, however, that neither of these suppositions is anywhere near the truth. The size of the individual droplets shows a variation of several hundred per cent, and their arrangement in the film is entirely irregular, being determined presumably by the presence of invisible condensation nuclei on the surface of the plate—a view that is strongly supported by the fact that successive deposits on the plate are seen under the microscope to preserve the same configuration with a surprising degree of accuracy. Further, if the size of the droplets were the determining factor in the production of the diffraction haloes, it would be difficult to understand why as they evaporate the rings in the halo should *contract* in size.

These facts necessitate an entirely different supposition regarding the element of regularity in the film which enables it to give rise to a recognisable system of coloured diffraction haloes. Measurements I have made seem to show that the droplets in the film—whether large or small—have practically all the same angle of contact with the surface of the plate, this angle of contact diminishing as the film evaporates. The formation of the coloured haloes is, on this view, due to the passage of the light *through* the minute lens-shaped droplets; the maximum deviation of the light determined by the common angle of contact fixes the position of the outermost achromatic halo, and the colour-sequence following

within it would be practically the same for all the droplets irrespective of their size. This would also furnish a satisfactory explanation of the contraction of the halo as the film evaporates.

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*Nature (London)* **109** 105–106 (1922)

# The colours of tempered steel

The well-known and characteristic tints that appear on the surface of a tarnishable metal when it is heated in contact with air have been usually regarded as interference colours due to the formation of a thin film of oxide on the surface of the metal. The correctness of this explanation has, however, recently been questioned (A Mallock, *Proc. R. Soc. London*, 1918), and rightly so, as a continuous film on a strongly reflected surface cannot on optical principles be expected to exhibit such vivid colours as those observed.

I have recently made some observations which shed a new light on this subject. It is found that the *missing colours* complementary to the tints seen by reflected light appear as light *scattered* or *diffracted* from the surface of the metal. In other words, if a plate of blue-tempered steel be held in a beam of light and viewed in such a direction that the regularly reflected light does not reach the eye, the metal shows a straw-yellow colour, and not the usual blue. It will be understood that the scattered light, being distributed over a large solid angle, appears much feebler than the regularly reflected colour, and in order to observe the effect satisfactorily the metal should have a smoothly polished surface before being heated up. Scratches and other irregularities show the ordinary colour of the film, and not the complementary tint. The most attractive effects are those exhibited by a heated copper plate, both on account of the vividness of the colours and on account of the ease with which the surface can be given a satisfactory polish.

It is clear from the observations mentioned above that the colours under discussion are in the nature of *diffraction effects* arising from a film which is not continuous, but has a close-grained structure. Interesting effects are observed when the surface of the illuminated plate is viewed through a nicol, the colour and intensity of the scattered, as well as of the regularly reflected, beams varying as the nicol is rotated about its axis. The most striking effect is obtained when the direction of observation is nearly parallel to the surface of the plate. The scattered light in this case is nearly completely polarised, and the colour of the regularly reflected light changes nearly to its complementary when the nicol is turned through 90°. The phenomena strongly recall to mind the observations of R W Wood on the colours of a frilled collodion film on a silvered surface, which have been discussed by the late Lord Rayleigh (*Philos. Mag.*, November, 1917), and it seems probable that the explanation of the phenomena will ultimately be found to be somewhat similar in the two cases.

*C V RAMAN*

210 Bowbazaar Street
Calcutta, India
11 October

*Nature (London)* **108** 242 (1921)

# A method of improving visibility of distant objects

The idea may have been suggested before, but I believe it is not generally known and appreciated how very much the power of distinguishing detail in a distant object, and especially of perceiving it in its natural colours, may be improved by the simple device of fitting a small Nicol's prism in the eye-piece end of the observing telescope. The Nicol serves to cut off a great deal of the blue atmospheric "haze" which usually envelops a distant view, and mostly consists of polarised light. Details which are usually lost in the haze, such as the colour of distant rocks or of the vegetation growing upon them, then stands out in a very striking way.

It may also be worth mentioning that the visibility of the horizon at sea, especially in a haze, may often be wonderfully improved by a similar device. In this case the result is due in part to a suppression of the reflection from the surface of the water as seen through the Nicol's prism.

It is hoped that these observations will not be merely a scientific curiosity, but may find a practical application.

*C V RAMAN*

S S Narkunda, near Aden
18 September

*Nature* (*London*) **110** 700–701 (1922)

# The spectrum of neutral helium

A most significant feature of the success of the quantum theory in explaining the sequence of radiation-frequencies forming the Balmer type of series in the spectra of hydrogen and ionised helium is that it also offers an intelligible explanation of the differences in the intensities of the successive lines in the sequence, and that its postulates are not inconsistent with the known facts regarding the sizes of the atoms in their normal states. The fundamental assumption in the theory is that the states of the atom represented by increasing quantum numbers depart more and more from the normal state, and the greater intensities of the earlier lines in a sequence are readily understood as due to the greater probability of transitions actually occurring between states represented by smaller quantum numbers.

Any attempt to build up a theory of spectra which ignores these fundamental considerations must be received with caution. The remark just made appears to be particularly applicable to Dr Silberstein's attempt (*Nature*, August 19) to explain the spectrum of neutral helium on the assumption of the independence of the electrons. Looking over the list of frequencies given in his letter, and comparing them with the maps and tables of the helium spectrum contained in Prof. Fowler's report, it is noticed at once that the well-known intense yellow line of helium at $\lambda 5876$, which is the first member of the diffuse series of doublets, is given by Dr Silberstein the formula 9/6·15/6, while other lines which are of vanishingly small intensity in comparison with it are assigned formulae with much smaller quantum numbers. For example, the doublet at $\lambda 3652$, which is the seventh in the sharp series and so faint that it fails to appear in the photographic reproduction of the spectrum, is assigned the formula 6/4·9/5. Similarly, the first diffuse singlet at $\lambda 6678$ gets the formula 9/6·24/7, while the fifth in the same series is indicated by 7/5·19/5, that is, by much smaller quantum numbers, while it is actually a far fainter line than the other.

These facts naturally lead one to question whether Dr Silberstein's proposed new combination principle has any real physical basis or significance. To settle this point, I undertook a careful survey of the figures and carried out a series of computations with the aid of my research student Mr A S Ganesan, and have come to the conclusion that the approximate agreements between the calculated and actual frequencies are merely fortuitous arithmetical coincidences. This is clear from the following facts brought out by a survey of the figures:

(1) The proposed combination formula with its freedom of choice of four numbers gives a very large number of lines out of which it is possible to pick out a

few coinciding approximately with practically any arbitrary series of frequencies which may be proposed, the accuracy of fit increasing as the quantum numbers chosen are increased.

2 The coincidences between the calculated and observed frequencies are most numerous and accurate precisely in the region where the density of either series of frequencies is greatest, which is what we should expect according to the laws of chance.

(3) It is not, in general, possible to get a good fit for the earlier members of a line-series except by using large quantum numbers. This is what we should expect if the coincidences were fortuitous, as the frequency-differences between successive lines are greatest in the beginning of a series.

(4) More than one combination of quantum numbers will fit a given line tolerably well. For example, the $D_3$ line of helium is also represented fairly well by 13·21/5·12.

(5) The quantum numbers giving the best fit do not fall into any regular sequence when arranged either according to the frequencies of the lines or their intensities, nor do they show any characteristic differences for the singlet and doublet series.

Needless to say, the foregoing remarks apply with even greater force to the case of the lithium atom when a choice of six numbers is permitted.

Finally, it may be remarked that the Rydberg constant 109723 chosen by Dr Silberstein is appropriate only to the case of the ionised helium atom in which only one electron is coupled to the nucleus. If both electrons exert reactions on the nucleus and move simultaneously, the value of the Rydberg constant cannot remain the same in general.

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210 Bowbazaar Street, Calcutta
18 October 1922

*Astrophys. J.* **57** 243–247 (1923)

# On the spectrum of neutral helium—I*

C V RAMAN and A S GANESAN

ABSTRACT

*Silberstein's formula for the lines of neutral helium*—As objections to the validity of the formula

$$\nu = 4N\left\{\left(\frac{1}{n_1^2}-\frac{1}{m_1^2}\right)+\left(\frac{1}{n_2^2}-\frac{1}{m_2^2}\right)\right\}$$

it is pointed out that the ionizing potential computed by means of this formula does not agree with that given by experiments; that no definite principle of selection has been given to indicate observable lines; that there is no arrangement of lines as regards series relationship, division into singlet and doublet systems, or of intensities. The difference between the actual and the calculated frequencies is shown to be the same as that to be expected on the assumption that the coincidences are purely accidental; and the same frequencies can be obtained with different sets of numbers.

In his paper on this subject contributed to the September number of the *Astrophysical Journal* (**56**, 119, 1922), Dr Silberstein has suggested a formula for the explanation of the spectrum of neutral helium. This formula may be written as follows:

$$\nu = 4N\left\{\left(\frac{1}{n_1^2}-\frac{1}{m_1^2}\right)+\left(\frac{1}{n_2^2}-\frac{1}{m_2^2}\right)\right\},$$

where $n_1, n_2, m_1, m_2$ are four independent integers such that $m_1 > n_1$ and $m_2 > n_2$. "This formula amounts, obviously, to putting $\nu = \nu_1 + \nu_2$ where $\nu_1$ and $\nu_2$ are frequencies of any two lines of ionized helium, observed or only theoretical. This would then be a new kind of combination principle, the sum of the frequencies belonging to one atomic system (ionized helium) giving the frequency for another system (neutral helium)." With this formula Dr Silberstein has succeeded in explaining 84 of the total 111 observed lines of the helium spectrum, to a very close approximation.

Now, the question is whether the formula has any physical basis or significance. Silberstein derives it on the assumption that "the mutual perturbation of the two electrons is practically nil or negligible", and in support of this says that "as a matter of fact we have no good evidence that the electrons, especially as trabants of the nucleus within the atom, do act upon each other at

all, and some bold modern physicist, encouraged by the recognized prohibition to radiate, might deny the electrons the right to interact while they are busy obeying the orders of the central body driving them around on stationary orbits".

But there are some very weighty objections against the formula and the results derived therefrom.

The ionization potential as calculated with this formula is quite different from that which is actually observed. The ionization potential is the work per unit charge required in moving an electron out of the influence of the atom. On the assumption that the two electrons do not act on each other, the ionization potential would obviously be

$$V = \frac{4 \times 109723{\cdot}2}{8102} = 54{\cdot}2 \text{ volts.}$$

But actually it is known that to doubly ionize the helium atom about 80 volts are necessary.

The formula, as it stands, represents a very large number of lines and there is absolutely no principle of selection followed which prima facie indicates the lines to be chosen and those to be rejected. As a matter of fact, the selection actually made appears to be more or less haphazard, the only aim being to choose such numbers as happen to give values coinciding most closely with the observed values. The want of a definite principle of selection is a strong objection against accepting the proposed formula as having any physical significance.

Again, as is well known, the observed lines of the helium spectrum fall into certain definite series which Silberstein dismisses as empirical, but which nevertheless have a definite physical meaning. Thus in each series as we pass from the less to the more refrangible part of the spectrum the lines become more and more crowded, grow fainter and fainter, and tend to converge to a definite limit. But Silberstein's formula shows no arrangement either with reference to series relationships or with reference to singlet and doublet systems of lines.

Nor does it suggest any arrangement with regard to the intensity of the lines. Bohr's theory, which has successfully explained the spectra of hydrogen and ionized helium, suggests that the probability of electrons remaining in distant orbits is very small and that more electrons are to be found in orbits closer to the nucleus. And from the analogy with the spectra of hydrogen and ionized helium, we should expect that more intense lines are obtained by using comparatively smaller quantum numbers than for the less intense ones. In Silberstein's table of values some of the first lines of great intensities are not to be seen at all (probably no combination within the limits prescribed for $m$ and $n$ gives them), and of the rest, more frequently than not, the more intense lines are obtained by larger quantum numbers than the less intense ones. Table 1 gives some examples.

Many of the lines can be obtained by numbers other than those chosen by Silberstein (confined, of course, between his limits $m \ngtr 32$ and $n \ngtr 9$), and in some

Table 1

| Observed $\nu$ | Intensity | Quantum numbers, and calculated $\nu$ |
|---|---|---|
| 9231·86 }<br>9230·83 } | 200 (P.d.) | Not included |
| 25708·63 | 10 " | Not included |
| 31361·12 | 8 " | $\left(\frac{16}{4}\cdot\frac{19}{8}\right)$ 31358 |
| 33944·75 | 6 " | $\left(\frac{15}{4}\cdot\frac{30}{7}\right)$ 33950 |
| 37537·5 | 1 " | $\left(\frac{4}{3}\cdot\frac{18}{5}\right)$ 37536 |
| 37798·22 | " | $\left(\frac{4}{3}\cdot\frac{20}{5}\right)$ 37794 |
| 41150 | 5 (S.d.) | Not included |
| 21211·35 | 3 " | $\left(\frac{6}{5}\cdot\frac{16}{5}\right)$ 21205·5 |
| 27374·48 | 1 " | $\left(\frac{6}{4}\cdot\frac{9}{5}\right)$ 27377 |
| 4857·34 | 20 (S.s.) | $\left(\frac{10}{7}\cdot\frac{15}{4}\right)$ 4856·7 |
| 22528·65 | 1 " | $\left(\frac{7}{5}\cdot\frac{11}{5}\right)$ 22527·4 |

cases these agree more closely with the observed values than those given by Silberstein. Thus to give a few examples (table 2).

The average difference between the actual frequencies of the helium lines and the values nearest to them given by Silberstein's formula is exactly what we should expect from the simple theory of probability on assuming the coincidences to be fortuitous. Thus for instance, if $m \ngtr 32$ and $n \ngtr 9$, the total number of lines obtained with Silberstein's formula, lying between $\nu = 28000$ and $\nu = 29000$, is found to be 161. If these lines were evenly distributed between these limits, the interval between successive lines would be $1000/161 = 6{\cdot}1$ and the maximum difference of any value of $\nu$ between 28000 and 29000, chosen at random, and the nearest number in the list would be 3·0. The minimum error being zero, the average error would be 1·5. Taking the 19 observed lines of helium between these limits and finding for each the nearest number from the list, the average actual error is found to be 2·6; that is, actually greater than the number indicated on the

**Table 2**

| Observed | Silberstein's values | Other values |
|---|---|---|
| 24830 | $\left(\frac{13}{4}\cdot\frac{n}{n}\right)$ 24834 | $\left(\frac{10}{11}\cdot\frac{11}{9}\right)$ 24833 |
| 26938 | $\left(\frac{30}{4}\cdot\frac{n}{n}\right)$ 26943 | $\left(\frac{16}{5}\cdot\frac{20}{6}\right)$ 26935·4 |
| 28574 | $\left(\frac{4}{3}\cdot\frac{16}{7}\right)$ 28577 | $\left(\frac{11}{4}\cdot\frac{26}{9}\right)$ 28572·8 |
| 36208 | $\left(\frac{5}{4}\cdot\frac{20}{4}\right)$ 36208·7 | $\left(\frac{10}{4}\cdot\frac{10}{5}\right)$ 36208·7 |
| 36592 | $\left(\frac{5}{4}\cdot\frac{25}{4}\right)$ 36603 | $\left(\frac{17}{4}\cdot\frac{17}{6}\right)$ 36585 |
| | | $\left(\frac{13}{4}\cdot\frac{32}{6}\right)$ 36596·7 |
| 19824 | $\left(\frac{5}{4}\cdot\frac{14}{6}\right)$ 19827 | $\left(\frac{6}{}\cdot\frac{23}{9}\right)$ 19828 |
| | | $\left(\frac{14}{5}\cdot\frac{22}{9}\right)$ 19828 |
| | | $\left(\frac{28}{6}\cdot\frac{24}{7}\right)$ 19827 |
| 20358 | $\left(\frac{5}{4}\cdot\frac{16}{6}\right)$ 20352 | $\left(\frac{17}{5.}\cdot\frac{20}{9}\right)$ 20358·3 |
| | | $\left(\frac{11}{5}\cdot\frac{32}{8}\right)$ 20357·6 |
| 27508 | $\left(\frac{9}{4}\cdot\frac{18}{8}\right)$ 27516 | $\left(\frac{11}{4}\cdot\frac{16}{9}\right)$ 27507·6 |

assumption that the coincidences are purely fortuitous. This is what we should expect; for, owing to the non-even distribution of the numbers, there are certain gaps with differences of about 20 to 25, and some 3 of the 19 lines falling in these gaps eventually increase the average error. From Silberstein's table the average error for the 15 lines which he has given between these limits is about 3·5. This is due to the fact that there are combinations other than Silberstein's that are nearer the observed values.

These facts clearly show that coincidences between the actual frequencies and those given by Silberstein's formula must all be regarded as purely fortuitous and having no physical significance whatever.

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*Astrophys. J.* **59** 61–63 (1924)

# On the spectrum of neutral helium—II[1]

C V RAMAN and A S GANESAN

ABSTRACT

*A rejoinder to Dr Silberstein's reply*—The two points raised by Dr Silberstein in reply to the criticisms of his combination formula are answered. The figures have been recalculated, taking the maximum limit of the quantum numbers to be that proposed by Dr Silberstein himself, and the number of fortuitous coincidences to be expected between the observed lines of the helium spectrum and those given by the formula is calculated and found to agree fairly well with the actual number. The view that the coincidences noticed are due to mere chance is maintained.

In his rejoinder[2] to our criticism of his paper[3] on this subject Dr Silberstein has attempted to dispose of our contention that the approximate coincidences between some of the lines of the helium spectrum and those given by his combination formula are purely fortuitous. He raises two points in his rejoinder. The first is that we took the limits of the quantum numbers to be

$$3 \leqq n \leqq 9 \quad \text{and} \quad 4 \leqq m \leqq 32 \tag{1}$$

and thus obtained a high estimate of the probability of chance coincidences. To find whether this reply meets our objection, we have recalculated our figures, confining ourselves to the limits

$$3 \leqq n \leqq 8 \quad \text{and} \quad 4 \leqq m \leqq 20 \tag{2}$$

now proposed by Dr Silberstein himself as suitable for a test of the correctness of his views. Further to clinch the matter, a calculation has been made, as explained below, of the number of fortuitous coincidences to be expected.

The first step in the work is to make a table of all the lines given by the combination formula, satisfying the conditions (2) and lying in the frequency interval

$$19800 < \gamma < 37800. \tag{3}$$

We find there are 760 lines, not 631 as per Silberstein. It is found further that the lines are not distributed uniformly throughout the whole interval of frequency.

[1]Reprinted courtesy of (author) and *The Astrophysical Journal*, published by the University of Chicago Press; © (1924) The American Astronomical Society.

[2]*Astrophys. J.* **57**, 240 1923.

[3]*Ibid.*, **56**, 119, 1922.

There are numerous small gaps and a small number of relatively large gaps. Thus one-third the number of gaps is less than 10, about two-thirds the number are less than 25, while only 10 per cent of the gaps exceed 50. Within the frequency limits given by (3), which are separated by a gap of 18,000, there are 96 observed lines of the helium spectrum. To determine how many chance coincidences may be expected between these and the 760 lines given by the formula, we may imagine the latter to be represented by points on a straight line, and that 96 shots are fired at random so as to hit the line. If there are $N$ gaps of average interval $x$, the expectation of the number of shots that will hit some of these gaps is evidently

$$\frac{Nx}{18000} \times 96.$$

Further, if any shot falls within a frequency gap $x$, the maximum distance between it and the nearest point on the straight line is $x/2$. Actually all distances between 0 and $x/2$ are equally probable, and if $y$ be the number of shots that may be expected to hit the gap $x$, we may divide them further into $n$ groups of $y/n$ shots each, each group hitting the line at distances of $x/[2(n+1)]$, $2x/[2(n+1)], \ldots nx/[2(n+1)]$ from the nearest point. In practice it is sufficient to take $n$ moderately large, say 5. In this way, by taking the different gaps in the line, it is possible to work out the number of shots that may be expected to hit the line within a specified distance from the nearest point. The argument may be illustrated by the following example. The number of frequency gaps between 20 and 25 is 83. The aggregate gap interval is 1925. The number of lines that may be expected to fall within this gap is

$$\frac{1925}{18000} \times 96 = 10.$$

**Table 1**

| Permissible error | Fortuitous fits | | Permissible error | Fortuitous fits | | Permissible error | Fortuitous fits | |
|---|---|---|---|---|---|---|---|---|
| | Calculated | Actual | | Calculated | Actual | | Calculated | Actual |
| 1 | 3 | 7 | 11 | 61 | 65 | 21 | 85 | 87 |
| 2 | 9 | 15 | 12 | 65 | 69 | 22 | 87 | 87 |
| 3 | 17 | 24 | 13 | 68 | 70 | 23 | 88 | 87 |
| 4 | 27 | 33 | 14 | 72 | 73 | 24 | 89 | 88 |
| 5 | 33 | 42 | 15 | 73 | 74 | 25 | 90 | 89 |
| 6 | 43 | 47 | 16 | 77 | 76 | 26 | 92 | 89 |
| 7 | 49 | 49 | 17 | 79 | 79 | 27 | 93 | 90 |
| 8 | 54 | 51 | 18 | 81 | 80 | 28 | 93 | 92 |
| 9 | 56 | 55 | 19 | 82 | 83 | 29 | 95 | 92 |
| 10 | 60 | 63 | 20 | 83 | 87 | 30 | 96 | 93 |

Of these 10 lines we can expect 2 to fit the nearest one with an error of 1·9, 2 with an error of 3·8, 2 with 5·7, 2 with 7·6, and 2 with an error of 9·5. Calculating in this way, we have constructed table 1, giving the relations between the permissible error and the fits calculated and actual.

The table shows that except for small errors less than 3, the agreement between the calculated and the actual number of fits is nearly perfect. Hence, even confining ourselves to the conditions given in (2), we find that the coincidences are accidental, and our original contention still holds.

Silberstein says that "44 lines coincide with observed lines whose total is 96. The mean deviation is $\delta\nu = 2{\cdot}57$. Now a straightforward computation will show that the probability of such an event considered as a fortuitous set of coincidences is.... well below $10^{-13}$." It is a matter for regret that Dr Silberstein does not explain clearly in his reply to our criticism what his "straightforward method of computation" is. In view of the figures given above, it would seem that there must be some fundamental error which vitiates his method of calculation. Perhaps he has overlooked the fact that the theoretical lines according to his formula are not uniformly distributed, but fall into groups, and that this must profoundly affect the probability of random coincidences. But if his formula claims to explain the spectrum of helium, why should 44 frequencies alone coincide and not all the 96?

Calcutta
22 August 1923

*Nature (London)* **115** 946 (1925)

# Anomalous dispersion and multiplet lines in spectra

Recently, H B Dorgelo has carried out a series of measurements of the intensities of the components of multiple spectral lines (Dissertation, Utrecht, 1924) and obtained results of great interest. He found that the doublets of the sharp series of the alkalies had a 2:1 ratio of intensity, the triplet components of the sharp series of the alkaline earths had a 5:3:1 ratio, the triplets of the sharp series of a sextet system had a 4:3:2 ratio, and the triplets of an octet system a 5:4:3 ratio. The theoretical interpretation of this result has been discussed by Sommerfeld ("Atombau", Fourth Edition, p. 649) and by Ornstein and Burger (*Z. Phys.*, **24**, 41 (1924)). As illustrations of Dorgelo's work may be quoted the case of the triplets of manganese 6021, 6016, 6013, which show a 4:3:2 intensity ratio.

Probably the best known case of the existence of simple intensity relationships of this kind are the two D-lines of sodium, for which a 2:1 ratio has long been shown. The anomalous dispersion of sodium vapour has been extensively studied, notably by Roschdestwensky, who found (*Ann. Phys.* (*Leipzig*), **39** (1912)) that of two constants $a_1$, $a_2$ in the dispersion equation

$$n = 1 + \frac{a_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{a_2\lambda^2}{\lambda^2 - \lambda_2^2},$$

where $\lambda_1$ and $\lambda_2$ are the wavelengths of the two D-lines, $a_1$ was just twice as large as $a_2$. Dorgelo's work suggests that, in the case of multiplet lines, similar numerical relationships between the constants of anomalous dispersion should be found. Unfortunately, very little in the way of quantitative data on anomalous dispersion is available except in the case of the alkali metals.

Perhaps the best work in this direction is that of A S King at Mount Wilson, who with the electric furnace studied the anomalous dispersion of iron, chromium, titanium, and manganese lines (*Astrophys. J.*, **45**, 254 (1917)). An enlarged photograph of the anomalous dispersion due to the manganese triplet 4031, 4033, and 4035, which belongs to the sextet system, reproduced with King's paper, has been examined, and it is found that the dispersion constants $a_1$, $a_2$, and $a_3$ of the lines deduced from the photograph agree closely with the 4:3:2 ratio to be expected on theoretical grounds. In the case of the chromium triplets 5208, 5206, and 5204, Dorgelo obtained experimentally an intensity ratio of 100:72:45, while King's photographs give the ratio of anomalous dispersion to be roughly as 100:75:50, which is a fair agreement. A careful study of the original negatives secured at Mount Wilson may be suggested as likely to furnish further data regarding these interesting spectral relationships.

*C V RAMAN*
*S K DATTA*

*Astrophys. J.* **56** 29–33 (1922)

# On Einstein's aberration experiment*

C V RAMAN

ABSTRACT

*Aberration experiment proposed by Einstein to decide between the theories of light.*—According to Einstein, if light from swift canal rays were focused on a slit and the transmitted rays were made parallel and were then focused on cross hairs, the image of the slit would be shifted when a plane-parallel layer of a dispersing medium such as $CS_2$ is placed in the path of the parallel rays, because, due to the motion of the canal rays, the light rays on one side of the beam would have a shorter wavelength than those on the other side and hence, if the ordinary wave theory is correct, the wave front should be rotated. It is here shown, however, that this conclusion is based on a misconception of the behaviour of such wave fronts; that the position of the final image is independent of the relative wavelengths of the different light rays; and that, therefore, no shift is to be expected either according to the wave theory or according to any other theory of light. Hence the experiment cannot help discriminate between the rival theories.

In a recently published communication† Einstein has suggested an experiment which according to him furnishes a method for deciding between the two alternative hypotheses regarding the nature of the elementary process of light-emission now holding the field, viz., the classical or undulatory theory in which light-waves are regarded as given out continuously from the luminous atom, and the quantum theory in which light is emitted as the result of an explosive act involving a definite energy-change in the atom and uniquely determining its frequency. It is proposed here briefly to summarize Einstein's paper and then to examine his arguments. It will be shown that his discussion of the phenomena to be expected is at fault and that in reality no effect is indicated by either theory and hence the proposed experiment will not serve to differentiate between them.

The proposed experiment is based upon J Stark's well known observation that the light emitted by the luminous atoms in the *Kanalstrahlen* exhibits the Doppler effect, the magnitude of the shift of wavelength depending on the angle between the line of movement of the atoms and the direction of observation. Einstein points out that according to the classical or undulatory theory, light is emitted from the atom simultaneously in different directions with different frequencies, and proposes to test whether this is actually the case in the following way:

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†*Sitzungsber. Preuss. Akad. Wiss. Phys. Math. Kl.* 1921, p. 882.

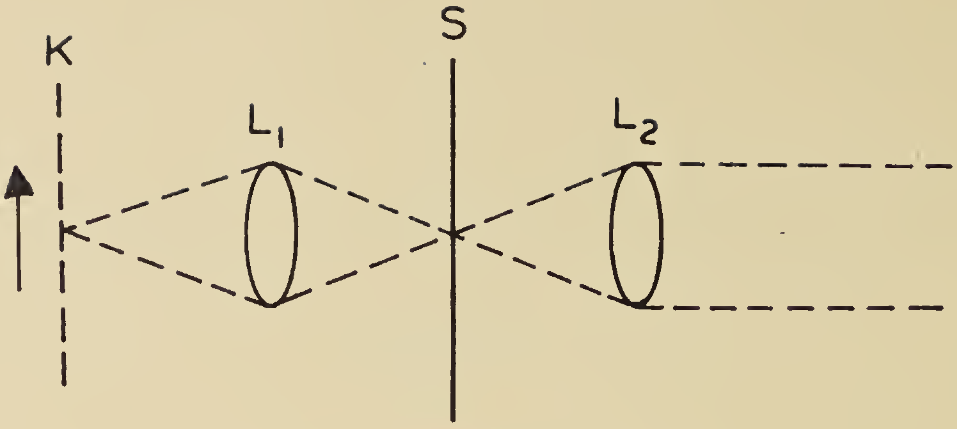


**Figure 1**

In figure 1, K represents a stream of canal rays $L_1$ is a lens which focuses an image of the same on a slit S contained in an opaque screen. The light passing through the slit falls on a collimating lens $L_2$, which renders the beam parallel. The collimated beam then falls upon the object-glass of a telescope focused for infinity (not shown in the figure). An image of the slit S will then be seen in the field of view. Einstein puts forward the proposition that if now a plane-parallel layer of a dispersing medium, e.g., carbon disulphide, be put in between the lens $L_2$ and the observing telescope, a displacement of the image of the slit is indicated by the principles of the wave theory, the deviation angle $\alpha$ being given by the formula:

$$\alpha = \frac{1}{\Delta} \cdot \frac{v}{c} \cdot \frac{dn}{\left(\frac{d\nu}{\nu}\right)} \tag{1}$$

where $v/c$ is the ratio of the velocity of the atoms in the *Kanalsrahlen* to the velocity of light, $l$ is the thickness and $n$ the refractive index of the dispersing medium, $\Delta$ is the distance $SL_2$; $KL_1$ and $L_1$ S are assumed to be equal and $\nu$ is the frequency of the light.

We now proceed to indicate the argument on which Einstein bases his derivation of formula (1) which is briefly as follows: The surfaces of constant phase in the light from a luminous atom on emergence from the lens $L_2$ are obviously plane. But since the light falling upon the upper and lower edges of the lens system is according to the Doppler principle of different wavelengths, the wave fronts after emergence from $L_2$ are not exactly parallel to each other but are slightly inclined in the manner of the leaves of a fan. If no dispersing medium be interposed, the wave fronts after passing through the object-glass of the observing telescope reach foci, which are not coincident but form an image of the slit of finite width in the same position as when the luminous atoms emitting the waves are at rest. If, however, between the lens $L_2$ and the observing telescope a plane layer of a dispersing medium be interposed, the conditions are altered. The upper and lower portions of the wave fronts correspond to different frequencies and therefore move forward with different velocities and hence the wave fronts swing round to a greater and greater extent proportionately with the length of the medium

traversed. Einstein then supposes that as the result of this swinging round of the wave fronts, the image of the slit in the observing telescope should suffer a corresponding deflection the magnitude of which is given by formula (1).

The error in Einstein's reasoning enters at the last stage, that is, in his assuming that the swinging round of the wave fronts involves a deflection of the image of the slit in the focal plane of the observing telescope. Referring to figure 1, we see that as a luminous atom moves along the stream of *Kanalstrahlen*, the waves emitted by it enter the slit $S$ only for a limited period of time depending on its width, that is, the disturbance passing through it forms a wave-group of limited extension. It is perfectly true, as pointed out by Einstein, that when the disturbance enters a dispersing medium, the wave fronts in it swing round steadily as they pass through it. But the position of the image of the slit is determined not by the inclination of any particular wave front to the axis of the optical system, but by the normal to the wave-group considered as a whole. It is easily shown that the latter remains fixed and does not turn round. The problem is exactly similar to that which arises in connection with Michelson's determination of the velocity of light in dispersive media by the revolving mirror method, and was very clearly dealt with by Willard Gibbs.* The group velocity is given by the relation

$$U = V - \lambda\frac{dV}{d\lambda}; \tag{2}$$

the individual wave planes in the group rotate with an angular velocity

$$\frac{dV}{d\lambda}\cdot\beta$$

where $\beta$ is the angle between successive wave planes; they also move forward through the group with a velocity $(V-U)$, and in the interval of time

$$\frac{\lambda}{(V-U)}$$

in which each wave plane moves into the position of the next in the group, it rotates through an angle

$$\beta\cdot\frac{dV}{d\lambda}\cdot\frac{\lambda}{(V-U)}$$

which by relation (2) is exactly $\beta$. Hence the second wave plane in passing a point moving with the group has exactly the same position which the first one had. Hence as the wave planes move out of the group and disappear and are followed by fresh ones, the inclination of the wave normal to the group considered as a

***Nature* (*London*), April 1886, p. 582, and *Scientific Papers*, 1 253.

whole remains fixed. In other words, after emerging from the layer of dispersing medium, the group would come to a focus in precisely the same position as when no dispersing medium is introduced.

The matter may also be viewed from another standpoint. Referring to figure 1, it is clear that what is observed in the field of view of the telescope are not the moving luminous atoms but the fixed slit illuminated by the finite width of the pencil of canal rays. An appropriate method of treating the problem is therefore to regard the slit as a secondary source of light in accordance with the Fresnel–Huyghens principle. If the slit S be very narrow it may be simply regarded as a source of cylindrical waves which diverge from it in all directions, and in the disturbance reaching the lens $L_2$, all differences in the frequencies at different points would cease to exist and the image of the slit would be formed in the focal plane of the observing telescope according to the ordinary laws of geometrical optics. No difference would therefore be made by the interposition of a plane-parallel layer of dispersing medium. If the slit S be wider, we may conceive it to be divided up into a large number of very small elements, the image of each of which is formed in a fixed position in the same way as in the case of a very narrow slit, and precisely identical results are reached.

That no displacement of the image in the slit in the observing telescope can occur on the introduction of a dispersing layer can also be shown on purely kinematical reasoning independent of any theory or hypothesis regarding the nature of light. To do this, we have merely to replace the stream of canal rays at K by a stationary source which emits light of different colour in different directions, e.g., a small dispersing prism from which a spectrum diverges and falls upon the lens $L_1$ and which after being focused at the slit S and passing through the lens $L_2$ is viewed through the observing telescope. It is clear that the introduction of a dispersing layer bounded by parallel faces will not influence the observed position of the source in the field of view.

Calcutta
20 March 1922

*Nature (London)* **109** 477–478 (1922)

# Einstein's aberration experiment

In the *Sitzungsberichte* of the Berlin Academy of December 8 last, which has recently come to hand, Einstein describes an ingenious arrangement which he suggests might serve to decide between the classical theory of light and the theory in which light is regarded as made up of single quanta of energy emitted discontinuously from luminous atoms. Figure 1 (reproduced from the paper)

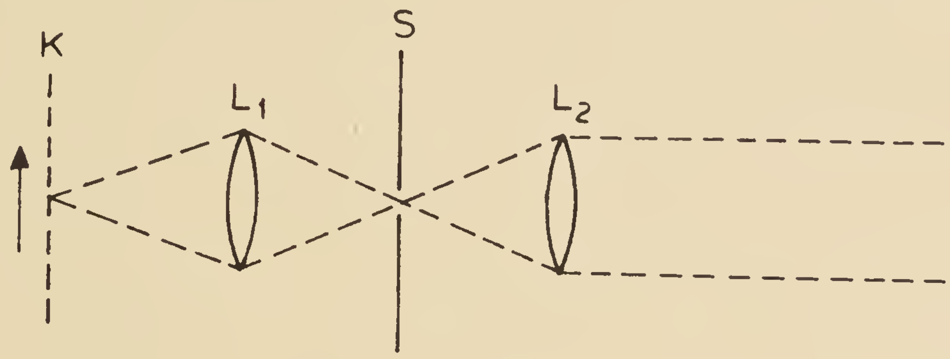


**Figure 1**

illustrates the proposed experiment. K is a stream of canal rays, $L_1$ is a focussing lens, S is a screen containing a slit which serves to isolate a definite pencil of light, and the lens $L_2$ renders the emergent beam parallel. The emergent pencil is observed through a telescope focussed for infinity, so that the image of the slit in the screen S would be seen sharply focussed in the field of view. Since the atoms in the canal rays emitting light are in motion, the Doppler effect comes into evidence, and the rays proceeding at any instant from individual luminous atoms in different directions should, according to the wave-theory of light, be of different frequencies. Einstein suggests that the rays passing through the slit S and incident on the upper and lower parts of the lens $L_2$ should consequently be of different frequencies. If, therefore, a layer of a dispersing medium such as carbon disulphide be placed between the lens $L_2$ and the observing telescope, the different rays would travel through it with different velocities. Hence the wave-front should suffer an aberration and the image of the slit seen in the focal plane should shift through an extent proportionate to the thickness of the dispersing layer introduced. Einstein conceives that according to the quantum theory of light, on the other hand, such displacement should not occur, and he believes that the proposed arrangement furnishes an *experimentum crucis* to decide between the rival theories.

I wish here to direct attention to a fallacy which is present in Einstein's reasoning and invalidates it. It is clear that in the proposed experiment what would be observed are not the moving luminous atoms but the fixed edges of the illuminated slit in S, and it is easily shown that even according to the principles of the wave-theory no aberration of the image of the latter could be expected. To make this evident we may conceive the slit to be extremely narrow, or in the alternative, if it be wide, regard it as divided up into a large number of very narrow elements each of which, according to Huyghens's principle, would operate as a secondary source of light. The light from any small portion of the lens $L_1$ arriving at the slit would spread out by diffraction in all directions in the form of cylindrical waves, so that the waves reaching $L_2$ would consist everywhere of *superposed* wave-fronts of all the frequencies reaching the slit, and not, as Einstein supposes, of different frequencies at different points of $L_2$. The waves diverging from S would thus pass through $L_2$ and the dispersing medium behind it according to the ordinary laws of geometrical optics, and no shift or aberration of the image of the slit would occur. The error in Einstein's reasoning lies in his having ignored the vitally important part which diffraction plays, according to the wave-theory of light, in the theory of the formation of images of illuminated apertures by optical instruments.

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16 March 1922

*Philos. Mag.* **43** 447–455 (1922)

# On the convection of light (Fizeau effect) in moving gases

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[Plates I and II]

## 1. Introduction

It was early in the last century that Arago tried his famous experiment with the prism to detect whether the aether in the interior of a material body and the light-waves travelling inside it are carried along with that body in its motion, and it was to explain the negative result of this experiment that Fresnel propounded his well-known hypothesis that the aether outside a moving body remains stationary while that inside it drifts along with it, though with a diminished velocity. He deduced a law according to which this diminished velocity is given by the relation

$$u' = \left(1 - \frac{1}{\mu^2}\right)u,$$

where $u$ is the velocity of the body and $\mu$ its refractive index. This result also explained why the aberration of the fixed stars was found by Airy and Hoek to be independent of the nature of the substance filling the telescope tube. And on account of its fundamental importance in the theory of optics, some of the most eminent physicists have devoted considerable energy to verifying the different aspects of this law. More recently it has gained additional importance on account of the fact that this law follows as a matter of course from Einstein's remarkable principle of the relativity of space and time†, and its experimental verification is now looked upon as one of the proofs of the correctness of his theorem of the addition of velocities and consequently of the special principle of relativity.

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*Communicated by the authors.
†See Cunningham's 'Theory of Relativity,' p. 61.

We are, in the present note, not concerned with the first part of Fresnel's law, which demands a fixity of the aether outside a moving body and which found support in the experiments of Sir O Lodge*, who failed to discover a drift of aether in the neighbourhood of moving matter, even in the narrow space between two revolving disks or a crevice in a massive sphere. The apparent disagreement of these with the experiments of Michelson and Morley†, which seem to contradict this fixity of the external aether, has been explained away by FitzGerald and Lorentz as being due to an inevitable change in the dimensions of the apparatus on account of its motion with the earth, an explanation very considerably simplified and made almost self-evident by the advent of the relativity principle‡.

But so far as the second part is concerned, the only evidence of a positive character in favour of this law of aether drift was furnished by the celebrated experiments of Fizeau, in which a delicate interference method was employed to detect and measure a change in the velocity of light on account of a velocity of about 7 metres per second of a column of water through which it was made to travel. This experiment was repeated by Michelson and Morley§ in 1886 with improved apparatus, and it demonstrated the surprisingly good agreement with theory. But the appearance of the elaborate electron theory of Lorentz made this agreement much less brilliant; for he showed that in dispersive media the convection coefficient was not $1-(1/\mu^2)$, but $1-(1/\mu^2)-(T/\mu)(\mathrm{d}\mu/\mathrm{d}T)$. This necessitated a more careful repetition of the experiment which has recently been accomplished by Zeeman‖, and the result is decidedly in favour of the new theory. The subject has been further followed up by him, and he has succeeded in overcoming the enormous experimental difficulties, and has actually determined for glass and quartz not only the Fresnel coefficient, but also the Lorentz correction for dispersion.

In view of this recent work and the accuracy which has been attained in the measurements, it is hardly necessary to refer to the work of Sir J J Thomson¶, which led to the result that an electromagnetic wave inside a moving body should drift with half the velocity of that body. In support of this, it was argued that the substances for which Fresnel's law was actually verified in a positive manner happened to be such that $1-(1/\mu^2)$ was in their case very nearly equal to $\frac{1}{2}$. But even before the work of Zeeman, it was quite evident from the negative results of the experiments with moving air conducted by Michelson and Morley that Thomson's value for the convection coefficient could not be correct.

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* *Philos. Trans. R. Soc. London Ser. A*, 1893, p. 727.

† *Philos. Mag.* 1887, p. 449.

‡ Cunningham, 'Relativity and Electron Theory,' p. 34.

§ *Am. J. Sci.*, **31**, 377 (1886).

‖ *K Akad. Amst. Proc.* **17**, 445 (1914); **18**, 398 (1915); **22**, pp. 462 and 512 (1920).

¶ *Philos Mag.* April 1880.

While, therefore, all the available evidence is strongly in favour of Fresnel's value or, rather, Lorentz's corrected value, at least in the case of solids and liquids, it is highly desirable that it should be confirmed in the case of gases also. A special grant secured from the Calcutta University through the kind offices of Sir Asutosh Mookerjee has enabled the writers to undertake this research, and it is proposed to give here a preliminary account of the progress of the work and of the difficulties encountered.

But before we do so it might be of interest to deduce Fresnel's law for gases from slightly different considerations. The late Lord Rayleigh* in discussing the scattering of light by small particles and in referring the blue colour of the sky to the molecules of air, deduced an expression for the refractive index of the gas in terms of the molecular constants. Following the same line of argument, we may also deduce on Doppler's Principle an expression for the altered refractive index when the air molecules are set in motion with a constant velocity $v$ in a definite direction. On this principle, the incident light of wavelength $\lambda$ is received by the molecules as light of wavelength

$$\frac{1}{1-\frac{v}{b}},$$

where $b$ is the velocity of light in the medium without these scattering molecules. Thus, in the notation of Lord Rayleigh, the expression for the vibration scattered from the molecule in a direction making an angle $\theta$ with that of the primary vibration is

$$\frac{D'-D}{D}\cdot\frac{\pi T}{r\lambda^2}\left(1-\frac{v}{b}\right)^2 \sin\theta\cos\frac{2\pi}{\lambda}(bt-r).$$

And considering the particles which occupy a thin stratum d$x$ perpendicular to the primary ray $x$, the resultant, at a point on the incident ray, of all the secondary vibrations which issue from this stratum is

$$n\mathrm{d}x\int_x^\infty \frac{D'-D}{D}\cdot\frac{\pi T}{r\lambda^2}\left(1-\frac{v}{b}\right)^2\cos\frac{2\pi}{\lambda}(bt-r)2\pi r\mathrm{d}r$$

$$= n\mathrm{d}x\frac{D'-D}{D}\cdot\frac{\pi T}{\lambda}\left(1-\frac{2v}{b}\right)\sin\frac{2\pi}{\lambda}(bt-x).$$

This, combined with the primary wave $\cos(2\pi/\lambda)(bt-x)$, will give $\cos(2\pi/\lambda)(bt-x-\delta')$, where

$$\delta' = nT\mathrm{d}x\frac{D'-D}{2D}\left(1-\frac{2v}{b}\right).$$

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*Scientific Papers*, **4**, 395.

If $\mu'$ be the effective refractive index of the medium as modified by the moving molecules, that of the medium without the particles being taken as unity,

$$\delta' = (\mu' - 1)\mathrm{d}x.$$

And therefore

$$\mu' - 1 = nT\frac{D' - D}{2D}\left(1 - \frac{2v}{b}\right).$$

But it has been shown by Lord Rayleigh that if $\mu$ be the refractive index with the molecules when they do not have the velocity $v$,

$$\mu - 1 = nT\frac{D' - D}{2D};$$

$$\therefore \quad \mu' - 1 = (\mu - 1)\left(1 - \frac{2v}{b}\right);$$

$$\text{i.e.,} \quad \mu - \mu' = 2(\mu - 1)\frac{v}{b},$$

which, in terms of the corresponding velocities of light $V$ and $V'$, reduces to

$$V' - V = v\cdot\frac{2(\mu - 1)}{\mu^2}.$$

In the case when $\mu$ does not much differ from 1, as in air, this becomes

$$V' - V = v\frac{(\mu + 1)(\mu - 1)}{\mu^2}[\text{since } \mu + 1 = 2]$$

$$= v\left(1 - \frac{1}{\mu^2}\right).$$

## 2. Experimental methods

It was proposed to employ the same optical arrangements as had been used by Fizeau and which constitute in effect a compensated Rayleigh Interferometer, and which are eminently suited for this work. Michelson's arrangement might have been better, but owing to the great length of path necessary, the attempt to observe the fringes with it did not prove to be a success. A large refracting telescope with a 7 inch object-glass and a focal length of over 7 feet was available from the observatory of the Indian Association for the Cultivation of Science, and was made use of in the work. The plan of the optical arrangement is shown in figure 1, from which it is clear that if $l$ be the length of each of the tubes AB and CD, the shift of the fringes observed when a current of air is sent round with a

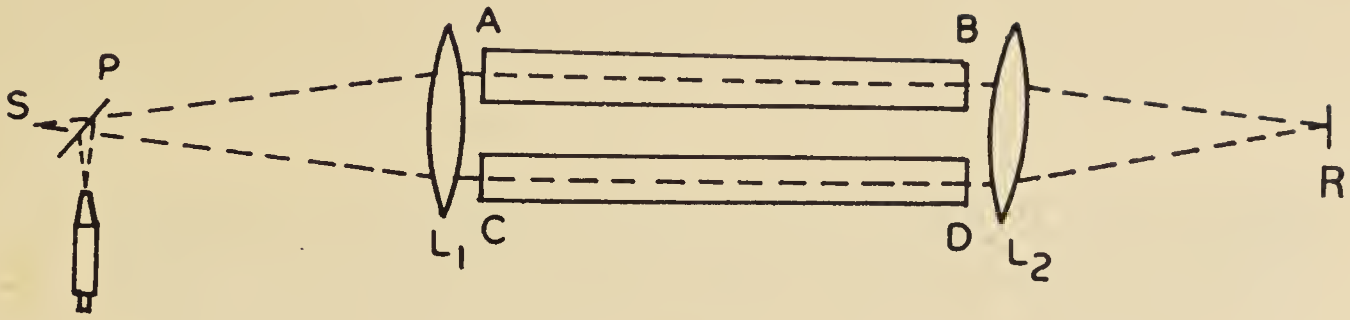


**Figure 1**

velocity $v$ in the direction of the arrows will be

$$\frac{4lv\theta}{\lambda V}$$

of the distance between two consecutive maxima or minima, where $\theta$ is the convection coefficient and $V$ is the velocity of light in the medium. If, however, the current is first allowed to flow as in the figure and then suddenly reversed, the total shift will be double this or equal to $8lv\theta/\lambda V$. It is evident, therefore, that what is required in order to increase the shift and render it measurable is that both $l$ and $v$ should be increased. Consequently we proposed to utilize all the space available to us and make the tubes about 200 feet in length. For driving the current it was proposed to utilize a Pressure Blower driven by a 3-h.p. gas engine (figure 3, plate I). With this we expected a velocity of air of over 50 metres per sec in the tubes. The amount of shift of the fringes which we could therefore expect became

$$\frac{8 \times 200 \times 30 \times 5000 \times \left(1 - \dfrac{1}{(1.0003)^2}\right)}{5 \times 10^{-5} \times 3 \times 10^{10}},$$

or about one-tenth of a fringe-width, which should be easily measurable with sufficient accuracy. In Michelson and Morley's experiment the size of the apparatus was very much smaller and the velocity of air not more than about 25 metres per sec, so that the observed shift was, as stated by them, certainly less than $\frac{1}{100}$ of a fringe and probably less than $\frac{1}{200}$.

The actual setting up of the apparatus was taken in hand about July 1920, and the site chosen was the compound of the Indian Association. One of the out-houses served to house the optical parts at the observing end, and a small hut was erected at the other end to receive the reflecting arrangement $L_2R$ (figure 1). The big telescope body with the object-glass was mounted on solid brick-work pillars erected for the purpose, and adjustments had to be provided for both vertical and horizontal movements. Some of these are shown in figure 3 (plate I). A fine spectrometer slit was mounted at the eyepiece end of the telescope, and the thin plate-glass, P, was also mounted in the same tube. A microscope was used to observe the fringes. For the other end, another shorter telescope (4 inch objective)

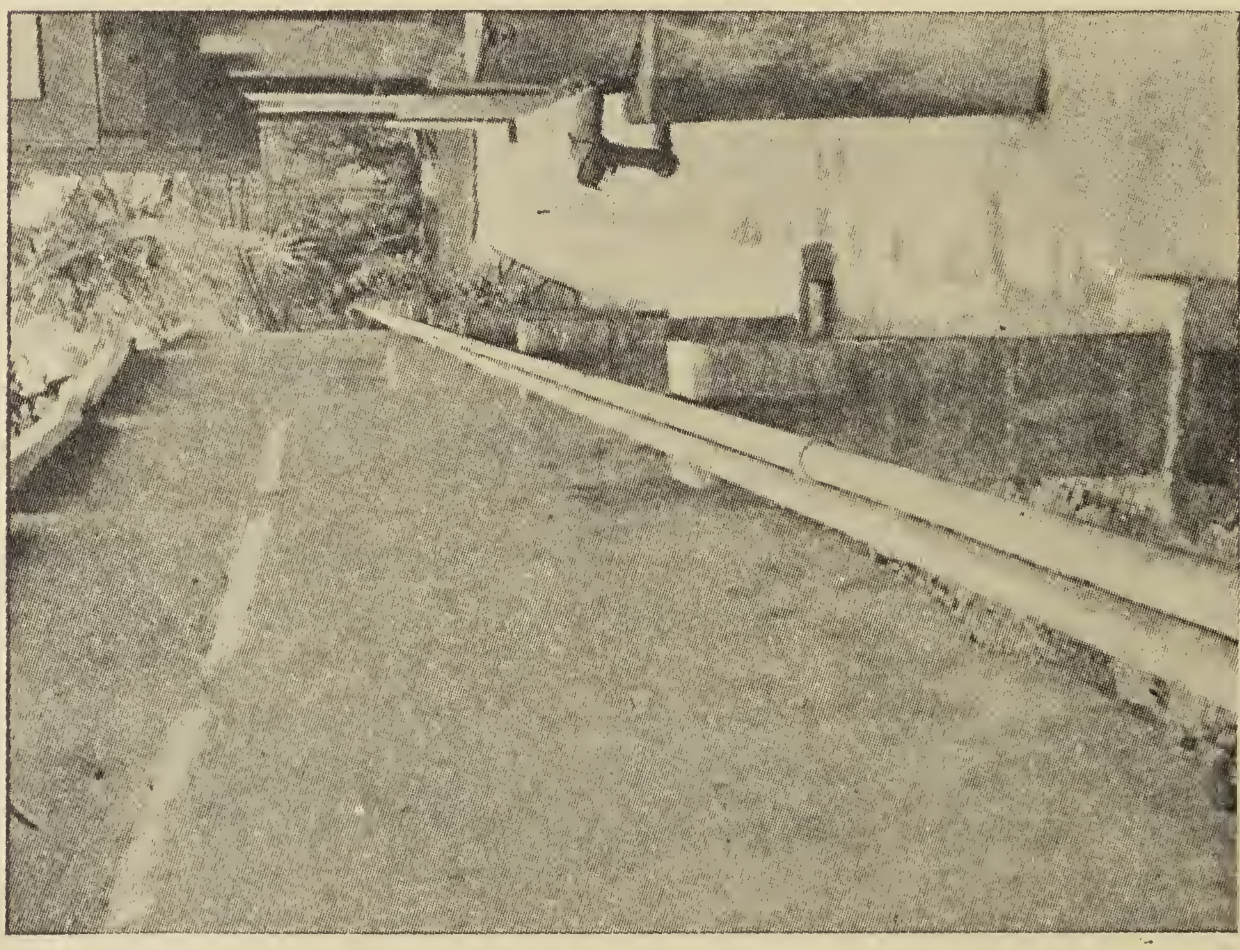

**Figure 2.** Showing 200-foot pipe-line.

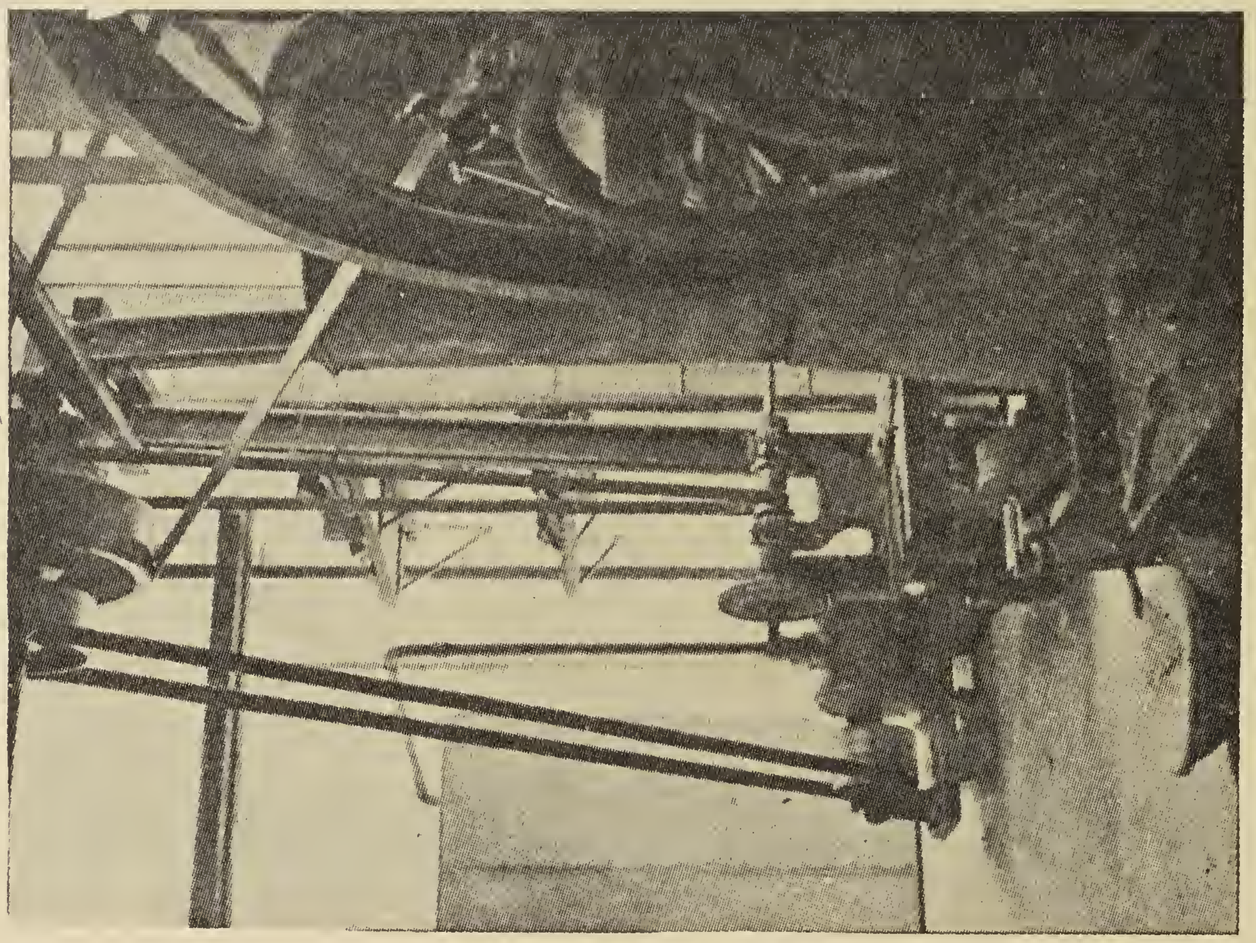

**Figure 3.** Showing engine, blower and reservoir.

**Plate I**

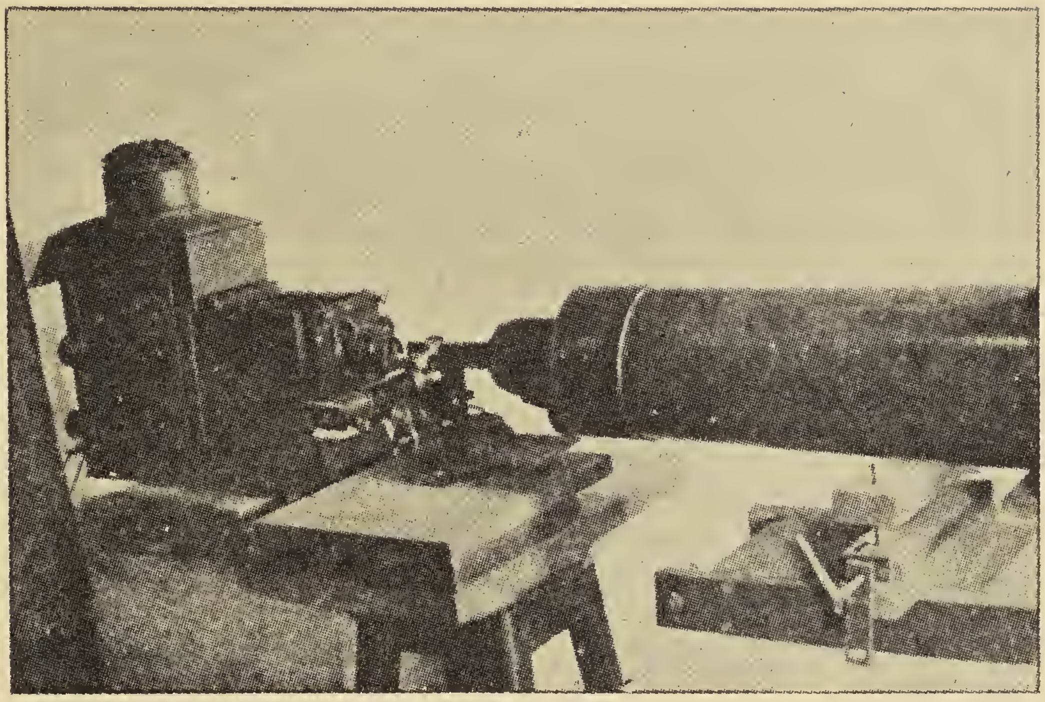

**Figure 4.** Showing observation end.

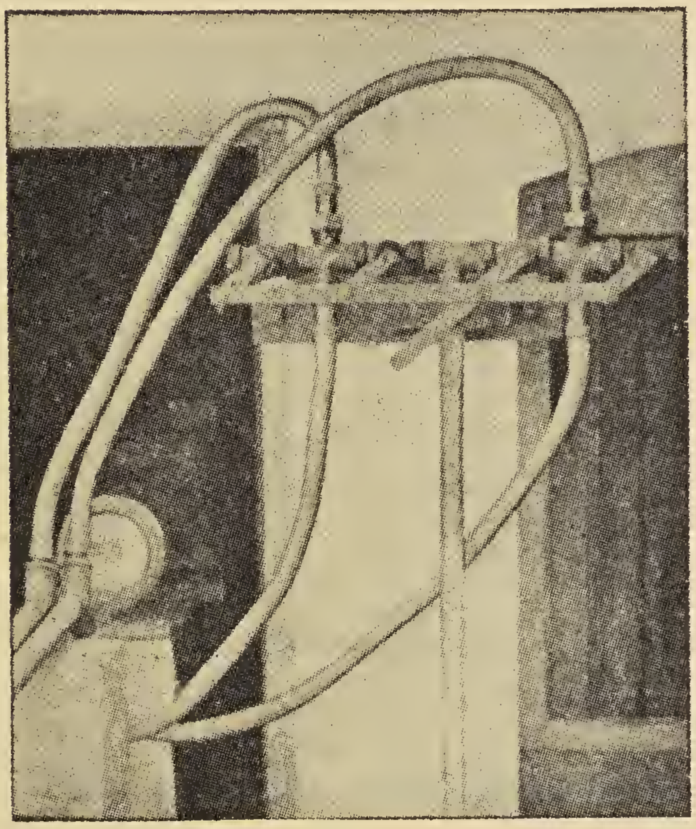

**Figure 5.** Reversing gear.

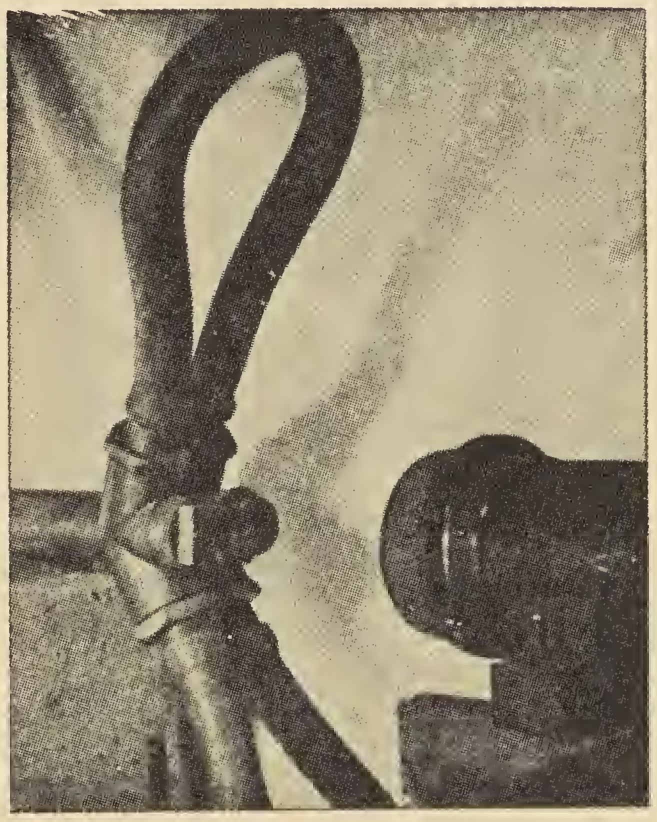

**Figure 6.** Showing further end of pipe-line.

**Plate II**

was similarly mounted on two pillars, and the eyepiece was in this case replaced by one of the front-silvered Michelson interferometer mirrors, so mounted as to allow of accurate adjustment of its plane. But the real difficulty lay in the fixing of the tubes. Galvanized-iron pipes of $1\frac{1}{2}$ in. internal bore were selected, and each piece was carefully straightened. Brick pillars were erected at intervals of about 10 ft., and their tops carefully aligned by means of a theodolite so that the deviations of the top line from the mean did not exceed $\frac{1}{10}$ of an inch for any individual pillar. This having been accomplished, the pipes were assembled and placed on top of the pillars, so that ultimately there was a pair of tubes quite straight from end to end and separated from each other by a constant distance throughout. There was no appreciable sag of the pipe between the pillars. This seems to have been avoided automatically by having such a considerable continuous length. This was finally tested by placing a small hole near one end and illuminating it by a strong beam of light. Observing at the other end, the whole of the aperture should be uniformly illuminated if there was no bend anywhere. Unfortunately this was very difficult to secure, for the slightest cutting-off of the light anywhere cast a tremendously large shadow at the great distance where it was being observed, and it was not at all easy to locate where the fault lay. The width of the diffraction fringes at the edge of the shadow, however, gave a rough and ready indication of the approximate position of the fault, and it was with the help of these that finally the major part of the aperture was freed from all obstacles. A parallel beam of light now sent down any one of the tubes passed unobstructed and filled almost the whole aperture at the other end.

The ends of the tubes were closed by windows of about $\frac{3}{4}$ in. diameter covered by thick interferometer plates secured between leather washers very much after the manner employed by Zeeman*, and the current of air was led into the pipe by two side tubes inclined at about 60° to the axis of the pipe. The end pieces for carrying the windows and these inlet tubes were cast in brass, and can be seen in figure 6 (plate II).

This method of leading the current of air into and out of the tubes is calculated to cause the least disturbance in the path of the light.

The manner of connecting the two tubes with each other at one end and with the arrangement of cocks to facilitate the reversing the direction of flow of air at the other, is shown in figures 5 and 6 (plate II). This connection is by means of lead pipes, and the reversing arrangement consists of four cocks connected by means of a handle, so that at any time two alternate cocks are open and the other two simultaneously closed. The central vertical pipe in the photograph leads to the blower, which was used rather as an exhauster, so that the air of the atmosphere entered one of the open cocks of the reversing arrangement, passed

***Loc. cit.*

through one pipe to the other end, and returned through the other pipe and thence into the blower. This method was followed in order to avoid the temperature changes which would have certainly occurred within the tubes if the compressed air from the blower had been led into them. The temperature of the whole of the air in the tubes was thus kept equal to that of the external atmosphere.

With all these arrangements complete in January 1921, the two telescopes were carefully adjusted so that a fine slit sent two parallel beams down each tube, and the reflecting mirror at the other end sent them back still strictly parallel. It is evident that the slightest want of adjustment in the direction or the parallelism of the beams was enough to stop all light from passing through the tubes, and even the smallest angle between the two surfaces of each plate closing the ends of the tubes was inadmissible. But when these adjustments were carefully carried out, the fringes obtained were excellent and surprisingly steady, though the light itself "boiled" very badly, except in the cool mornings. The heat of the sun during the day caused variations in the temperature of the air inside the pipes, and the convection currents deflected the light in all sorts of ways. This was to some extent avoided by covering the entire length of the pipes by bamboo screens, but the most satisfactory results were obtained only in the mornings or an hour or two after the sun had gone down.

To test whether or not the paths traversed by the two interfering beams were identical, the method suggested by Michelson was employed. A plane-parallel plate of glass was inserted in front of one of the ends of the tube, and the effect of its rotation on the fringes was observed in the microscope. With the final adjustments no displacement of the fringes could be observed by this means, and there seemed to be no reason to doubt that the paths were really identical and not merely parallel.

The current of air was now turned on, but it was at once apparent that the air was not flowing at a constant rate, but was being driven through the tubes in puffs; and although nothing happened to the fringes which, whenever visible, appeared in the same position as when no current of air was flowing, yet the spots of light themselves "boiled" very badly indeed. An attempt was made to overcome this difficulty by inserting a reservoir of air to steady the motion, and, after some preliminary trials with a brickwork and cement reservoir which developed a leak and proved unsatisfactory, a large wooden box, 4 ft. × 4 ft. × 6 ft., covered over with galvanized-iron sheets was installed, and inserted in the line of flow of the air between the pipe-line and the blower (see figure 3, plate I). This successfully checked the oscillations in the speed of the current of air, and the spot of light in the microscope containing the interference fringes could be steadily seen, even with the current of air running.

To measure the velocity of the air-current, a pair of Pitot tubes with water-manometer were inserted in the channel between the pipe-line and steadying reservoir.

## 3. Results

When the arrangements including the steadying reservoir were complete in April 1921, it was found that the engine and blower were unequal to the task of drawing the air through the system at the originally estimated velocity of 50 metres per second, and that a speed of only about 20 metres per second could be attained. The hot weather which had then commenced also made the temperature conditions in the tubes very unfavourable for systematic work. Nevertheless, on two evenings when observations were made after a smart shower following an April nor'wester, the fringes were seen very steadily, and appeared to show a slight but unmistakable shift on reversing the direction of the air-current. On the second occasion an attempt was made to estimate the magnitude of the shift by setting a cross wire on the fringes and comparing the shift observed on reversing the air-current with that produced by flexure of the microscope tube by a known small load. The shift was estimated to be about $\frac{1}{20}$ part of a fringe, which was of the right order of magnitude and in the direction indicated by theory. Subsequent attempts to confirm these observations and measurements under less favourable conditions proved unfruitful, as the fringes then showed a distinct *rotation* as a whole when the air-current was reversed. The rotation proved to be a very troublesome and disturbing factor, and before the cause of it could be ascertained and removed, the work had to be suspended, owing to the departure of one of us for Europe. While, therefore, the results so far obtained cannot be regarded as entirely conclusive, they hold out a distinct promise of success when the work is resumed under more favourable conditions, particularly if a more powerful blower with steady electric drive can be obtained and installed.

*Nature (London)* **107** 747 (1921)

# Conical refraction in biaxial crystals

An arrangement for demonstrating conical refraction usually found in laboratories is a piece of aragonite crystal mounted inside a little tube which has one end covered with a metal foil pierced by a number of pin-holes, and an eye-lens in a focussing mount at the other end. When the tube is directed against a luminous object and the eye-lens focussed on the pin-holes through the crystal suitably oriented they are seen as luminous rings of light. Writers on physical optics who describe this experiment refer to it as illustrating *internal* conical refraction—that is, as due to the fact that the Fresnel wave-surface has a tangent-plane which touches it along a circle. I wish to point out that this is really an error. A little consideration will show that as the eye-lens is focussed on the pin-holes, which may be as small as we please, we are concerned here with the waves *diverging* from them in all directions within the crystal, and the observed effect is due to the fact that the two sheets of the wave-front intersect at a conical point. In other words, the experiment really illustrates *external* conical refraction. This is confirmed by the fact that an extended source of light may be used without interfering with the success of the experiment.

A remarkable effect is observed if, with the tube pointed towards an open window, the eye-piece is steadily drawn back from the crystal. It will be noticed that a well-defined image of each pin-hole may be traced behind the crystal for a distance of several centimetres. The formation of this continuous image by a crystalline plate with parallel faces cannot be explained on geometrical principles, and is of great interest. The effect appears to be due to the dimpled form of the wave-front within the crystal, and is being further investigated by Mr V S Tamma and myself.

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22 Oxford Road
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4 August

*Philos. Mag.* **43** 510–513 (1922)

# On a new optical property of biaxial crystals

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## 1. Introduction

A fact of singular interest and importance in the optics of crystalline media which appears hitherto to have been overlooked is that a plate of a biaxial crystal bounded by parallel faces is capable of focussing divergent rays proceeding from a distant light-source and forming *real* images of it. Very simple apparatus will suffice to observe this phenomenon. The incandescent filament of a tiny 2-volt lamp or an illuminated pin-hole serves as a suitable source of light. At some distance from it is placed a crystal of aragonite cut and polished with parallel faces at right angles to the bisectrix of the acute angle between the optic axes. On suitably orienting the crystal and examining the pencil of light which has passed through it, a real erect unpolarized image of the luminous filament may easily be picked up and traced continuously away from the crystal for a considerable distance. There is no difficulty in receiving and observing the image directly on a plate of ground glass if desired.

The image is sharp and bright and practically achromatic if the object and the place of observation are both within a few centimetres of the crystal, one on each side. As either the object or the place of observation is drawn away from the crystal the image spreads out into a spectrum. Using monochromatic light, however, it can be seen that the image remains quite well defined and sufficiently bright to be traced for a distance of some 30 or 40 cm from the crystal. The effects are specially beautiful if observed in the light of a mercury vapour lamp. Separate images of the pin-hole corresponding to the yellow, green, and violet radiations of the mercury vapour may be seen, and images corresponding to the fainter components of the line spectrum may also be observed by cutting out the superfluous light with suitable colour filters. A simple slit with a plate of aragonite crystal and an eye-lens to observe the image thus functions as a little

*Communicated by Prof. A W Porter, F.R.S.

spectroscope, which has quite a marked dispersion in the region of shorter wavelengths.

It should be remarked that these optical images formed by an aragonite plate differ from those formed by an ordinary converging lens in several respects. The images in the present case are *real, erect*, and of *unit magnification* irrespective of the distance of either object or image from the crystal. Further, the image is *continuous*, that is, it may be observed anywhere in the prolongation of a certain line for a considerable distance from the crystal and not merely at a single point as in the case of the images formed by a lens. Also the images appear sharply defined in a field of diffuse light, showing that only part of the energy passing through the crystal is brought to a focus. The object being fixed, the image moves when the orientation of the crystal is altered, but not when the plate is moved in its own plane. The focussing property, in other words, appears to be related to a fixed direction within the crystal. In order that the image may be within the field of observation it is necessary, in fact, that the bundle of light-rays should pass through the crystal roughly in this fixed direction, which appears to be that of either axis of single ray velocity in the crystal.

## 2. Explanation of the phenomenon

The fact last mentioned suggests a mode of approach to a theoretical explanation of the phenomenon. As already mentioned, the object may be placed at a distance from the crystal, but for simplicity we shall first consider the case in which the object is a point source of light placed just on one of its surfaces. The form of the wave-fronts in the disturbance diverging within the crystal is the Fresnel surface of two sheets which, as is well known, has a singular point in the direction of either axis of single-ray velocity. If we resolve the wave-front into the group of plane waves of which it is the envelope, we see at once that the singularity is really the crossing point of an infinite number of plane waves the normals to which generate the surface of a cone, and hence the intensity of the disturbance at the singular point must be very great in comparison with that at other points on the wave. On emergence from the crystal the singularity persists, and, since in its immediate neighbourhood the wave-front is approximately symmetrical in shape about the axis of the cone of normals, it can easily be seen that the advancing front would retain the same general configuration exhibiting a singularity or concentration of luminosity along its course, as indicated in figure 1.

We have thus in effect a continuous image of the source along a line. When the source instead of being placed on the surface is removed to a distance from the crystal, the waves which diverge from it are in the first instance spherical, but on entry into the crystal these divide at once into two sheets, the points of intersections of which must be in the nature of foci or concentrations of luminosity in the wave-front. On emergence of the waves from the crystal the

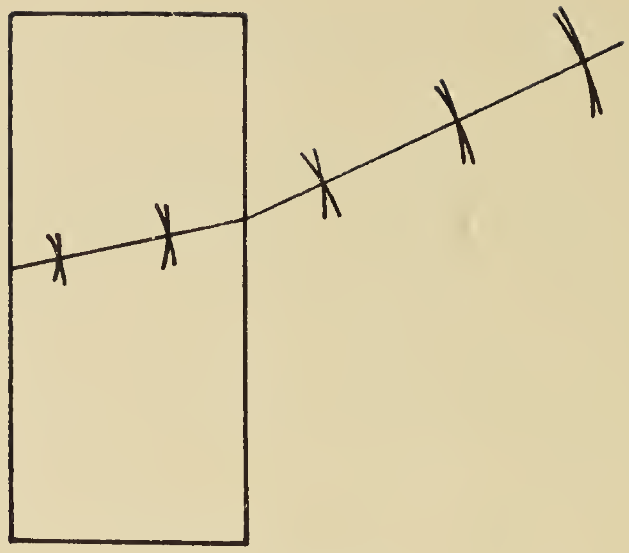

**Figure 1**

same effect is propagated outwards, giving a continuous focus or image of the source in much the same way as in the case illustrated in figure 1. When the luminous source is of finite dimensions we have an image corresponding to each point of it, and it is easy to see that we should have a complete picture built up which would be of the same dimensions as the source and similarly oriented.

The spectral dispersion of the image is also easily understood. For, the direction in which the singularity travels within the crystal being inclined to the normal to the plate, the direction in which the singularity travels on emergence from the crystal depends on the refractive index and hence must make a greater angle with the normal to the plate for the shorter wavelengths. This is exactly what is actually observed. In an exact quantitative discussion the fact that the direction of the axis of single-ray velocity in the crystal varies with the wavelength must also be taken into account. A fuller discussion of this point and of the resolving power of the crystalline plate regarded as a spectroscope in itself would be of interest. This would obviously require a determination of the distribution of luminosity in and around the point of singularity in the wave-front. It is hoped at an early opportunity to carry out a detailed investigation on these points.

## 3. Summary and conclusion

(*a*) The paper describes a new optical effect observed with a biaxial crystalline plate, viz., that such a plate though bounded by parallel faces, is capable of forming on one side of it when suitably oriented a *real* erect image of a source of light such as a luminous filament placed at some distance from it on the other side.

(*b*) The characteristics of this image-forming property are set out in detail, a noteworthy feature being the spectral resolution of the image which occurs when either the object or the image is at a considerable distance from the crystal.

(*c*) A theoretical explanation of the phenomenon is suggested, viz., that the singular point in the Fresnel wave-surface is a locus of maximum intensity, and,

owing to the fact that the wave-form on emergence from the crystal retains its general character, we get a continuous concentration of luminosity along a line any point of which is in effect an optical image of the source.

The investigation described in this paper was carried out in part in the senior author's laboratory at Calcutta, and in part at University College, London, during the visit to England of the senior author, who wishes to express his cordial thanks to Prof. A W Porter, F.R.S., for his hospitality and kind interest in providing the necessary facilities for the work.

*Nature (London)* **113** 127 (1925)

# The effect of dispersion on the interference figures of crystals

Crystallographers are familiar with the fact that the colour-lines in the figures shown by crystal-sections between crossed nicols in the polarisation microscope often deviate to a notable extent from the so-called "isochromatic lines" discussed in the text-books of physical optics. The difference is attributed to the dispersion of the optic axes, and its character enables the type of dispersion to be determined (for details, see for example Tutton's "Crystallography"). Though the phenomenon is thus well known and of considerable importance in practical work with crystals, I have found no record of any attempt to determine *theoretically* the form of the true isochromatic lines for any specified dispersion. Possibly it has been thought that the task would be too complicated and laborious to be worth undertaking. It may, therefore, be worthwhile to point out a fairly simple way of approaching the matter.

The general principle determining the observed position and colour-distribution of interference-fringes in white light is that the *group*-velocity and the group-refractive index should be considered, and not the wave-velocity and the wave-refractive index. Thus, for example, if a thick parallel plate of glass be placed in the path of one of the interfering beams of a Michelson interferometer, and white light is used, fringes may be observed when the retarding plate is compensated by increasing the air-path of the other beam; they are then less distinct but far more numerous than without the plate, and the colour-distribution is determined by the fact that the compensation for different group-wavelengths occurs for different retardations.

Taking now the case of a crystal-plate between crossed nicols, the so-called "isochromatic surface" is derived from the equation $\rho(\mu_1 - \mu_2) = \text{constant}$, where $\rho$ is the linear-path within the crystal and $\mu_1$, $\mu_2$ are the two wave-refractive indices. If, however, instead of $\mu_1$, $\mu_2$, we consider the group-indices $\bar{\mu}_1$ $\bar{\mu}_2$, the sections of the surface $\rho(\bar{\mu}_1 - \bar{\mu}_2) = \text{constant}$ would give the lines for which the relative retardation has specified values for any given *group* in the spectrum, and these may be expected to follow much more closely the colour-lines actually observed in the polariscope.

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21 November

*Trans. Opt. Soc. (London)* **26** 289–292 (1925)

# The optical properties of amethyst quartz

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MS received, 7th May, 1925. Read, 11th June, 1925

**Abstract.** Circularly-polarized waves of light, in passing through a section-plate of amethyst quartz would obviously be retarded unequally in the alternate laminae having right- and left-handed optical activities. Diffraction effects must therefore arise, and these can actually be observed even in almost colourless varieties of the quartz. The section-plate behaves in effect as a phase-change diffraction grating, and the question of the visibility of its structure with and without the aid of polarizers and analysers is discussed in the light of Abbe's theory of microscopic vision.

## Introduction

The periodic alternation of layers rotating the plane of polarization in opposite directions, so beautifully shown by amethyst quartz when a section-plate cut normal to the optic axis is viewed between nicols, has often been observed and described. Some fine illustrations of the type of sectorial repeated twinning characteristic of amethyst quartz are reproduced as plates XXV and XXVI of Tutton's new and very attractively written book, *Natural History of Crystals*, in which several pages are devoted to a description of amethyst, and his account will no doubt stimulate interest and inspire fresh studies of the properties of this remarkable substance. The view now generally accepted is that the twinning is of the simple Brazilian type in which the plane of twinning is parallel to a pair of faces of the second-order hexagonal prism, the actual surfaces of junction of oppositely active parts being, however, very varied in their distribution and character. The effects observed in the polarization microscope are explained in a simple way as due to the superposition of the opposite rotations of the plane of polarization produced by the two types of quartz. Parts of the section-plate in which equal thicknesses of the oppositely active parts occur would appear quite black between crossed nicols, other regions exhibiting illumination and colour varying with the thickness of the two kinds traversed by the light. The effect of rotating either the polarizer or the analyser on the structure seen is similarly elucidated.

We wish to draw attention to another class of optical effects arising from the structure of amethyst, and to suggest a new viewpoint from which to consider the phenomena observed in the polarization microscope with this substance.

## Diffraction effects

Consider the case of a section-plate cut normal to the optic axis, the alternate strips of oppositely active parts being assumed to be of equal width and separated by planes normal to the plate. If the incident waves of light be unpolarized, they may be resolved into two wave-fronts circularly polarized in opposite senses but having no determinate phase-relation which may therefore be considered separately. An incident circularly-polarized wave, assumed to be initially plane, traverses the alternate strips with different velocities, and on emergence from the plate must therefore become corrugated. The corrugated wave-front should, according to well known principles*, give rise to diffraction effects and should in fact, at a sufficient distance from the crystal, resolve into sets of plane waves corresponding to a central undeviated pencil and diffraction spectra of the first, second, and higher orders. The same result should occur when the boundaries of separation between the twins are not plane or parallel to the optic axis, or if the alternate strips are not equal in width, the distribution of intensity in the diffraction spectra being alone influenced by these factors.

That amethyst does give rise to diffraction effects as indicated by theory is easily verified by observation. For this purpose the polarizing and analysing nicols are not necessary and may be removed. A narrow source, e.g., a "Pointolite" lamp, should be used, and the light so adjusted that it passes normally through the plate. Coloured diffraction fringes, arranged periodically and running parallel to the lines of the structure, are then seen in the field behind the plate. The fringes (which are of the Fresnel type) are shown just as conspicuously by nearly colourless varieties of amethyst as by the more deeply tinted ones, and this circumstance, together with the fact that they disappear completely when the viewing microscope or eyepiece is focussed on the plate, proves clearly that the diffraction effect is due to the periodic change of phase produced by the structure and not to any periodic variation of transparency. The diffraction spectra of the Fraunhofer type due to the structure may also be seen when the observing eyepiece is focussed on the image of the source of the light formed by the objective.

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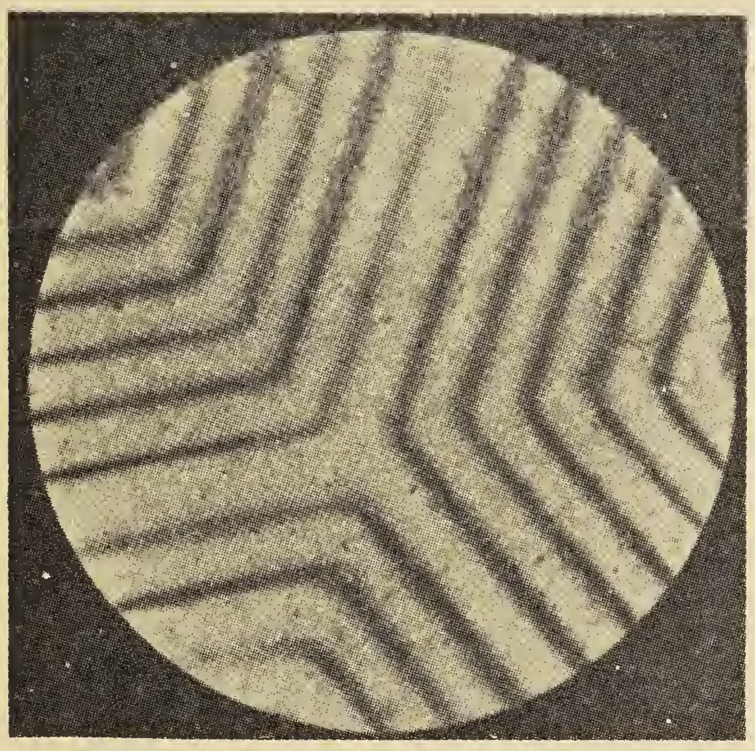
*Rayleigh, "On the Dynamical Theory of Gratings," *Scientific Papers*, 5, p. 388.



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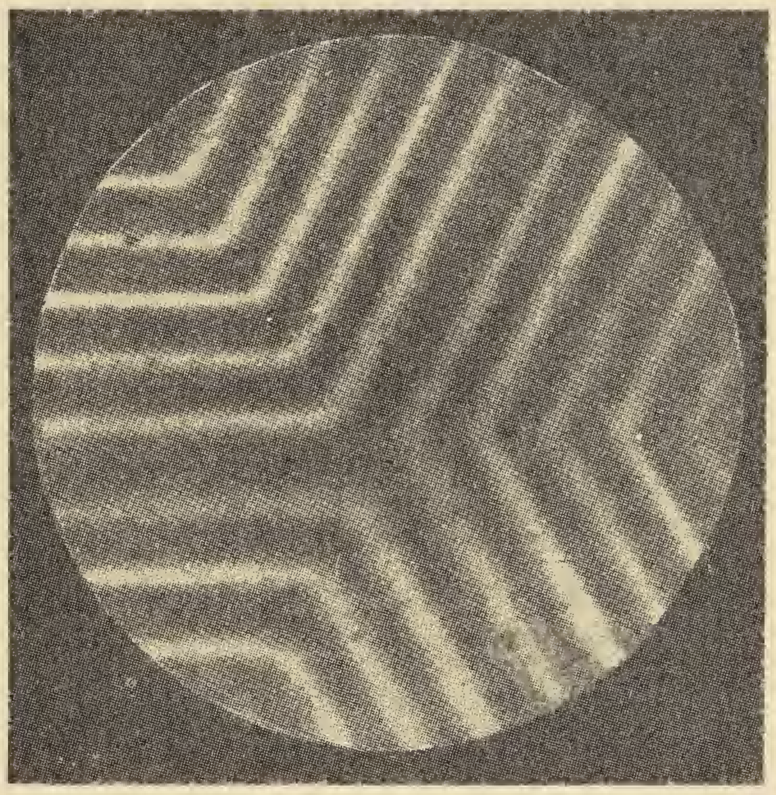

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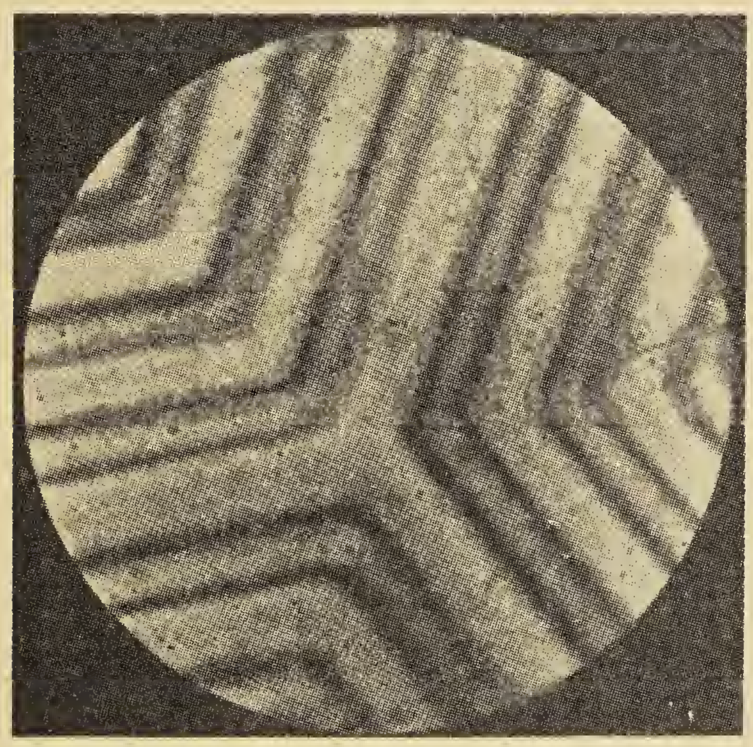

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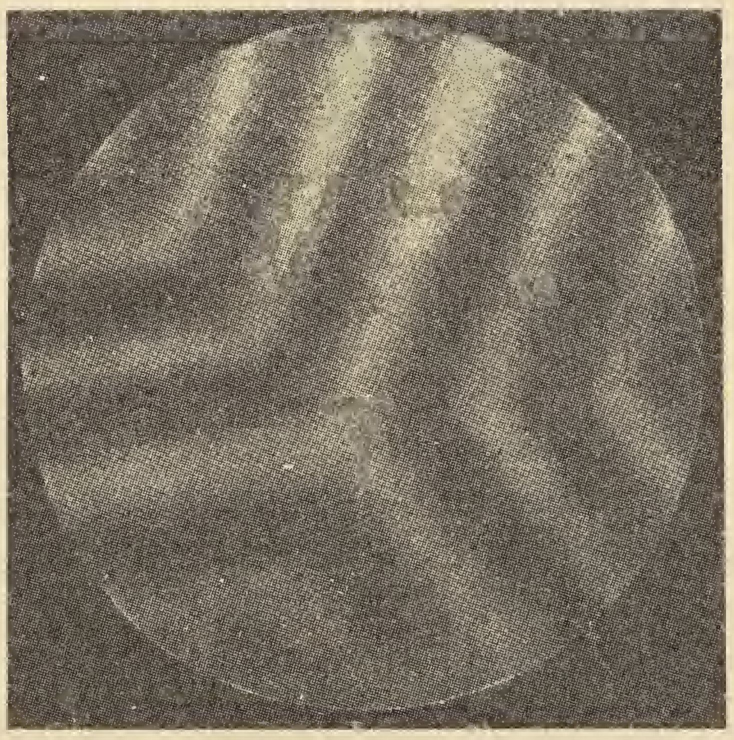

(4)

**Figures 1–4**

## Amethyst as phase-change grating

Since a section-plate of amethyst quartz is thus in effect a phase-change diffraction grating, its appearance in the polariscope may be interpreted in the light of the Abbe theory of microscopic vision, and in fact furnishes very convenient illustrations of that theory. The first point worthy of remark is that little or no structure is observable when the polarizer and analyser (either or both) are absent, though, as we have seen, they are not required to enable the structure to give rise to diffraction effects. This has a bearing on the general question of the visibility of phase-change structures. Lummer and Rieche* have discussed the case of a *sudden* change of phase, and found that the boundary at which this occurs should be visible as a distinct minimum of intensity. This is borne out by observations with structures having sharply-defined laminar boundaries†, but when the boundaries are less sharp, their visibility is greatly prejudiced and becomes practically zero. In the present case, the phase-changes arise in traversing a considerable thickness of the crystal, and even if the boundaries between the oppositely active parts of the crystal were perfectly plane and normal to the surface of the section, the wave-front on emergence from the crystal cannot present any *discontinuous* changes of phase over its area. The failure to observe any marked structures in the absence of the polarizer or analyser is thus intelligible. When the polarizer and analyser are put in, the structure ceases to be merely a phase-change grating and becomes one in which there is also a periodicity of effective transparency, and an enormous increase in the visibility of the structure is the result. An improvement in the visibility of the structure of twinned crystals of potassium chlorate when polarized light is used has been noted by Lord Rayleigh‡.

## Visibility of structure

The matter may also be regarded in another way. When the incident light is unpolarized, it may be resolved into two components circularly polarized in opposite senses and having no determinate phase-relations, and the diffraction spectra given by them are merely superposed. If, however, the incident light be polarized, the two sets of circularly-polarized diffraction spectra are in a definite relation of phase which is different from that of the central undeviated pencils, since the accelerated parts of one wave correspond to the retarded parts of the other and vice versa. Hence, the introduction of an analyser enables the relative

*Lummer and Rieche, *Bildentstehung im Mikroskop*, F. Vieweg und Sohn (1910), pp. 74–76.
†N K Sur, *Proc. Indian Assoc. Cultiv. Sci.*, 7 (1922) 125–144.
‡Rayleigh, *Proc. R. Soc. London*, **A102** (1923) 669–670.

intensity of the central image and of the diffraction spectra to be varied, and therefore, in accordance with the Abbe theory, also the appearance of the structure under the microscope.

The four pictures reproduced in the plate are photographs in the polarization microscope of a practically colourless piece of amethyst quartz 2·5 mm thick. They were taken with analyser and polarizer in different relative positions, and using monochromatic light of different wavelengths. The differences in the appearance under the varying conditions are sufficiently striking. In figure 1, the nicols were crossed and green light was used. In figure 2, the nicols were parallel and violet light was used. Figures 3 and 4 correspond to intermediate positions of the nicols, figure 3 being obtained with violet and figure 4 with green light.

*Trans. Opt. Soc. Am.* **27** 51–55 (1925)

# On Brewster's bands—Part I

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MS received, 9th September, 1925. Read, 15th October, 1925

**Abstract.** The paper considers the explanation of Brewster's bands and other allied phenomena from the new and very suggestive standpoint proposed by Schuster (*Philos. Mag.* Oct. 1924). When monochromatic light is reflected by or transmitted through two parallel plates in succession, we have a superposition of the Haidinger ring-systems due to the two plates in the sense that the observed intensity in any given direction is the product of the intensities due to either plate separately. Illustrations showing the effect of such superposition in various cases are reproduced with the paper, differential and summational fringe-systems of various orders being observable. When non-homogeneous light is used, the Haidinger rings disappear and along with them also the superposition pattern, leaving only a uniform illumination in the field, except in the special case of the differential system of the first order for two plates of equal thickness. A simple geometrical explanation is thus forthcoming why Brewster's bands can be observed even in non homogeneous light with thick plates in this case.

## 1. Introduction

As is well known, coloured interference bands are observed when an extended source of light is viewed in transmission through or by reflection from two glass plates in succession when these are of equal thickness and are held at a suitable small inclination with each other. These bands, first observed by Sir David Brewster, form the basis of the very convenient interference refractometer developed by Jamin, and their explanation is therefore a matter of considerable interest and importance. In a recent and very interesting paper*, Sir Arthur Schuster has proposed a way of regarding the effects observed in this case which is different from that usually given. If we use an extended source of highly monochromatic light, and view it by transmission through or a reflection from a single plate, we should get the well known Haidinger rings, or interference-curves of constant obliquity representing the variation of the transmitting or reflecting power—as the case may be—of the plate for light of a specific wavelength in

*Schuster, *Philos. Mag.* **48** (Oct. 1924) 609–619.

different directions. If two plates are used in combination, Schuster suggests that we may regard each of them as producing its own set of Haidinger rings, but that these are superposed in the sense that the observed brightness in any specific direction is determined by multiplying the fractions of the intensity of the original source transmitted—or reflected, as the case may be—by the two plates in succession. For, if we consider a wave-front travelling out from the source in a specific direction, the two plates act on it in succession, and to find their resultant effect, we have only to multiply their transmitting or reflecting powers for the specific angles of incidence concerned. The joint effect of the two plates is thus merely to superpose their interference-patterns in the sense indicated; when the plates are inclined to each other, the centres of the two ring-systems would of course not be coincident. In his paper, Schuster has endeavoured to trace the geometric character of the complex pattern formed by the two non-concentric sets of rings.

While the treatment outlined by Schuster is of undoubted elegance and simplicity, it must be pointed out that his paper does not actually offer an explanation of Brewster's bands. For, as is well known, it is not necessary to use highly monochromatic light to observe these bands, and they may be seen even with white light and thick plates. Under these conditions, the Haidinger rings due to a parallel plate cannot be observed at all, and it is by no means obvious without explanation why, when each plate separately gives merely uniform illumination, their superposition should give any observable fluctuations of intensity in the field. To furnish an explanation of this and to extend the theory from the new standpoint in other directions is the purpose of the present paper. In part I we shall consider the geometric character of the patterns formed by the superposition of non-concentric ring-systems and explain why Brewster's bands can be observed in non-homogeneous light provided the plates are of equal thickness. In part II we shall consider the explanation of the general (elliptic) form of Brewster's bands observed in monochromatic light with plates inclined at any angle. In part III the character of Brewster's bands as observed with plates of doubly refracting crystals will be discussed.

## 2. The geometric character of the superposed patterns

To illustrate the effect of multiplying the distribution of light-intensity in two sets of rings when their centres are not coincident, we have prepared the prints reproduced in figures 1 to 6. Of these, figures 1 to 4 represent the superposition of two *identical* ring-systems with gradually increasing distance between their centres. Figures 5 and 6 represent the effect of superposing two *dissimilar* ring-systems, the centres of which are moderately removed from each other in figure 5, and still farther apart in figure 6. The figures were prepared by the very simple expedient of printing twice on bromide paper from negatives on which alternate

bright and dark rings were spaced in the manner of a Fresnel zone-plate. The same negative was used twice with a suitable displacement for preparing figures 1, 2, 3 and 4. For preparing figures 5 and 6, two different negatives with ring-systems of unequal size were successively printed on the same sheet of bromide paper.

The law of spacing of the Haidinger rings in a parallel plate of refractive index $\mu$ and thickness $t$ is $2\mu t \cos r = n\lambda$, where $r$ is the angle of refraction within the plate. If $i$ be the angle of emergence from the plate, the law may be written in the form

$$2t\sqrt{\mu^2 - \sin^2 i} = n\lambda.$$

For moderately small values of $i$, the law of spacing of the rings is very similar to that on a Fresnel zone-plate. Hence figures 1 to 4 may be regarded as correctly representing the effect of superposing the Haidinger ring-systems of two plates of *equal* thickness moderately inclined to each other, while figures 5 and 6 may be regarded as doing the same for two plates of *unequal* thickness.

The characteristic feature shown in figures 1 to 3 is the system of rectilinear equidistant bands running transverse to the line joining the centres of the ring-systems. These bands are very broad in figure 1, corresponding to a small inclination of the plates, and become closer and closer together with increasing inclination of the plates until, as in figure 4, they are so close together as to be lost to the eye in the general structure of the field. At the same time, other features appear in the superposition pattern, viz., the two external circular fringe-systems shown in figure 3, and the internal circular fringe-system shown in figure 4. The significance of these other features we shall consider presently. A noteworthy feature of the rectilinear bands is that the central bright one passing through the point midway between the centres of the two systems is always fixed in position and orientation.

We have now to consider the manner in which the superposition patterns are formed. Taking first the case in which the two sets of rings are identical in size, we may, following Schuster, take the centre of each system to be a point of maximum intensity; the radii of the rings having maximum intensity are (approximately) in the ratio of the square roots of successive integral numbers. Referred to a system of co-ordinates in which the line joining the two centres is the axis of $x$ and the origin is a point midway between the centres, the rings are represented by the equations

$$y^2 + (x - a)^2 = R^2 m, \quad y^2 + (x + a)^2 = R^2 n,$$

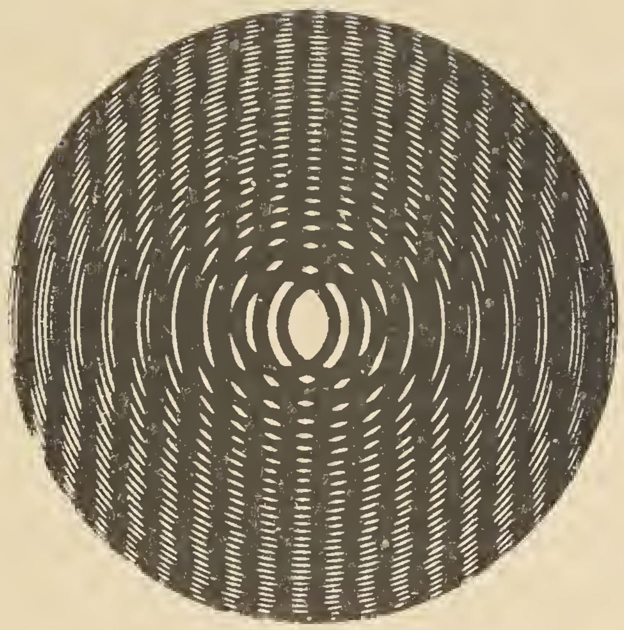
where $m$ and $n$ are integers and $R$ is a linear constant. From these equations, we readily obtain by addition and subtraction

$$4ax = R^2(n - m), \tag{1}$$

$$y^2 + x^2 + a^2 = \tfrac{1}{2}R^2(m + n), \tag{2}$$

$$y^2 + x^2 + 6ax + a^2 = R^2(2n - m), \tag{3}$$

(1)



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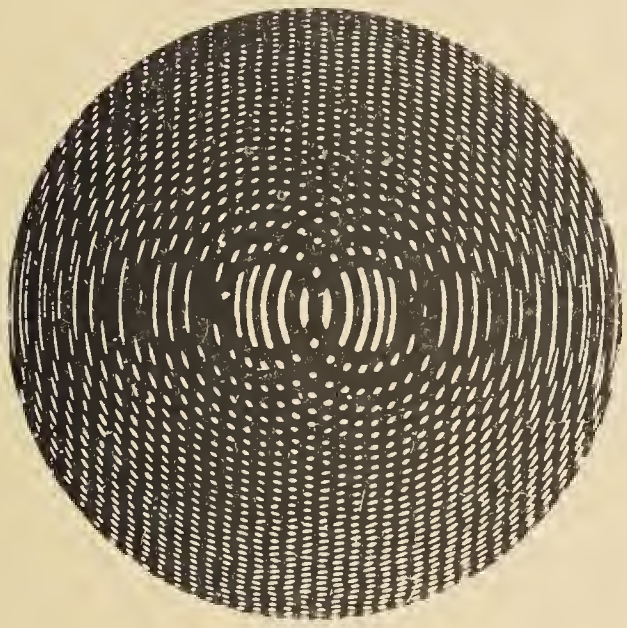

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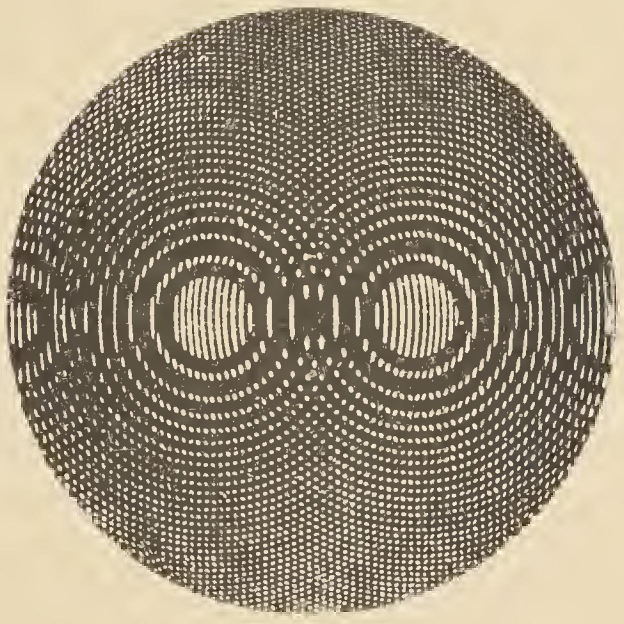

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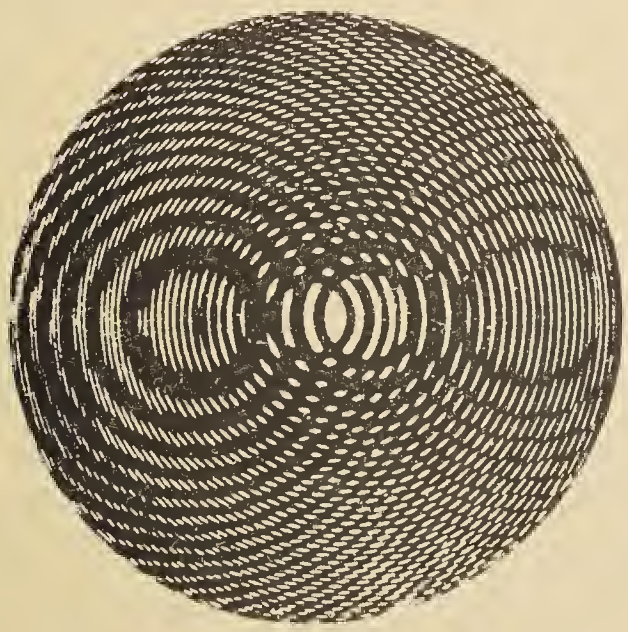

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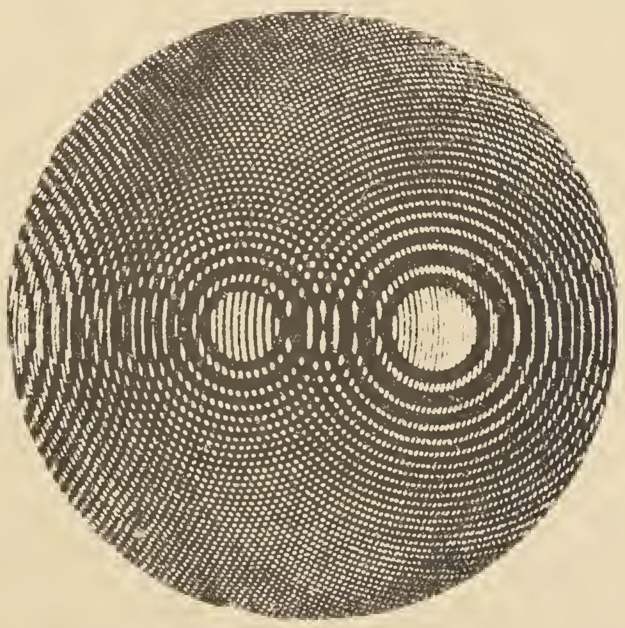

(6)

**Figures 1–6**

$$y^2 + x^2 - 6ax + a^2 = R^2(2m - n), \tag{4}$$

and so on. Equation (1) represents a set of straight lines parallel to the $x$-axis. Equation (2) represents a set of circles with their centres at the mid-point between the centres of the ring-systems. Equations (3) and (4) represent sets of circles with their centres at $x = \pm 3a$, and so on. These loci contain points where the maxima of intensity of one set of rings coincide with the maxima of the other set and the minima of one set with the minima of the other. Alternating between these, we have other loci of similar shape where the maxima of one set coincide with the minima of the other set, and vice versa. Since the intensity in the superposed patterns is the product of the intensity in the patterns due to the two plates separately, the loci mentioned represent lines along which there is alternately a general strengthening and weakening of the ring-pattern. This is the feature exhibited in figures 1 to 4. The systems represented by equations (2), (3), and (4) become prominent in the superposition pattern only when the rectilinear fringes given by (1) are so narrow as not to obscure the structure of the field.

The case of the superposition of two ring-systems of unequal size may be similarly discussed. We have as the equations of the ring-systems

$$y^2 + (x - a)^2 = R_1^2 m, \quad y^2 + (x + a)^2 = R_2^2 n,$$

where $R_1$ and $R_2$ are now different linear constants. Adding and subtracting we have

$$(R_1^2 - R_2^2)(x^2 + y^2 + a^2) + 2(R_1^2 + R_2^2)ax = R_1^2 R_2^2(n - m), \tag{5}$$

$$(R_1^2 + R_2^2)(x^2 + y^2 + a^2) + 2(R_1^2 - R_2^2)ax = R_1^2 R_2^2(m + n), \tag{6}$$

$$(2R_1^2 - R_2^2)(x^2 + y^2 + a^2) + 2(2R_1^2 + R_2^2)ax = R_1^2 R_2^2(2n - m), \tag{7}$$

$$(2R_2^2 - R_1^2)(x^2 + y^2 + a^2) - 2(2R_2^2 + R_1^2)ax = R_1^2 R_2^2(2m - n), \tag{8}$$

and so on. These equations also represent sets of concentric circles, the centres lying on the $x$-axis.

## 3. Phenomena in non-homogeneous light

We are now in a position to discuss the effect of using non-homogeneous light on the appearance of the superposed patterns. An alteration of wavelength changes the linear constant of the Haidinger ring-systems, which consequently expand or contract, as the case may be. If we were considering only each plate separately, the fluctuations of intensity in the field would be wiped out completely, and we would get only uniform illumination. With two plates, as the wavelength is altered, a radial movement of each ring-system occurs and the superposition patterns also alter. The *circular* fringe-systems formed by superposition, whose equations are given in (2), (3), (4), (5), (6), (7), and (8), move radially from their respective centres,

and when any appreciable range of wavelengths is used, they are completely wiped out and disappear. But the rectilinear fringes given in equation (1), which correspond to the differential-pattern for two plates of *equal thickness*, behave otherwise. For the central bright fringe of this system cuts the circular rings orthogonally and therefore remains fixed in position, and the same is also true *approximately* of the other fringes running parallel to it on either side. From equation (1) it can be seen that an alteration of $R$ simply means a change of the width of the rectilinear fringes. Thus in white light the system would remain visible, the central fringes being achromatic and the outer ones coloured. It is only in highly monochromatic light that we can expect to see the circular fringe-systems corresponding to the summationals and differentials of higher order. As a matter of fact, using two plane-parallel glass plates 1 mm thick and sodium light, the Haidinger rings and also the superposition pattern consisting of the rectilinear fringes corresponding to the differential system, and the circular fringes corresponding to the summationals and the differentials of higher order, can be readily observed. We have studied these features and will describe them more fully in part II.

The first differential system for two plates of equal thickness appears in equation (1) as a system of straight lines. This is only the case when the Haidinger rings are taken to be spaced exactly according to the law of the square roots of the natural numbers. Actually, as is well known, the Haidinger rings in a glass plate are spaced somewhat differently, the successive rings becoming closer and closer together only up to about an angle of emergence of 45° and then widening out again. As a consequence of this, the differential pattern is not really a system of straight fringes throughout the field, and actually consists of a system of closed ellipses. The modification of the theory necessary on this account and the special phenomena observed in crystalline plates will be considered more fully in parts II and III of the paper*.

We have to thank Mr Asutosh Dey for preparing the illustrations appearing with this paper.

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*Not published.

*Proc. Phys. Soc. London* **38** 350–353 (1926)

# On the diffraction of light by spherical obstacles

PROFESSOR C V RAMAN, F.R.S.
and
Mr K S KRISHNAN

ABSTRACT

The diffraction of light inside the shadow, thrown by a small source of light, of a sphere and a circular disc of the same diameter, was studied, with special reference to the relative intensities of the central bright spots. With the source at about 2 metres from the obstacles, with a quarter-inch polished steel ball, the bright spot could be detected visually up to 3 cm behind the obstacle, while with a steel disc of the same diameter, with the edges perfectly sharp, smooth and circular, the spot could be traced up to 2 cm.

The relative intensities of the two spots were studied at different distances behind the obstacles, qualitatively by photography and quantitatively by visual photometry. At small distances behind the obstacles, the spot inside the shadow of the sphere is much feebler than the disc-spot, however approximating to the latter as we reach farther back from the obstacles, but even at 100 cm remaining appreciably feebler.

A general explanation is suggested.

## I. Introduction

It has long been known* that at the centre of the shadow of a spherical obstacle thrown by a small source of light there is a bright spot similar to that found in the shadow of a circular disc; and, in fact, a spherical obstacle is often used instead of a disc to demonstrate the formation of the bright spot at the centre of the shadow of a circular boundary. It is usually assumed by experimenters† that at a point on the axis of the shadow a circular disc and a sphere of equal radius would give practically identical results. This, however, is not actually the case, and it is the purpose of this paper to draw attention to the notable differences that exist between the effects observed in the two cases.

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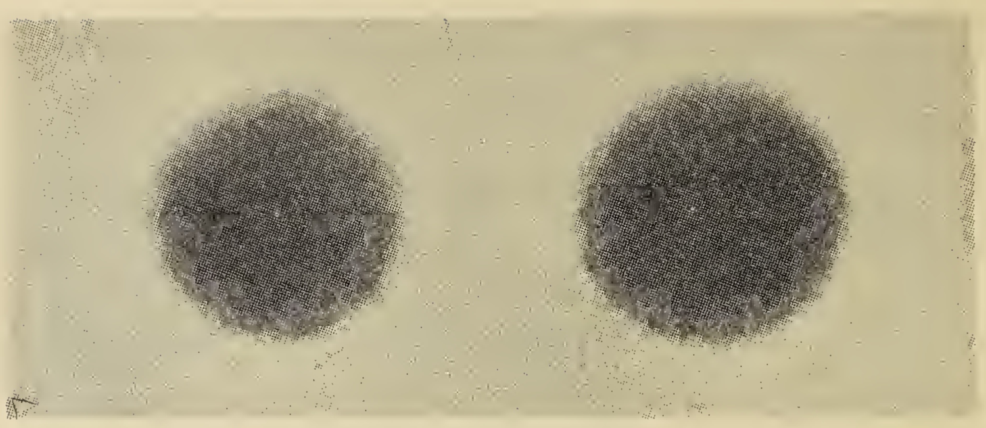
*Rayleigh, *Scientific Papers*, 5, p. 112, *see* also A O Rankine, *Proc. Phys. Soc. London*, **37**, p. 267 (1925).
†Hufford, *Phys. Rev.* 7, p. 545 (1916).

## II. The intensity of the bright spot

To compare the effects obtained in the shadow of a spherical obstacle and a circular disc of equal size, it is convenient to mount them side by side on a glass plate, so that the bright spots at the centres of their shadow may be seen at the same time. In most of our observations we used a quarter-inch (diameter) steel ball and an accurately made steel disc of the same size, cut on the lathe so as to have a sharp circular edge of razor-like smoothness and sharpness. They were attached by specks of wax, with sufficient space between them, to a glass plate, and held at a distance of about two metres from the source.

The diffraction patterns within the shadow of disc and sphere were seen simultaneously through a lens of sufficiently wide field of view. When a bright source of light is used, it is convenient to use a plate with two holes cut in it, to correspond with the shadow of the sphere and the disc, and place it in the field of view so as to cut off all extraneous light except that diffracted into the region of shadow. The removal of the glare outside the region of shadow is very helpful, and with this arrangement it is possible to trace the bright spots in the centre of the shadow up to within 3 cm of the object in the case of the sphere, and to less than 2 cm in the case of the disc, thus testifying to the accuracy of the edges. A series of photographs were taken of the diffraction patterns with the source of light 179 cm in front of the obstacles, and with the object plane of the camera at different distances behind them. Some of these are reproduced here (figures 1, 2, 3, 4). In the photographs the diffraction pattern on the right corresponds to the sphere and that on the left to the disc. We can easily see that the central white spots in the case of the sphere are much less bright than in the case of the disc. Thus, in figure 1, which corresponds to a distance of about 11 cm behind the obstacles, the spot in the case of the sphere is invisible in the photograph. At 13 cm, as shown in figure 2, it is just visible. At 25 cm, (figure 3) it is still much feebler than for the disc, and at 40 cm (figure 4) the difference in intensity of the two bright spots is still conspicuous. Further, we see in the photographs that the general illumination within the geometrical shadow is much greater for the disc than for the sphere. The spots in the shadow of the sphere were distinctly reddish in comparison with those for the disc, and the photographic intensity thus differed more than the visual intensity.

A quantitative study of the relative intensities of the central white spots of the two diffraction patterns was made with the help of an Abney rotating sector photometer placed just in front of the obstacles, and looking for the diffraction patterns through the eye-glass with the two apertures in its focal plane, mentioned already. The source of light was at a distance of 232 cm in front of the obstacles. The results are shown in the graph on p. 351 (figure 5). Owing to the difference in colour, some uncertainty arises in the visual estimates of equality of intensity. Further, for short distances behind the obstacles the comparison was by no means easy, owing to the spots having a very small size, and appearing against



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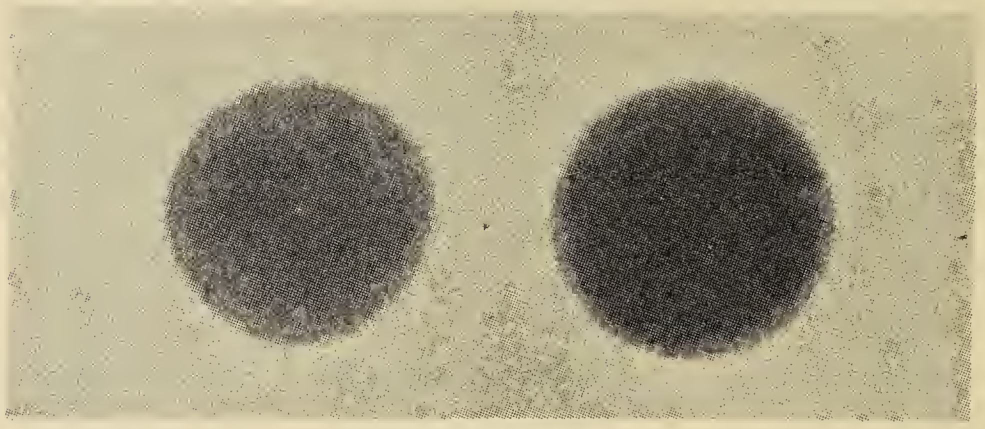

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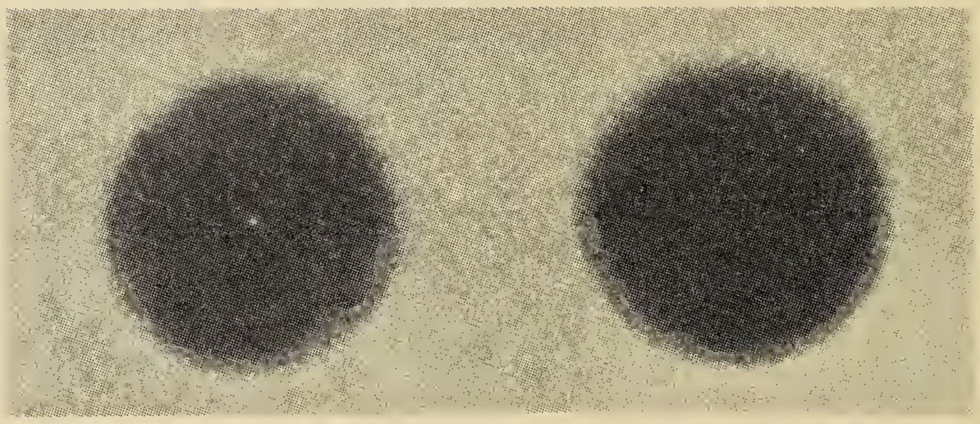

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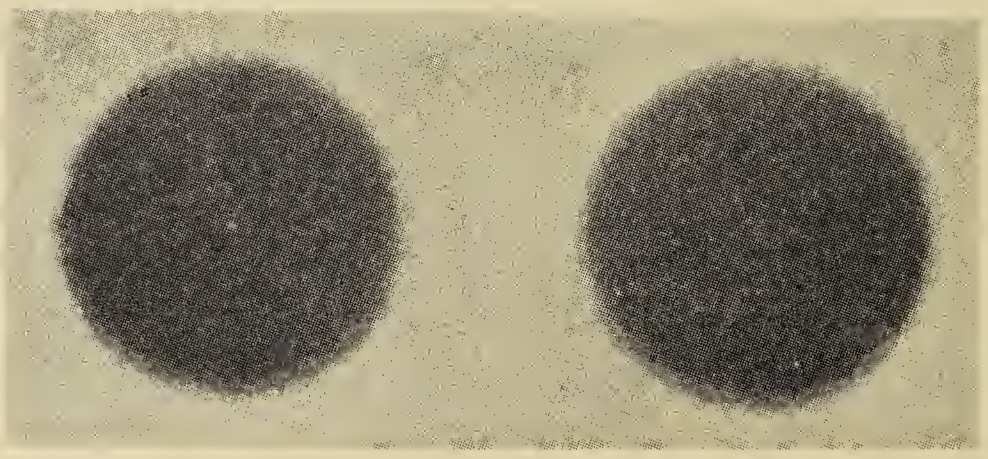

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**Figures 1–4**

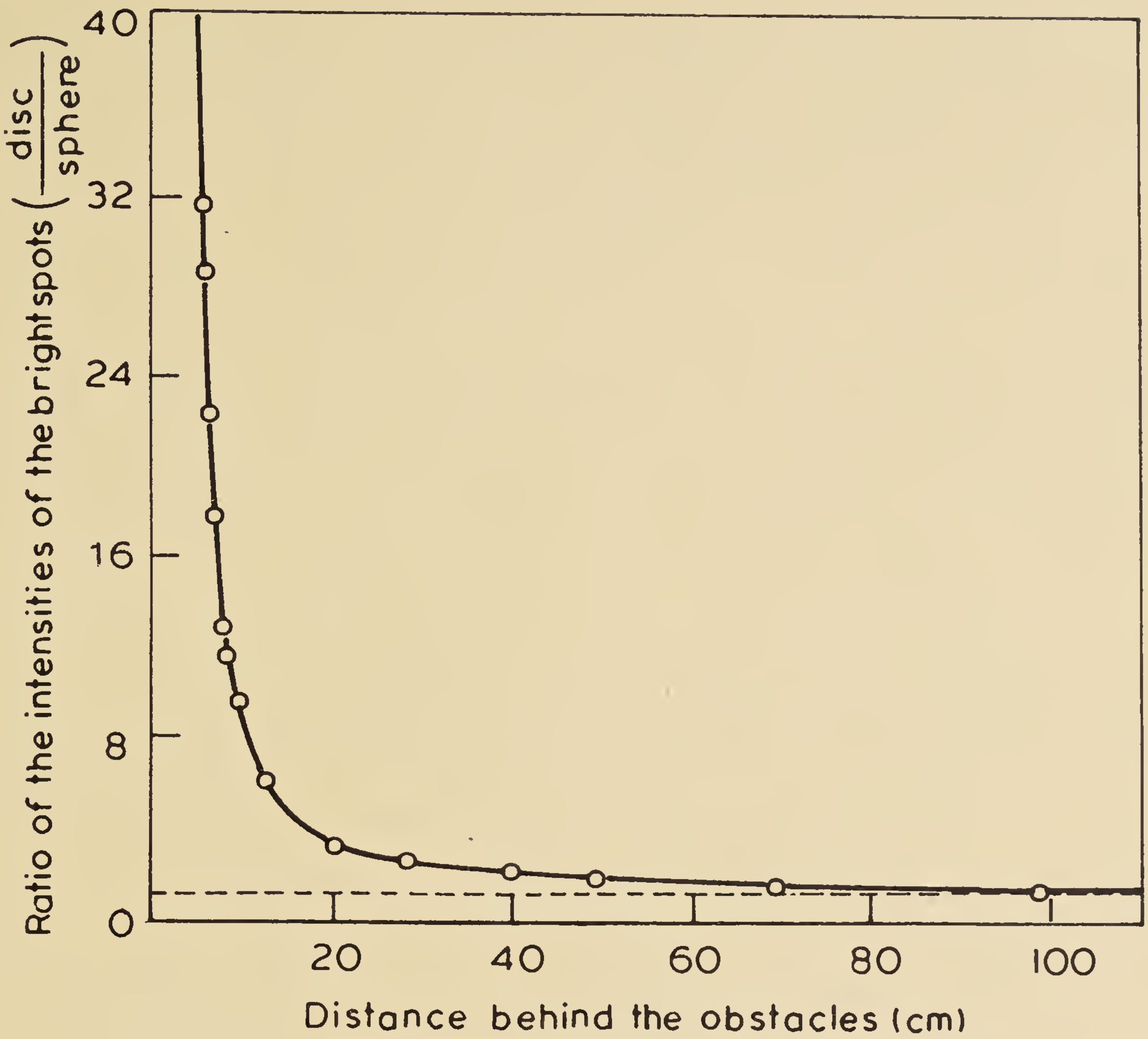



**Figure 5**

a luminous background. Owing to these circumstances, the measurements shown in the graph are only approximate. Nevertheless, they sufficiently indicate the general character of the phenomenon. The dotted line in figure 5 is the asymptote to the curve, and is slightly above the line of equality of intensities.

## III. Discussion of results

Without going into the mathematical theory of diffraction by a spherical obstacle, it is not difficult to give a general physical explanation of the above experimental results. In the case of the disc (figure 6a) the rays diffracted by the illuminated edge reach the point of observation directly. In the case of the sphere, however, the position is somewhat different. Drawing tangent cones enveloping

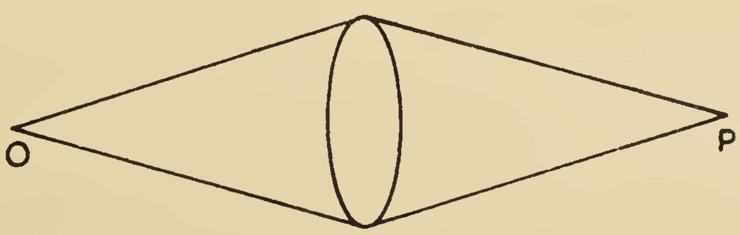



**Figure 6a**

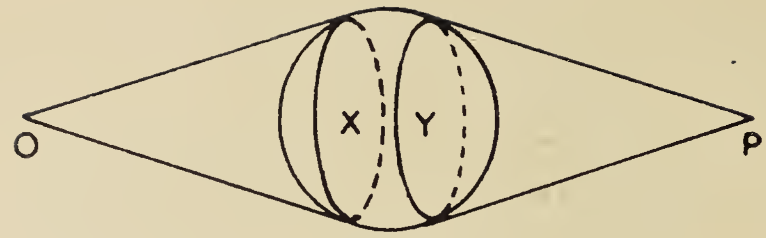



**Figure 6b**

the spherical obstacle, with the source and the point of observation as apexes (figure 6b), we see that they now touch the sphere at different circles of contact, X and Y respectively. Thus, the circle of contact Y, from which diffracted rays originating at the surface can reach the point of observation, lies within the region of geometrical shadow, and not at its edge, as in the case of the disc. The disturbance incident on the surface of the sphere has to creep round it, as it were, over the arc XY before the rays diffracted out by the sphere can reach the point of observation, and must suffer a very considerable diminution in the process. Thus, we can see that the intensity of the central white spot in the diffraction pattern of a sphere will be less than in the case of the disc at the same distance behind by a quantity depending on the length of the arc XY between the circles of contact of the tangentially incident and diffracted rays. Now the length of this arc will be the greater the nearer the point of observation approaches the sphere, so that the intensity of the sphere-spot, as compared with the disc-spot, ought to decrease as we approach the obstacles. Proceeding in opposite direction, the intensity at large distances will approach that of the disc, but still will be smaller than the latter by an amount which will depend on the distance of the source from the obstacle.

That the foregoing way of viewing the matter is not fanciful, but is really a statement of the physical processes occurring in the case, is evident from the following observations. A microscope is focussed tangentially on the circle of contact X already mentioned, which appears as a luminous edge in the field of view. If now the microscope is shifted laterally into the region of the geometrical shadow, we find that it has also to be drawn back longitudinally towards Y in order to keep the diffracting edge of the sphere in focus, whereas in the case of the disc such longitudinal movement is not found to be required. Further, the luminous edge of the sphere is found to diminish in brightness much more rapidly than in the case of the disc when the observer's eye is moved laterally into the region of shadow. Similar differences are also found when we compare the diffraction into the region of shadow by a sharp straight edge and by the edge of a cylinder.

We recognise that the explanation we have offered is only qualitative. The reality of the effects described is, however, unquestionable, and we have no doubt that a quantitative explanation will be forthcoming when the diffraction problem is considered on the basis of the electromagnetic theory for the case of the large sphere. This problem has been handled by Poincaré, Nicholson, Macdonald, Bromwich, G N Watson and others. The paper by Macdonald, on "The

Diffraction of Electric Waves Round a Perfectly Reflecting Obstacle,"* might in particular be referred to, as the analysis contained in it approaches most closely to the point of view from which we have explained our experimental results. The formulae given by Macdonald are, however, not in a form capable of immediate application to the problem without considerable labour. As the experimental work was completed last summer, and as we are at present engaged on other work, we have thought it best not to defer publication of the results any longer.

***Philos. Trans. R. Soc. London*, **A210**, 113 (1910). For other references *see* Bateman, "Electrical and Optical Wave Motion."

*Philos. Mag.* **50** 812–815 (1925)

# On the nature of the disturbance in the second medium in total reflection

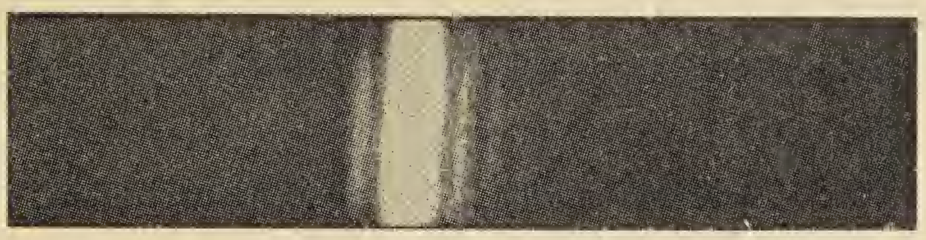
PROFESSOR C V RAMAN, F.R.S.
(University of Calcutta)*

[Plate I]

Some five years ago[†], an investigation was carried out in the author's laboratory of the diffraction phenomena observed when a pencil of monochromatic light is incident at slightly less than the critical angle on the boundary between two media (say, glass and air) and emerges into the second medium almost grazing the surface. The results were very interesting and suggestive, and when communicating the paper describing them for publication, I had intended to pursue the matter further and to investigate by the same methods the cases in which the incidence is made equal to the critical angle or is increased even further. But other work intervened and the subject was laid aside. Sir Arthur Schuster's recent paper on Total Reflection[‡] recalled the subject to mind, and induced me to make some further observations. The present paper describes the experimental facts which are significant enough, and are fairly easily understood from the point of view adopted in Dr Chuckerbutti's paper, but do not seem to fit in with Sir Arthur Schuster's way of looking at the matter.

It is useful briefly to recall Dr Chuckerbutti's work. Owing to the enormous dispersion which occurs when light is incident on a glass-air boundary at or near the critical angle, it is necessary to use highly monochromatic light, e.g., the green or violet line of the mercury arc: the observations are readily made on an ordinary spectrometer. A pencil of light whose aperture is limited by parallel straight edges[§] passes through a prism of glass, is incident on its second face and emerges nearly grazing the surface. Owing to the narrowness of the emergent pencil, it is considerably spread out by diffraction, and the Fraunhofer pattern seen in the observing telescope shows some remarkable features. It is strongly asymmetrical,

*Communicated by the author.
[†]B N Chuckerbutti, *Proc. R. Soc. London* **A99** p. 503 (1921).
[‡]*Proc. R. Soc. London* **A107** p. 15 (1925).
[§]As noted by Dr Chuckerbutti, the place at which the limitation of aperture occurs is not a matter of importance so long as such limitation occurs before actual emergence of the refracted rays from the boundary.



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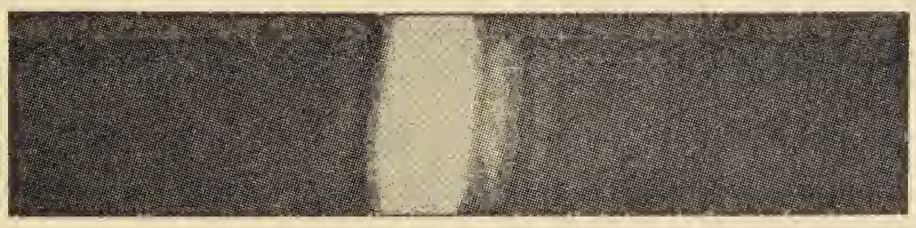

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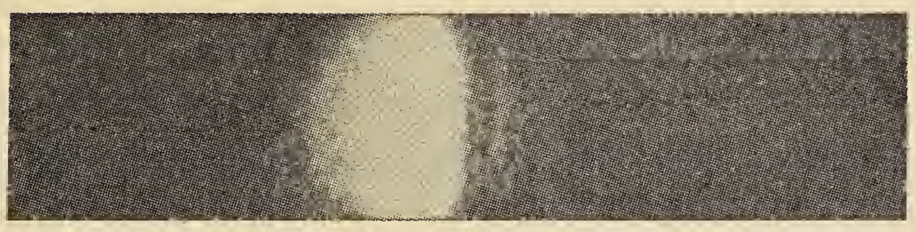

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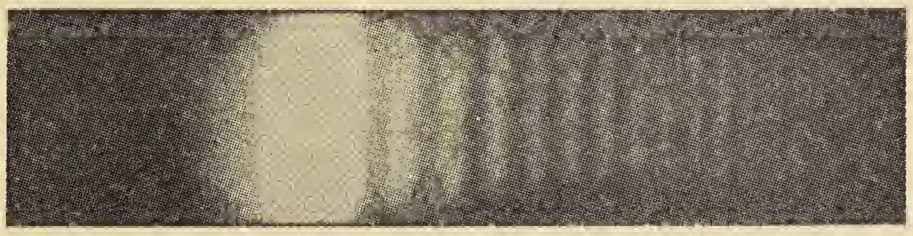

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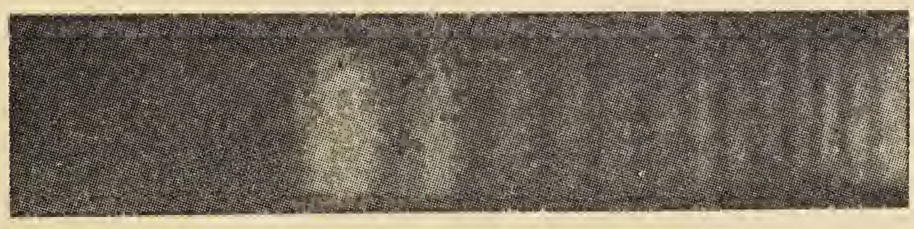

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**Figures 1–5.** Illustrating total reflection phenomena.

**Plate I**

the bands on one side of the pattern being wider, fewer in number, and much fainter than the corresponding bands on the other side of the pattern. These features are clearly shown in figures 1, 2, 3, in plate I which represent the green line of mercury as spread out by diffraction at gradually increasing angles of

incidence. In figure 1 the incidence was at appreciably less than the critical angle, in figure 2 it was slightly less, while figure 3 corresponds practically to the case of critical incidence. Panchromatic plates were used, and owing to their comparative insensitiveness to the green rays, long exposures were necessary. Visually, the diffraction bands are broad and quite conspicuous.

When the incidence is increased beyond the critical angle, the principal band in the diffraction-pattern moves out and disappears from the field, followed in succession by one or more of the others accompanying it, but a great many of the long train of bands remain in the field and are just as distinctly visible as ever. Visually with the green of the mercury arc, some thirty or forty of the bands may be seen and counted, the particularly astonishing feature being the slowness of the rate at which the intensity in successive bands falls off. For photographing these stages the violet line of the mercury arc and Ilford Zenith plates proved very convenient. Figure 4 shows the stage at which the incidence is just at the critical angle and the principal band is still visible. In figure 5 it is at more than the critical angle, and the principal band has moved out of the field altogether, nevertheless the long trail of dark and bright bands in the pattern behind it remains as conspicuous as ever. They can be traced even with angles of incidence considerably in excess of the critical angle.

The phenomena may be explained in the following simple way. If $a$ be the aperture of the illuminated surface, $i$ the angle of incidence, $\theta$ the angle of diffraction, $\mu$ the refractive index of glass, the positions of the dark bands in the diffraction pattern are given by the equation tested in Dr Chuckerbutti's work

$$Z = \pi a/\lambda \cdot (\mu \sin i - \sin \theta) = \pm n\pi,$$

where $n$ is an integer. If $\mu \sin i < 1$, both positive and negative values of $Z$ are admissible, but if $\mu \sin i > 1$, which corresponds to the case of "total" reflection according to geometrical optics, only positive values of $Z$ are possible and correspond to the portion of the diffraction-pattern which continues to be visible.

The rate at which the successive fringes diminish in intensity is certainly far less than in proportion to $\sin^2 Z/Z^2$. An additional factor, namely the increasing width of the diffracted beams as $\theta$ alters, has also to be considered as shown by Dr Chuckerbutti's measurements, and the successive bands should have intensities proportional to $\cos^2 \theta \cdot \sin^2 Z/Z^2$. This would account for the large number of bands actually seen in the cases of critical incidence and total reflection.

To find out what was happening in the immediate vicinity of the surface at which "total" reflection occurs, a microscope placed with its axis nearly parallel to the surface was focussed on it. It was then observed that the two edges of the illuminated area of the surface appeared as extremely fine luminous lines. To focus the front and rear edges, the microscope had, of course, to be in different positions. *Both* the edges were thus found to take part to an equal extent in the

observed diffraction effects. Experiment does not thus lend any support to Sir Arthur Schuster's suggestion that one of the edges plays a privileged part in total reflection phenomena.

Dr Chuckerbutti in his paper had already pointed out the analogy between the case of refraction by a surface at less than the critical angle and the case of oblique transmission through a plane aperture bounded by straight edges, and Sir Arthur Schuster has based his discussion of total reflection on an extension of this analogy. The present writer has found experimentally, however, that the analogy breaks down when the incidence in the case of the plane aperture becomes excessively oblique. One of the edges then begins to screen the other from the infalling waves, and the diffraction-pattern becomes excessively feeble and almost disappears when this stage is reached and passed. In the case of total reflection, however, this is not true, and as observation shows, part of the diffraction-pattern remains visible just as conspicuously as ever. The analogy between the case of total reflection and that of an aperture in an opaque screen bounded by two straight edges is therefore wholly invalid.

The penetration of the disturbance into the second medium in the case of a beam of finite width illustrated in figure 5 is also seen conspicuously when, instead of a single aperture, a system of parallel equidistant apertures forming a diffraction-grating limits the incident beam. A long train of diffraction-spectra may then be observed emerging into the second medium, even when the angle of incidence is such that total reflection should occur according to the laws of geometrical optics. Also, in the case of the Lummer–Gehrcke interferometer studied by Dr Chuckerbutti, I have found that the well-known bands in this instrument can be quite distinctly seen even when the light traversing the plate is incident on its surface at an angle exceeding the critical angle.

## Summary

Photographs have been secured showing the penetration of the disturbance into the second medium which occurs by reason of the limitation of aperture when a pencil of light of finite width is incident at more than the critical angle on the boundary between two media. A long train of diffraction bands can be seen in which the intensity of the successive bands diminishes relatively very slowly. The phenomenon can also be observed in the Lummer Plate. A simple explanation is offered of the results which fit in very well with Dr Chuckerbutti's earlier work on the case in which the incidence is at less than the critical angle, but do not support the considerations advanced by Schuster.

210 Bowbazaar Street, Calcutta, India
14 May 1925

*Proc. Indian Assoc. Cult. Sci.* 9 271–286 (1926)

# On the total reflection of light

PROFESSOR C V RAMAN, F.R.S.

## 1. Introduction

It has long been known that the explanation of the phenomenon of total reflection of light on the principles of the wave-theory involves the existence in the second medium of a disturbance which penetrates beyond the boundary to a depth depending on the angle of incidence and diminishing exponentially as the perpendicular distance of the point of observation from the boundary is increased. Stokes* showed how the expression for this disturbance which he designated as a *superficial undulation* may be derived directly from the Fresnel formulae for the intensities of the reflected and refracted beams of light, and applied the same method to the investigation of the appearance of the central spot in Newton's Rings formed beyond the critical angle of incidence. A discussion of the problem on the principles of the electromagnetic theory is given in Drude's[†] Theory of Optics, where the question of the flow of energy in the second medium is also considered on the basis of Poynting's Theorem. That the superficial disturbance in the second medium must be a physical reality is indicated by the consideration that it is closely related to the changes of phase occurring in total reflection and that the same theory which predicts it also gives a quantitative explanation of the elliptic polarisation actually observed when light plane-polarised in any azimuth is totally reflected.[‡] Further, the phenomena of Newton's Rings beyond the critical angle, already mentioned, and the fact that small particles placed in contact with the boundary in the second medium are observed to scatter light when viewed through a microscope are usually regarded as confirming the theory. Some doubt has however been thrown on the usual treatment in a recent theoretical paper[§] by Sir Arthur Schuster who appears to

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*Math. Phys. Papers*, **2**, 57.

[†] English Translation, p. 299.

[‡] So far as the writer is aware, no measurements of the *absolute* change of phase of the light-vector for the two principal components taken *separately* have been made for the case of total reflection at any angle. For the case of total reflection at grazing incidence however, Bevan has made observations by the method of Lloyd's interference-fringes (*Philos. Mag.*, Oct. 1907), which are in agreement with theory.

[§] *Proc. R. Soc. London* **A107**, p. 15.

hold that the assumption made in the theory of an infinitely extended surface is essentially illegitimate. Moreover, the present writer has recently shown* that by using intense monochromatic light (the green or violet line of the mercury arc), and an ordinary spectroscope, the light emerging from the second face of a prism on which the light was incident at an angle greater than the critical angle could be readily observed. Photographs showing the disturbance emerging into the second medium were published, and they clearly indicated that the effect observed was due to the limitation of the aperture of the pencil incident on the surface and was thus primarily a phenomenon of diffraction. Similar effects were also observed with a Lummer-Gehrcke plate when light was incident within the plate at an angle greater than the critical angle. These effects clearly indicate that diffraction does play a part in the phenomena of total reflection, and it becomes necessary to consider the matter afresh in the light of the new experimental evidence now available. It is proposed in this paper to consider, *de novo*, the phenomena of total reflection from the point of view of diffraction theory.

## 2. Application of the Fresnel–Huyghens principle

In the general explanation of total reflection first given on the principles of the wave-theory by Huyghens, the elementary parts of the boundary between the two media are regarded as the source of secondary wavelets emerging into both media. That there is no refracted wave in the second medium though the boundary is fully illuminated is a consequence of the fact that no common envelope can be drawn to the wavelets emerging into it. There is little doubt that the more recondite phenomena accompanying total reflection may also be explained by following up Huyghens's original line of thought and applying the principle of interference. In particular, the disturbance existing in the second medium at points close to the boundary, and the diffraction effects arising from the finiteness of the illuminated area should both be capable of determination in this way.

The first step in such a treatment is the marking out of the Fresnel zones on the boundary between the two media. When this is of limited area and the point at which the effect is to be determined is far away from it, the Fresnel zones obviously become parallel rectilinear strips on the surface, and the determination of the integrated effect due to all the zones follows the ordinary methods of diffraction theory. We find in fact that the surface on which light is incident beyond the critical angle and is "totally" reflected sends out into the second medium streamers of light giving rise to diffraction-patterns in the usual way. These diffraction-patterns differ however from those of the ordinary kind in being

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* *Philos. Mag.*, 6th Series, **50**, 812.

strongly asymmetrical in character and also "truncated," that is to say, they consist only of certain outlying and relatively faint parts of the diffraction-patterns associated with the forms of aperture used, the principal and relatively intense parts being absent. For, none of the Fresnel zones included within the area correspond to a pole or region of stationary phase. Since the light thus streaming into the second medium represents energy, the reflection occurring at the boundary technically ceases to be total, though practically, the departure from totality is negligibly small, unless the aperture is very small or the incidence is only slightly greater than the critical angle. The streamers of light emerging into the second medium, have, as in all cases of diffraction, their origin at the margins of the diffracting area. The front and rear parts of the boundary are, at distant points, equally operative. The observations of Dr Chuckerbutti* and those of the present writer already quoted on the effects observed with a surface bounded by parallel edges entirely agree with these indications of theory. One special feature which comes into prominence in these observations and which deserves to be emphasised is that the intensity of the diffraction-pattern is zero at all distant points lying in the plane of the boundary when produced, that is to say, at all points from which when viewed, the angular aperture of the illuminated area is zero. As we may move away from this plane, the diffraction-pattern steadily gains in intensity. This may be regarded as an effect due to the variation of the "obliquity-factor" of diffraction, and is, in fact, thus explained in the papers already quoted. It has the influence of altering enormously the relative intensities of the diffraction-bands and making them very different from those calculated in the usual way.

## 3. Disturbance at points close to the surface

The same method of treatment may be applied to the other case of interest, namely, the effect at a point in the second medium very close to the illuminated area. It is a fact of observation that for angles of incidence exceeding the critical angle, the illumination dies away very quickly as we move away from the surface and the chief interest is thus in determining the effect at points lying within a distance of a few wavelengths from it. At such small distances, the usual approximate methods of finding the effect due to the Fresnel zone and of integrating the same over the whole of the surface of resolution are not quite rigorous. Nevertheless, as will be shown below, they may be applied with success to the elucidation of the particular case under consideration. In fact, even by merely considering the geometrical form of the Fresnel zones, a considerable insight into the problem may be obtained.

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* *Proc. R. Soc. London* **A99**, 503, 1921.

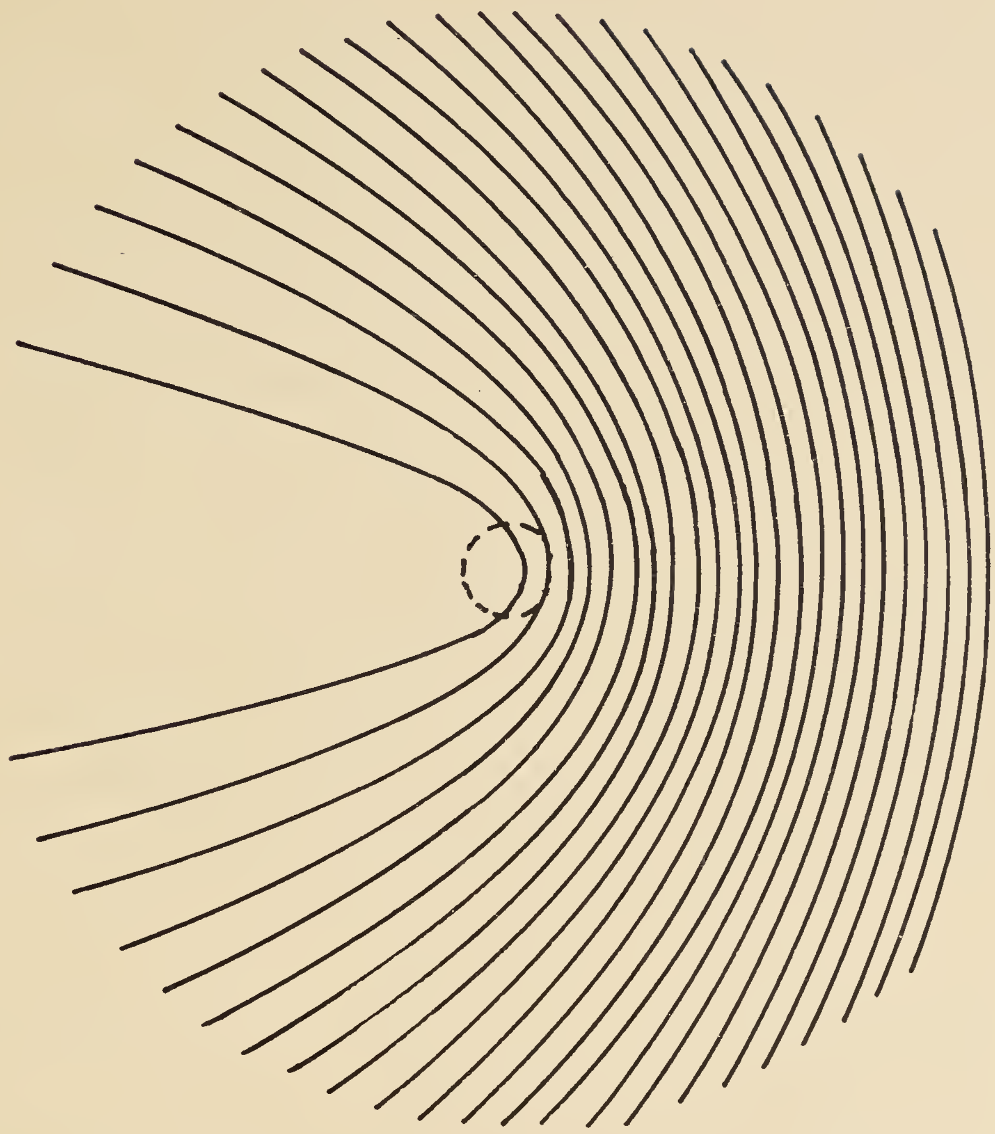

**Figure 1.** Fresnel zones on surface. Incidence = 45° = critical angle. Point of observation is on the surface.

The form of the Fresnel zones over the surface for any angle of incidence and for any point of observation may be readily mapped out in the following way. From the point of observation, a perpendicular may be dropped on the surface, and round its foot as centre, a set of circles spaced at half-period intervals from the point of observation are drawn. Crossing these are drawn a set of equidistant straight lines perpendicular to the plane of incidence and spaced at such intervals that the distance from one straight line to the next corresponds to a change of phase of the incident waves of half a period. The circles may be numbered, commencing from the centre outwards, 0, 1, 2, 3, 4, 5, etc. The straight line passing through the centre may be numbered 0, and those to the right of it, 1, 2, 3, 4, etc., and those to the left of it − 1, − 2, − 3, − 4, etc. The points of intersections of the circles and straight lines are then marked with the sum of the index-numbers corresponding to the particular circle and straight line cutting at each such point. These index-numbers represent the total difference of path between the secondary waves reaching the point of observation from the nearest element of the surface and from any other. Smooth curves may now be drawn free-hand or with the aid of a flexible steel strip through all the points having identical index-numbers. Very instructive diagrams may be obtained in this way for any specified angle of

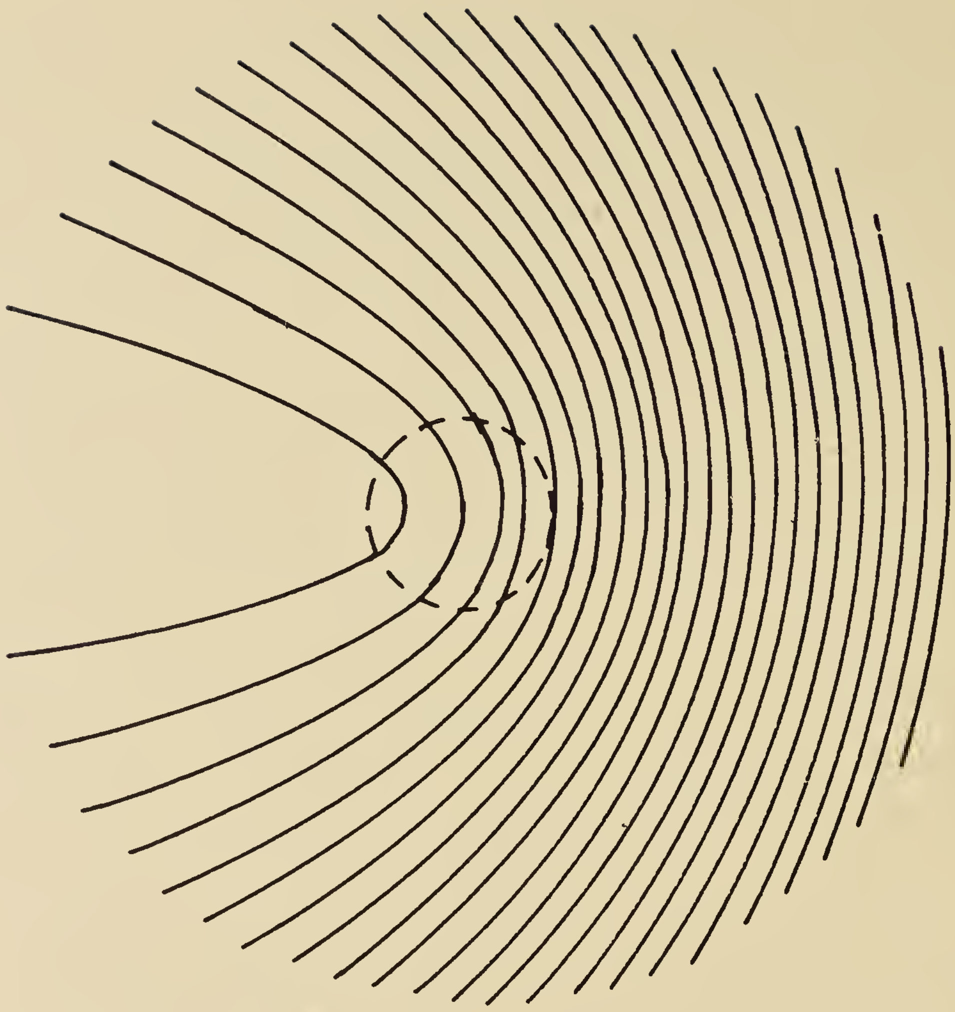

**Figure 2.** Fresnel zones on surface. Incidence = 45° = critical angle. Point of observation is $\lambda$ above the surface.

incidence and for any assigned value of the perpendicular distance from the boundary, and they give accurately the form of the Fresnel zones.

The geometrical form of the curves drawn in this way shows the following general features. The zones for all angles of incidence in excess of the critical angle are approximately hyperbolic in form. The fact that they are not closed curves indicates that for no point of observation does the surface present any pole or region of stationary phase. The curvature of the lines is most marked for points of observation near the surface; as the distance is increased, the lines become more and more nearly straight. The spacing of the zones draws a striking diminution as we pass in the plane of incidence from negative to positive values of $x$, that is, from the left to the right of the foot of the perpendicular drawn from the point of observation. This change in the spacing is the more sudden, the smaller is the distance of the point of observation measured perpendicularly from the boundary. It is largest at the critical angle and diminishes with increasing angle of incidence. Figures 1, 2, 3 and 4 represent the form of the Fresnel zones for particular cases and illustrate the foregoing remarks. Figures 1 and 2 represent the case of incidence at the critical angle 45° and figures 3 and 4 for incidence at 60°. The refractive index $\mu$ is taken as 1.414. In figure 1 and figure 3 the point of observation is on the surface. In figure 2 it is $\lambda$ above from the surface and in figure 4, $4\lambda$ above the surface. The centre of the smallest unit index circle drawn in each

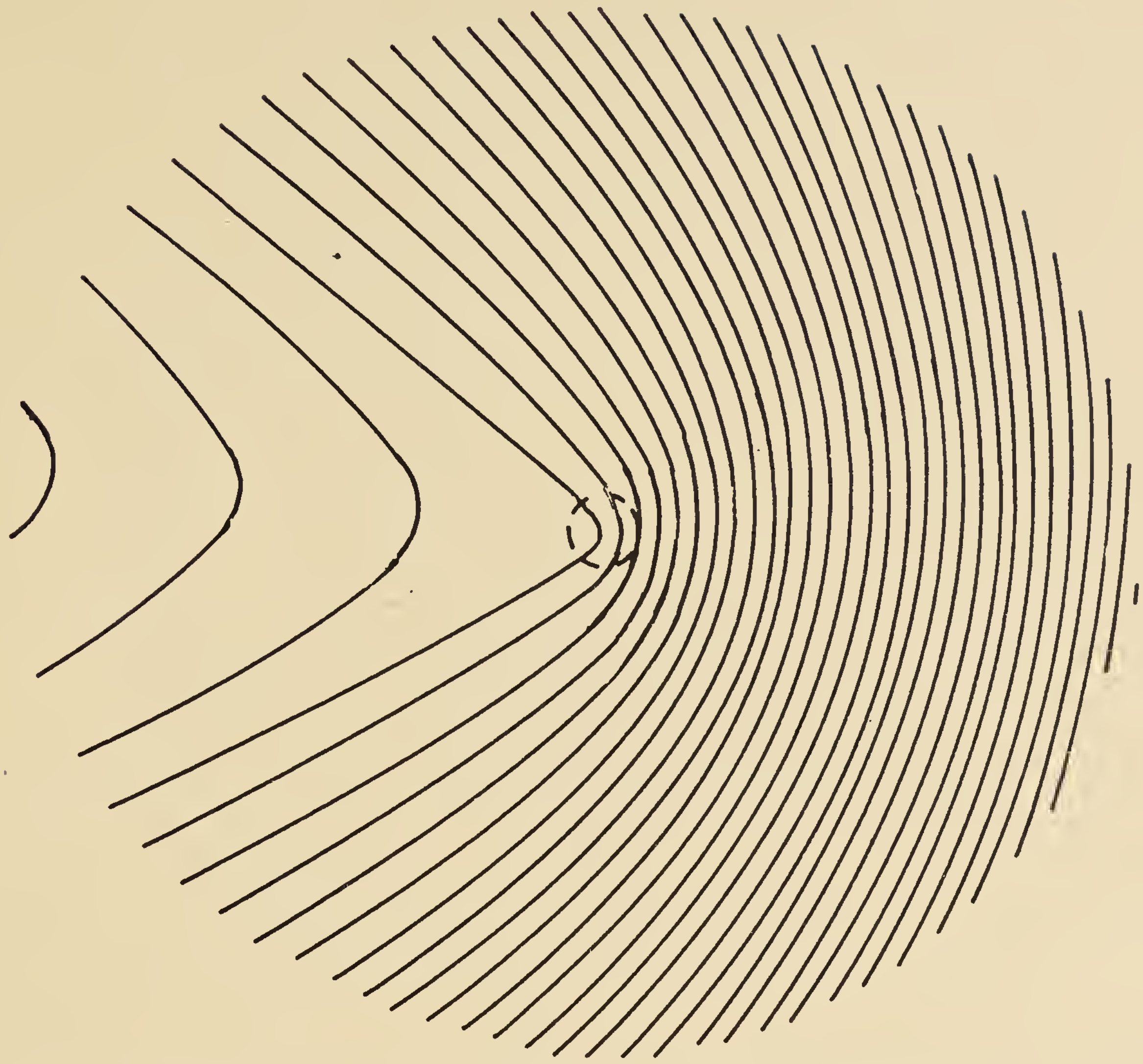

**Figure 3.** Fresnel zones on surface. Incidence = 60°. Point of observation is on the surface.

case is the foot of the perpendicular from the point of observation.

When we proceed to sum up the effects of the different Fresnel zones, taking into account the varying distances from the point of observation and the varying obliquities, we should obtain an idea of the way in which the residual effect observed in the second medium varies with the point of observation. Viewed in this way, it is seen that the penetration of the disturbance into the second medium in "total reflection" may always be regarded as a diffraction-effect. It is important however to determine what part of the effect arises from the outermost parts of the surface and what part from the area closest to the point of observation.

If the Fresnel zones had been uniformly spaced to the right and left of the foot of the perpendicular from the point of observation, they would have annulled each other's effects and given zero as the resultant disturbance. Actually however, as we have seen, there is a change in the spacing as we pass from left to right which is the more sudden, the closer we approach the surface between the two media. The summation over the Fresnel zones would therefore give a resultant effect which is the larger, the more nearly the point of observation approaches the surface. This effect arises from the part of the surface nearest the point of observation, and may be identified with the "superficial undulation" of Stokes and other writers. Since the change in the spacing of the Fresnel zones is most

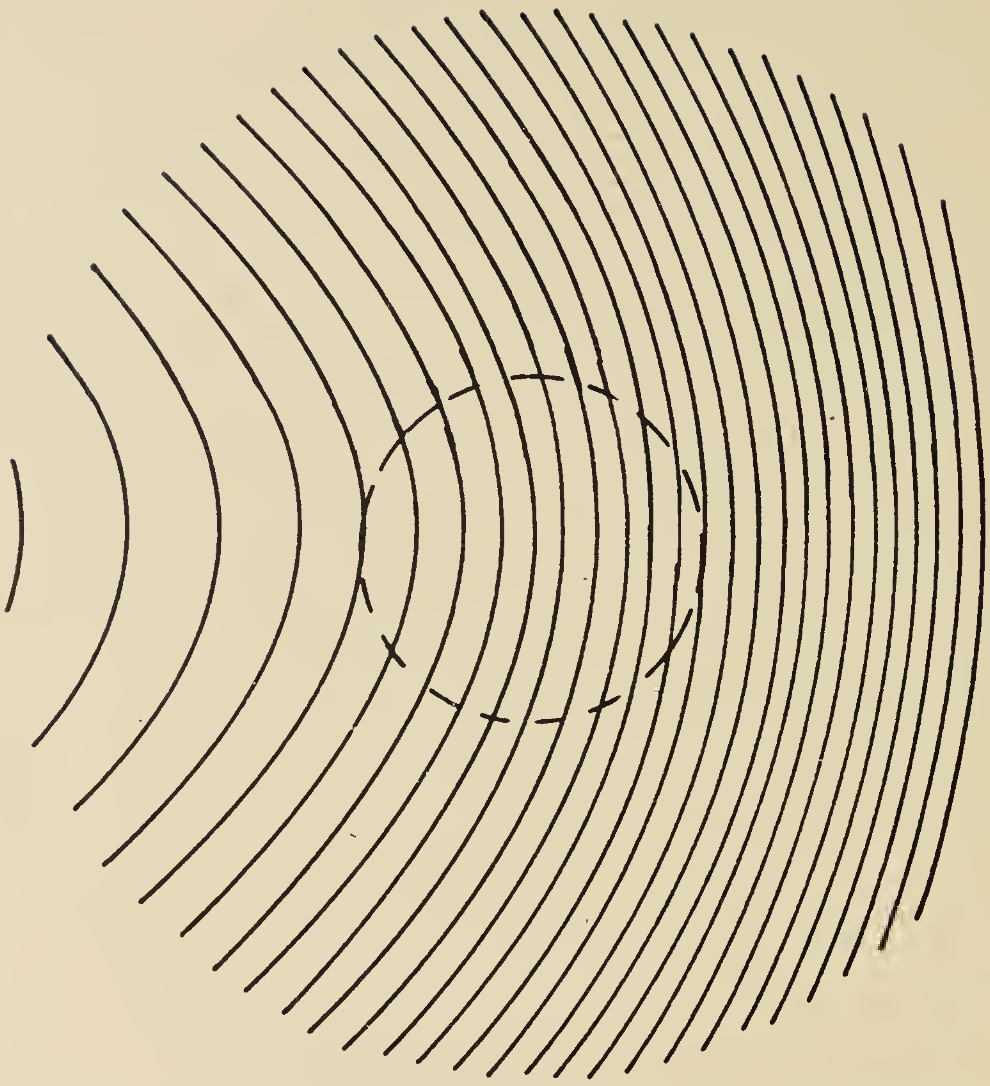

**Figure 4.** Fresnel zones on surface. Incidence at 60°. Point of observation 4$\lambda$ above the surface.

marked when the incidence is just at the critical angle and diminishes rapidly as the incidence is increased, we should expect the resultant effect to diminish in the same way. This is in agreement with the "superficial undulation" formula. We have already seen that the obliquity-factor of diffraction becomes vanishingly small when the angular aperture of the surface as viewed from the point of observation approaches zero. When the point of observation is sufficiently close to the surface, the obliquity becomes practically 90° for all the elements of the surface except those nearest to it. It follows that when the integration is carried out over all the Fresnel zones, the marginal parts of the surface contribute nothing and may be neglected.

## Analytical treatment of the problem

The preceding discussion indicates that the superficial undulation in the second medium is a diffraction-effect which arises, not from the margins of the illuminated area, but from the part of the area nearest the point of observation. This may be confirmed by mathematical analysis which indeed shows that the expected effect diminishes exponentially with the distance according to the law already derived from other considerations.

The diffraction integral expressing the effect at any point may easily be written down if we know the law of the secondary wave. The elementary disturbances arising from an area held obliquely to the wave-front have been expressed mathematically in Kirchoff's well-known formulation of Huyghens's principle. As has been remarked by various writers, however, Kirchoff's expression is not a unique solution of the problem, as an infinite number of formulae for the law of the secondary wave may be written down, all of which express correctly the disturbance in free space arising from specified light-sources. In our case, we are, moreover, dealing not with free space, but with the effects observed in the vicinity of a surface of separation between two media. The law of the secondary wave for this case has yet to be determined. For our present purpose, it is sufficient to proceed in the usual simple way and assume that the amplitude of the secondary wave is proportional to the area of the element from which it is sent out and inversely as the product of the wavelength and the distance of the element from the point of observation, and ignore all consideration of the obliquity factor. Let the surface be taken as coinciding with the $xy$ plane, and the plane of incidence be taken as the $xz$ plane. Further, let the point of observation be assumed to be on the $Z$-axis at a distance $Z$, from the origin, the latter being thus on the surface at the foot of the perpendicular drawn from the point of observation. Let $r$ be the distance of an elementary area on the surface from the origin.

An element of area on the surface is $r\mathrm{d}r\mathrm{d}\theta$ and the resultant effect is

$$\text{Const.} \int_0^\infty \int_0^{2\pi} \frac{A}{\lambda(z^2+r^2)^{1/2}} \cos\frac{2\pi}{\lambda}\{Vt-(z^2+r^2)^{1/2}-r\mu\sin\phi\cos\theta+\varepsilon\} r\mathrm{d}r\mathrm{d}\theta,$$

where $\mu$ is the refractive index of the first medium, the second medium being assumed to be free space, $\phi$ is the angle of incidence on the surface, and $\theta$ is the angle which the radius vector $r$ drawn on the surface makes with the plane of incidence. $\varepsilon$ is the phase difference between the primary disturbance and the secondary waves to which it gives rise. The integral is assumed to be taken over a sufficiently extended area. It is obvious from physical considerations that the expression must give results which differ entirely in character according as

$$\mu\sin\phi \lesseqgtr 1,$$

that is, according as the incidence is less or greater than the critical angle. This agrees, as we shall see presently, with the actual results of integration.

Integrating with respect to $\theta$ and writing

$$2\pi/\lambda\cdot(Vt+\varepsilon)=\chi$$

for shortness, the expression reduces to the form

$$\text{Const.} \int_0^\infty \frac{2\pi A}{\lambda(r^2+z^2)^{1/2}} \cos\{\chi-2\pi/\lambda\cdot(r^2+z^2)^{1/2}\} J_o(2\pi/\lambda\cdot r\mu\sin\phi) r\mathrm{d}r.$$

When $z$ is put equal to zero, that is, on the surface itself, the expression reduces to

$$\text{Const.}\frac{2\pi A}{\lambda}\left[\cos\chi\int_0^\infty \cos 2\pi r/\lambda\cdot J_0(2\pi r/\lambda\cdot\mu\sin\phi)\mathrm{d}r\right.$$
$$\left.+\sin\chi\int_0^\infty \sin 2\pi r/\lambda\cdot J_0(2\pi r/\lambda\cdot\mu\sin\phi)\mathrm{d}r\right].$$

The integrals appearing within the square brackets are well-known standard forms, the values of which depend on whether $\mu\sin\phi$ is greater or less than unity. If $\mu\sin\phi < 1$ the first integral vanishes and the second becomes equal to

$$(1-\mu^2\sin^2\phi)^{-1/2}$$

whereas if $\mu\sin\phi > 1$, the second integral vanishes and the first becomes equal to

$$(\mu^2\sin^2\phi-1)^{-1/2}.$$

The case $Z=0$ corresponds to the surface of separation and in order that our result might reduce to the primary disturbance on the surface, the constants expressing the law of secondary wave must be suitably chosen. It is necessary to assume different values for them in the two cases:

If $\mu\sin\phi < 1$, Const. $=\sqrt{1-\mu^2\sin^2\phi}$

and $\varepsilon=\pi/2$.

If $\mu\sin\phi > 1$, Const. $=\sqrt{\mu^2\sin^2\phi-1}$

and $\varepsilon=0$.

We shall now substitute these values in the general expression, considering the two cases separately.

*Case I. Incidence less than the critical angle and $\mu\sin\phi<1$.*

The expression for the light-disturbance given above involves the evaluation of two integrals, namely

$$\int_0^\infty \sin 2\pi/\lambda\cdot(r^2+Z^2)^{1/2}\cdot J_0(2\pi/\lambda\cdot r\mu\sin\phi)(r^2+Z^2)^{-1/2}r\mathrm{d}r$$

and

$$\int_0^\infty \cos 2\pi/\lambda\cdot(r^2+Z^2)^{1/2}\cdot J_0(2\pi/\lambda\cdot r\mu\sin\phi)(r^2+Z^2)^{-1/2}r\mathrm{d}r.$$

Using the well-known formulae

$$J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x,\quad\text{and}\quad J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$$

the two integrals under consideration are found to be special cases of a very general type of integral involving products of Bessel functions which has been discussed by Sonine.* We shall however adopt a different method of evaluation. Lamb† has proved the following formula:

$$\int_0^\infty -\exp[\alpha(\xi^2-\eta^2)^{1/2}]\cdot J_0(\beta\xi)\cdot(\xi^2-\eta^2)^{-1/2}\xi d\xi = \frac{\exp[-i\eta(\alpha^2+\beta^2)^{1/2}]}{(\alpha^2+\beta^2)^{1/2}}.$$

In this relation, write $\xi = r$ and $\eta^2 = -Z^2$. Also put

$$\beta = 2\pi/\lambda\cdot\mu\sin\phi \quad \text{and} \quad \alpha^2 = -(2\pi/\lambda)^2.$$

The equation then stands thus:

$$\int_0^\infty \exp[-2\pi i/\lambda(r^2+Z^2)^{1/2}]\cdot J_0(2\pi/\lambda\cdot r\cdot\mu\sin\phi)\cdot(r^2+Z^2)^{-1/2}r dr$$

$$= -i\cdot\lambda/2\pi\cdot(1-\mu^2\sin^2\phi)^{-1/2}\cdot\exp[-iZ\cdot 2\pi/\lambda\cdot(1-\mu^2\sin^2\phi)^{-1/2}].$$

Separating the real and imaginary parts we have

$$\int_0^\infty \sin 2\pi/\lambda\cdot(r^2+Z^2)^{1/2}\cdot J_0(2\pi/\lambda\cdot r\cdot\mu\sin\phi)\cdot(r^2+Z^2)^{-1/2}r dr$$

$$= \lambda/2\pi\cdot(1-\mu^2\sin^2\phi)^{-1/2}\cdot\cos\{2\pi/\lambda\cdot Z\cdot(1-\mu^2\sin^2\phi)^{1/2}\}$$

and

$$\int_0^\infty \cos 2\pi/\lambda\cdot(r^2+Z^2)^{1/2}\cdot J_0(2\pi/\lambda\cdot r\cdot\mu\sin\phi)\cdot(r^2+Z^2)^{-1/2}r dr$$

$$= -\lambda/2\pi\cdot(1-\mu^2\sin^2\phi)^{-1/2}\cdot\sin\{2\pi/\lambda\cdot Z\cdot(1-\mu^2\sin^2\phi)^{1/2}\}.$$

These results are confirmed by comparison with the general formulae given by Sonine and Nielsen. Substituting the values of the integrals in the expression for the disturbance in the second medium, we find that the latter reduces to

$$A\cos 2\pi/\lambda\cdot(Vt - x\mu\sin\phi - Z\sqrt{1-\mu^2\sin^2\phi})$$

which is of the same form as the ordinary expression for the refracted wave.

*Case II. Incidence at more than the critical angle, and* $\mu\sin\phi > 1$.

With the same substitutions as before, Lamb's formula now reads thus:

$$\int_0^\infty \exp[-2\pi i/\lambda\cdot(r^2+Z^2)^{1/2}]\cdot J_0(2\pi/\lambda\cdot r\cdot\mu\sin\phi)(r^2+Z^2)^{-1/2}r dr$$

$$= \lambda/2\pi\cdot(\mu^2\sin^2\phi-1)^{-1/2}\exp[-Z\cdot 2\pi/\lambda\cdot(\mu^2\sin^2\phi-1)^{1/2}].$$

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* *Math. Annalen, Band* **16**, p. 1. See also Nielsen, Cylinderfunction, 1904.

† *Philos. Trans. R. Soc. London* **A203**, 1904, p. 5.

Separating the real and imaginary parts, we have

$$\int_0^\infty \sin\cdot\frac{2\pi}{\lambda}\cdot(r^2+z^2)^{1/2}\cdot J_0(z\pi/\lambda\cdot r\cdot\mu\sin\phi)(r^2+z^2)^{-1/2}r\mathrm{d}r=0$$

and

$$\int_0^\infty \cos\frac{2\pi}{\lambda}(r^2+z^2)^{1/2}\cdot J_0 2\pi/\lambda\cdot r\cdot\mu\sin\phi)\cdot(r^2+z^2)^{-1/2}r\mathrm{d}r$$

$$=\lambda/2\pi\cdot(\mu^2\sin^2\phi-1)^{-1/2}\exp\left[-Z\frac{2\pi}{\lambda}(\mu^2\sin^2\phi-1)^{1/2}\right].$$

Substituting these values in the expression for the disturbance in the second medium when total reflection is occurring, we find that the latter reduces to

$$A\cdot\cos\frac{2\pi}{\lambda}\cdot(Vt-x\mu\sin\phi)\exp\left[-Z\frac{2\pi}{\lambda}(\mu^2\sin^2\phi-1)^{1/2}\right].$$

Our investigation thus leads to precisely the same law of exponential decay as that derived from the Fresnel formulae for the superficial disturbance in the second medium, and the view that the latter is a diffraction-effect arising from the immediately contiguous part of the surface is thus fully substantiated.

In evaluating the diffraction integral, the area of the surface was taken as infinite, and we found that the case $\mu\sin\phi=1$ marks a point of discontinuity at which the phase of the secondary waves alters suddenly by quarter of a period, and their amplitude becomes very large. This circumstance and the form of the Fresnel zones drawn in figures 1 and 2 show that when the incidence is exactly at the critical angle, the finite extent of the surface cannot be ignored and must be taken into account for a more exact discussion. When however, the incidence is increased beyond the critical angle, the marginal portions of the area cease to be of importance in determining the observed effect at points not far from the surface. The further discussion of the phenomena at or very near the critical incidence and close to the surface on the basis of the integrals already given is a problem worthy of investigation which must however be deferred for the present.

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# Huygens' principle and the phenomena of total reflection

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**Abstract.** In this paper the phenomena of total reflection are considered, *de novo*, from the standpoint of the principle of Huygens, no use whatever being made of the Fresnel formulae for reflection and refraction. Huygens' principle enables us to evaluate the disturbance appearing in the second medium when light is incident on the boundary between two media and is totally reflected into the first medium. The disturbance takes the form of a superficial wave moving parallel to the boundary. The existence of such a superficial wave is then shown to involve, as a necessary consequence, an *acceleration* of the reflected wave with reference to the incident wave, the acceleration being zero at critical incidence and increasing to half an oscillation at grazing incidence. The intensity of the superficial wave is at critical incidence greater for the component having the magnetic vector parallel to the surface, but diminishes more rapidly with increasing incidence than for the component having the electric vector parallel to the surface; the phase-advance reaches its maximum value correspondingly sooner. The phase-angle between the two components is evaluated and found to be an *acute* angle, in agreement with the classical treatment based on the Fresnel formulae, but in disagreement with the conclusions of Lord Kelvin and Schuster. The source of error in the Kelvin–Schuster treatment is pointed out. Experimental evidence regarding the magnitude of the phase-advance of each component separately is available and is in agreement with the classical theory.

Finally, a method is described by which the distribution of intensity, state of polarisation, and direction of flow of energy in the superficial wave may be studied experimentally.

## 1. Introduction

In chapter III of his celebrated treatise on light, Christian Huygens applied his principle of the superposition of elementary wavelets to the explanation of the phenomena of reflection and refraction, and showed that the absence of a refracted wave and the increased intensity of reflection for incidences exceeding the critical angle follow from his principle as simple consequences. The principle of Huygens is thus the natural avenue of approach to the phenomena of total reflection from the standpoint of wave-theory. Following Fresnel, however, the treatment of total reflection usually given is based on the formulae for reflection and refraction obtained by him, a suitable mathematical interpretation being given to the angles of refraction which become imaginary when the incidence exceeds the critical angle. While this method may be mathematically elegant, it

leaves the physical aspects of the problem somewhat obscure. Moreover, there has been some controversy about the actual magnitude and sign of the changes of phase occurring in total reflection. Lord Kelvin*, in his *Baltimore Lectures*, discussed the subject at great length on the basis of the mechanical theory of light and claimed that the classical interpretation accepted for 80 years was in error. His views were supported by Schuster[†] in his book on optics, while Bevan's observations[‡] on Lloyd's fringes in internal reflection and Drude's discussion[§] of the Fresnel formulae on the electromagnetic theory, on the other hand, appeared to support the classical interpretation. Prof. Schuster has, however, reiterated his views in the new edition of his book[‖], and in a recent paper[¶] claims that Drude's electromagnetic treatment is in error. In view of these facts, it would seem that an independent treatment of total reflection, based directly on physical principles, and making no use whatever of Fresnel's formulae must be of value. It is proposed in this paper to supply such a treatment.

## 2. The superficial wave in the second medium

The method of approach which we shall adopt is that indicated by the author in a recent paper**. We regard the disturbance in the second medium as arising from the superposition of the wavelets radiated from different elements of the bounding surface and determine it by evaluating the integral which expresses the result of such superposition.

In figure 1 the plane of incidence is taken to be the plane of the paper. The origin of co-ordinates $O$ is taken to be on the surface at which total reflection occurs, the latter coinciding with the $xy$-plane ($z = 0$), and the plane of incidence with the $xz$-plane. In accordance with the principle of Huygens, the effect at a point $P$ in the second medium (co-ordinates $x, o, z$) is to be found by superposing the effects of the wavelets radiated from different elements of the bounding surface.

Drop a perpendicular $PO_1$ on the surface and divide the surface into circular zones with $O_1$ as centre. Denoting $O_1P_1$ by $\rho$, where $P_1$ is any arbitrary point on the surface, we have $PP_1 = r = (z^2 + \rho^2)^{1/2}$. An element of area on the surface is

*Kelvin, *Baltimore Lectures*, 1904 (386–406).

†A. Schuster, *Theory of Optics*, 2nd edn., p. 54.

‡P V Bevan, *Philos. Mag.* **14** 1907 (503).

§P Drude, *Theory of Optics*, English translation, pp. 278–284.

‖A Schuster and J W Nicholson, *Theory of Optics*, 3rd edn., p. 54.

¶A Schuster, *Proc. R. Soc. London* **A107** 1925 (15).

**C V Raman, *Proc. Indian Assoc. Cultiv. Sci.*, **9** 1926 (271).

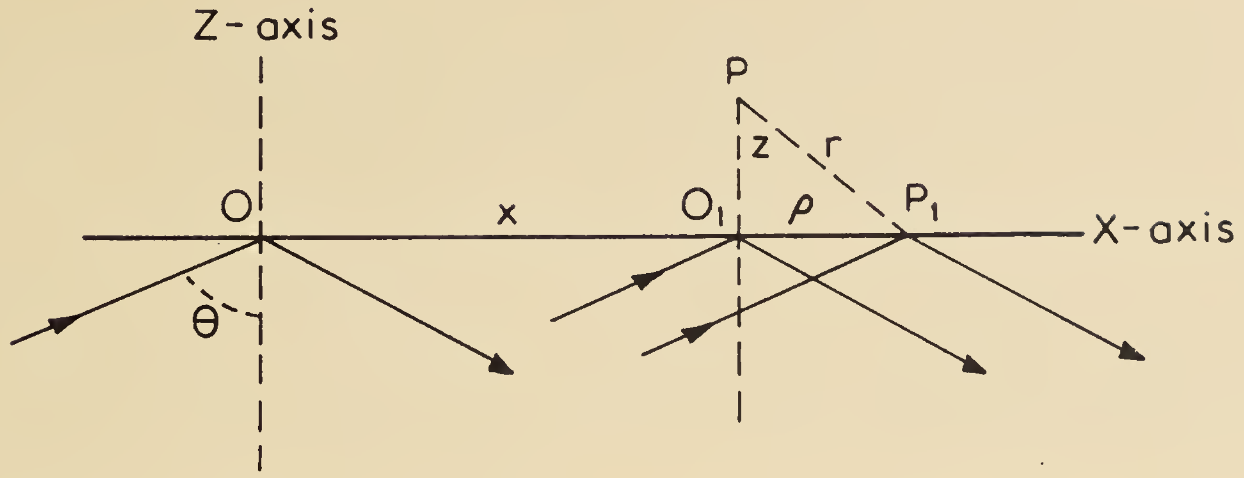


**Figure 1**

$\rho d\rho d\phi$ and the projection of $\rho$ on the $x$-axis is $\rho\cos\phi$. If a train of light-waves of period $T$ is incident on the boundary between two media in which the velocities of light are respectively $V_1$ and $V_2$, the refractive index of the second medium relatively to the first, $V_1/V_2$, being $n$, we may express the disturbance in the first medium due to the incident waves in the form

$$A\cos\left\{\frac{2\pi t}{T}-\frac{2\pi(x\sin\theta+z\cos\theta)}{V_1T}-\tfrac{1}{2}\delta\right\}, \tag{1}$$

where $\theta$ is the angle of incidence.

The disturbance due to the wavelet radiating from the elementary area $\rho d\rho d\phi$ at $P_1$ and reaching $P$ may be assumed to be of the form

$$\frac{k_1A}{V_2T}\cos\left\{\frac{2\pi t}{T}-\frac{2\pi(x+\rho\cos\phi)\sin\theta}{V_1T}-\frac{2\pi r}{V_2T}\right\}\frac{\rho d\rho d\phi}{r}. \tag{2}$$

It will be seen from (1) and (2) that we have assumed a difference of phase $\frac{1}{2}\delta$ (for the present undetermined) between the disturbance incident on any element of the surface and the secondary wavelet starting out from it. Such a difference of phase can conceivably exist, though, of course, its value may be zero in special cases. The numerical factor $k_1$ appearing in the amplitude of the wavelet is also for the present undetermined. The whole effect at the point $P$ is found by integrating (2) over the entire area of the surface. Integrating first with respect to $\phi$ between the limits 0 and $2\pi$, the result appears in the form

$$A_1\cos\frac{2\pi}{T}\left(t-\frac{x\sin\theta}{V_1}\right)+A_2\sin\frac{2\pi}{T}\left(t-\frac{x\sin\theta}{V_1}\right), \tag{3}$$

where

$$A_1=\frac{k_2}{V_2T}\int_0^\infty A\cdot J_0\left(\frac{2\pi\rho\sin\theta}{V_1T}\right)\cos\frac{2\pi r}{V_2T}\cdot\frac{\rho d\rho}{r}, \tag{4}$$

and

$$A_2 = \frac{k_2}{V_2 T}\int_0^\infty A\cdot J_0\left(\frac{2\pi\rho\sin\theta}{V_1 T}\right)\sin\frac{2\pi r}{V_2 T}\cdot\frac{\rho d\rho}{r}, \tag{5}$$

where $k_2$ is proportional to $k_1$ and is therefore undetermined for the present.

Integrals (4) and (5) are of standard types which have been evaluated by H Lamb*. When $\sin\theta$ is greater than $V_1/V_2$ or $n$, which is the condition for total reflection, the integral (5) vanishes, and the integral (4) reduces to the form

$$A_1 = \sigma A\cdot\exp\left(-\frac{2\pi}{TV_2}\cdot\sqrt{\frac{\sin^2\theta}{n^2}-1}\cdot z\right),$$

where $\sigma$ is a numerical constant which remains to be determined. The disturbance in the second medium is thus of the form

$$\sigma A\cdot\exp\left(-\frac{2\pi}{TV_2}\cdot\sqrt{\frac{\sin^2\theta}{n^2}-1}\cdot z\right)\cdot\cos\frac{2\pi}{T}\left(t-\frac{x\sin\theta}{nV_2}\right), \tag{6}$$

which expresses a wave of amplitude $\sigma A$ at the surface but decreasing exponentially with $z$ and propagated parallel to the surface along the $x$-axis. We shall consider the magnitude of the energy-flow in the superficial wave a little later, but it is obvious from (6) that it is entirely parallel to the surface, and thus the energy-flux *across* any element of area of the surface must be zero.

## 3. Change of phase in total reflection

Since there is no energy-flux *across* the boundary, it follows that the amplitudes of the incident and reflected waves must be equal. It can be easily seen, moreover, that the existence of a superficial wave of the form shown in (6) involves as a necessary consequence a difference in the phases of the incident and reflected waves at the boundary. To prove this, we have only to consider the continuity of the disturbance on the two sides of the boundary. It is convenient to consider separately the cases in which the electric vector and the magnetic vector in the incident waves are respectively parallel to the surface of separation between the two media.

*Case 1. Light polarised in the plane of incidence.* In this case we are concerned with the electric force $E_y$ parallel to $y$, and identify the amplitude $A$ appearing in our formulae with the amplitude of this vector. In order that $E_y$ may have the same value on both sides of the boundary, we must have, when $z = 0$, the sum of the electric forces in the incident and reflected waves equal to that in the

*H Lamb, *Proc. London Math. Soc.* 7 1909 (140).

superficial wave. In order to obtain this result, using the expressions (1) and (6) already derived for the incident and superficial waves respectively, we are compelled to assume that the electric force in the reflected wave is given by

$$A\cos\left\{\frac{2\pi t}{T}+\frac{2\pi(x\sin\theta - z\cos\theta)}{V_1 T}+\tfrac{1}{2}\delta\right\}, \tag{7}$$

and find that the factor $\sigma$ appearing in the amplitude of the superficial wave is connected with the phase-advance $\delta$ of the reflected wave relatively to the incident wave by the simple relation

$$\sigma = 2\cos\tfrac{1}{2}\delta. \tag{8}$$

We shall use $\sigma_s$ and $\delta_s$ to signify the values of $\sigma$ and $\delta$ in the present case. To find a second relation connecting them, we note that the component of the magnetic force $H_x$ parallel to the surface must also be continuous at the boundary. We must therefore have $(\partial E_y/\partial z)$ the same on both sides of the boundary. Differentiating (1), (6), and (7) with respect to $z$, and applying this condition, we obtain very readily

$$\sigma_s^2 = \frac{4\cos^2\theta}{1-n^2}, \text{ and } \tan\tfrac{1}{2}\delta_s = \frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}. \tag{9}$$

At the critical incidence $\sin\theta = n$, and therefore $\frac{1}{2}\delta_s$ is either zero or any multiple of $\pi$. The angle $\delta_s$ is either zero or any multiple of $2\pi$. We therefore take $\delta_s$ to be zero at the critical incidence. For incidences greater than the critical angle, $\tan\frac{1}{2}\delta_s$ is positive, and becomes infinite at grazing incidence. Accordingly, $\delta_s$ is positive and increases from 0 to $\pi$, as we pass from critical to grazing incidence.

*Case* 2. *Light polarised at right angles to the plane of incidence.* In this case the magnetic force is parallel to the $y$-axis, and we identify it with the vector $A$ appearing in our formula. Since $H_y$ must be the same on both sides of the boundary, we obtain the same expressions (7) for the reflected wave as in the preceding case, $A$ being now understood to refer to the magnetic force in the incident wave. Since the component of the electric force $E_x$ parallel to $x$ must be the same on both sides of the boundary, $(\partial H_y/\partial z)$ in the second medium must be $n^2$ times as large as its value in the first medium. Differentiating (1), (6), and (7) with respect to $z$, and applying this condition, we obtain a second relation between $\sigma_p$ and $\delta_p$ in addition to the relation $\sigma_p = 2\cos\frac{1}{2}\delta_p$ deduced as in the preceding case. In this way, we find

$$\sigma_p^2 = \frac{4n^4\cos^2\theta}{(n^4-n^2)+\sin^2\theta(1-n^4)} \text{ and } \tan\tfrac{1}{2}\delta_p = \frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}. \tag{10}$$

Arguing exactly as the preceding case, we see that the phase of the reflected wave is *advanced* relatively to that of the incident wave by a quantity $\delta_p$ which is zero at critical incidence and increases to $\pi$ at grazing incidence.

## 4. Difference of phase of components

Our investigation has shown that the incident and reflected waves are in the same phase at critical incidence, and are opposed in phase at grazing incidence. This is true both in Case 1 and Case 2 and hence the angle $(\delta_p - \delta_s)$, which represents the difference in the phase-advance in the two cases, is zero both at critical incidence and at grazing incidence. Since $n^2 < 1$, it can be seen from (9) and (10) that at intermediate incidences $\tan\frac{1}{2}\delta_p$ is greater than $\tan\frac{1}{2}\delta_s$. In other words, the phase-advance in Case 2 is greater than in Case 1, and $(\delta_p - \delta_s)$ is positive. From (9) and (10) we derive the following formulae:

$$\tan\delta_s = \frac{2\cos\theta\sqrt{\sin^2\theta - n^2}}{(n^2-1)+2\cos^2\theta}, \qquad \cos\delta_s = \frac{n^2-1+2\cos^2\theta}{1-n^2}, \tag{11}$$

$$\tan\delta_p = \frac{2n^2\cos\theta\sqrt{\sin^2\theta - n^2}}{(n^4+1)\cos^2\theta-(1-n^2)}, \quad \cos\delta_p = \frac{(n^4+1)\cos^2\theta-(1-n^2)}{(n^4-1)\cos^2\theta+(1-n^2)}, \tag{12}$$

$$\tan\tfrac{1}{2}(\delta_p-\delta_s) = \frac{\cos\theta\sqrt{\sin^2\theta-n^2}}{\sin^2\theta},$$

$$\cos(\delta_p-\delta_s) = \frac{2\sin^4\theta-(n^2+1)\sin^2\theta+n^2}{(1+n^2)\sin^2\theta-n^2}. \tag{13}$$

Formula (13) is identical with that derived by Drude. From our formulae it follows that $(\delta_p - \delta_s)$ for glass is always an acute angle, in agreement with what had generally been accepted as true. According to Kelvin and Schuster, however, the phase of the reflected wave is advanced in Case 1, and retarded in Case 2, and the difference in phase of the two components is represented by an obtuse angle. Since the method adopted in the foregoing treatment is based directly on first principles and gives the quantities under consideration in an entirely unambiguous manner, it would seem that the grounds on which Kelvin and Schuster base their criticism of the classical treatment must be invalid. We shall presently see that this is actually the case.

## 5. The Kelvin–Schuster treatment

For our purpose it is sufficient to examine the argument in the form in which it is presented by Schuster in his recent paper in which he criticises Drude's treatment, which is, in fact, substantially the same as that put forward by Kelvin. It rests on the supposed necessity for assuming that the well-known Fresnel coefficients of reflection for the parallel and perpendicular components of vibration must be both numerically and algebraically equal to each other in the limiting case of

normal incidence. The argument rests on a fallacy which will become clear when we recollect that in order to fix the direction of a ray in a unique manner, we require to know the positive direction of two *vectors*, namely the electric and magnetic vectors. The direction of the ray is perpendicular to both, and the phase of the oscillation, in it may be determined indifferently from the phase of either. In the case of normal incidence, the direction of the ray is reversed on reflection, so that if the electric vectors in the incident and reflected pencils are in the same direction, the magnetic vectors are opposed to each other, and *vice versa*. If, as proposed by Schuster, the positive direction of a vector is always to be so chosen that the positive directions of the vector for the incident and reflected pencils become coincident for normal incidence, we should be led to the absurd conclusion that the phase of a ray on normal reflection is electrically reversed but is magnetically unaltered. It is necessary, in fact, in order to obtain correct results, to assume a rule of signs for vectors lying in the plane of incidence which is exactly opposite to that suggested by Schuster. The scheme adopted by Drude is the correct one, and accordingly gives correct results. All ambiguity or difficulty may be avoided, however, by taking as the phase of a pencil of light, the phase of the vector (electric or magnetic as the case may be) which has a direction perpendicular to the plane of incidence. The direction of such vector remains invariable when the angle of incidence is altered, and no special convention as to sign is necessary. As a matter of fact, the Fresnel formulae for reflection and refraction can easily be derived by considering only the vector (electric or magnetic as the case may be) perpendicular to the plane of incidence and writing down the condition for its continuity at the boundary, as also for the equality of the normal flux of energy through unit area of the boundary on either side of it. The two Fresnel coefficients then appear with *opposite* signs, and since the phase of the reflected ray must be identical for the electric as for the magnetic vibration, the same result must also be valid when we consider the vectors lying in the plane of incidence.

The analytical necessity for so fixing the positive directions of vectors lying in the plane of incidence that the two Fresnel coefficients appear with opposite signs becomes very clear when we consider the case of a circularly-polarised ray incident normally on a surface and reflected from it. In this case, no doubt, both the components of the ray are reflected under the same conditions and it might seem, at first sight, that their relative phase should remain unaltered. In reality, however, owing to the reversal of the direction of the ray, if it has right-handed circular polarisation before reflection, it has left-handed circular polarisation after reflection, and this is analytically equivalent to a reversal of the phase of one of the components relatively to the other. It is precisely in order that the physical and analytical requirements of the case might both be complied with, that the signs of the vectors have to be taken in the manner chosen by Drude. The convention adopted by him is, in fact, merely the well-known Ampère rule of signs in another form. It has the effect of making vectors in the incident and reflected

rays, which lie in the plane of incidence and which are both in the same phase and parallel to each other at grazing incidence, continue to be in the same phase, though (geometrically) oppositely directed at normal incidence unless, of course, in the interval a real physical change in their relative phase has occurred.

## 6. Polarisation and intensity of superficial wave

We have already had occasion to consider the components of the electric and magnetic vectors parallel to the boundary in the superficial wave and their relation to the expressions (1), (6) and (7). The components of the electric and magnetic forces perpendicular to the surface may be similarly found by applying the boundary conditions. It appears that the components $E_y$ and $H_x$ in Case 1 differ in phase by a quarter of an oscillation, and the components $H_y$ and $E_x$ in Case 2 similarly differ in phase. Hence, in either case, the energy-flow along the axis of $z$ is zero. The components $H_z$ and $E_y$ in Case 1 and the components $E_z$ and $H_y$ in Case 2 have, however, identical phases, and therefore give rise to energy-flow parallel to the axis of $x$ proportional to $H_zE_y$ and $E_zH_y$ respectively. The ratio of the energy-flow in the two cases is found to be

$$\frac{n^2\sigma_s^2}{\sigma_p^2}, \quad \text{and } \textit{not } \frac{\sigma_s^2}{\sigma_p^2}.$$

Why the factor $n^2$ appears is most easily seen on considering the case in which the incidence is just at the critical angle; in this case $\sigma_s^2 = \sigma_p^2$, but in the emergent ray, which is parallel to the surface, the intensity of its $s$ component is $n^2$ times that of the $p$ component. In other words, since $n^2 < 1$, the superficial wave is polarised with the stronger component of the electric vector perpendicular to the surface.

When the expression for the flow of energy in the superficial wave is plotted against the angle of incidence $\theta$, it is found to fall off rapidly from its maximum value for critical incidence to zero value at grazing incidence. In other words, the intensity of the superficial wave falls off to zero as the angle of incidence is increased. $\sigma_p^2$ falls off more quickly than $\sigma_s^2$. Since $\sigma_p = 2\cos\frac{1}{2}\delta_p$ and $\sigma_s = 2\cos\frac{1}{2}\delta_s$, the intimate relation between this fall in intensity and the change of phase in reflection will be readily understood.

## 7. Experimental study of superficial wave

The existence of a superficial wave has been demonstrated in different ways. Stokes used the method of observing Newton's rings between a prism and a lens, when the angle of incidence exceeded the critical angle, and Quincke also investigated it by a method very similar in principle. By placing small particles in contact with the surface and thereby causing them to scatter light, the superficial

disturbance has also been made evident. This method is, however, only qualitative. The present writer has shown in a recent paper* that when a totally-reflecting surface is of finite extent, as indeed is inevitable, the superficial wave gives rise to diffraction effects which can be observed and photographed at great distances from the surface. The edges of the surface, in fact, act as sources of secondary radiation in the same way as the boundary of any fully illuminated diffracting aperture. In order, however, to observe the disturbance in the second medium at points very close to the surface itself and investigate it in detail, an entirely different method has been devised by the writer. This is indicated in figure 2.

AB is the hypotenuse of a right-angled glass prism ABC at which total reflection occurs. DE is a fresh "Gillette" razor-blade of which the sharp edge E is placed as nearly as possible parallel to the surface AB. A fine slow motion, such as that provided on interferometers, enables the razor-blade to be moved forward or backward so that the edge approaches or recedes from the face of the prism by fractions of a wavelength. The microscope M is focussed on the razor-edge; the latter is usually invisible unless some diffuse illumination is provided in the field of view. If the axis of the microscope be in the plane of incidence and the razor-blade be placed perpendicular to the plane of incidence and slowly advanced within a very small distance of the surface, it is seen as a luminous line in the microscope. The distance of approach or removal from the surface within which the luminosity of the edge is visible is a measure of the thickness of the layer within which the superficial disturbance in the second medium is sensible.

The principle of the method depends on the well-known property of a sharp and highly polished metallic edge to diffract a stream of light falling upon it

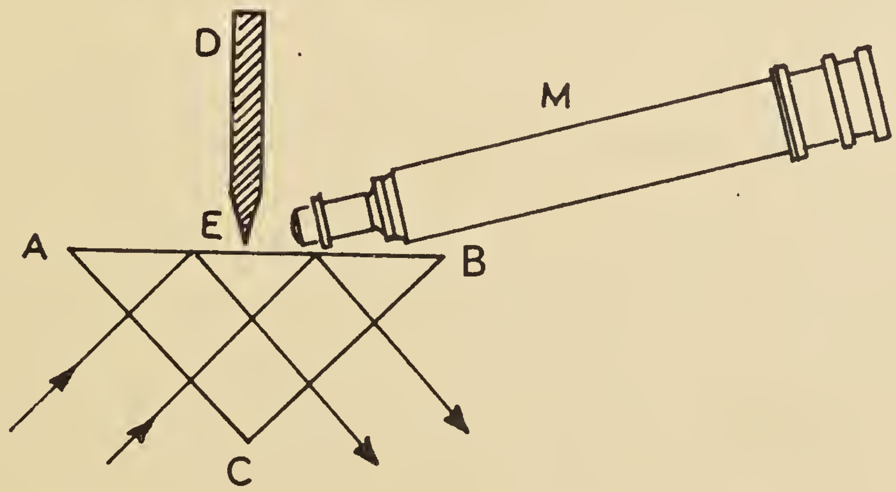


**Figure 2**

*C V Raman, *Philos. Mag.* **50** 1925 (812).

through considerable angles and to appear as a luminous line when viewed in suitable directions. Simple observations with a sharp razor-edge held in an ordinary parallel pencil of light show that the directions in which the luminosity of the edge is visible depend on the position of the edge relatively to the stream of rays. In the general case when the edge is inclined to the light-stream, the luminosity is visible only when the direction of observation is any generator of a cone having the edge for its axis and the direction of the undiffracted rays for another generator. When the stream of light falls normally on the diffracting edge, its luminosity is visible in any direction lying in a plane perpendicular to the edge.

From what has been stated above it is clear that the observations of the luminosity of the edge in the arrangement shown in figure 2, when it approaches sufficiently close to the surface, prove that there is actually a stream of energy in the second medium travelling parallel to the surface in the plane of incidence. The method enables the rate at which the intensity in the superficial wave decreases with increasing normal distance from the surface to be determined. It is found that, when the incidence is just at the critical angle, the intensity of the superficial wave is a maximum and is quite comparable with that of the incident and reflected beam. But as the incidence is increased the intensity falls off more or less quickly and becomes zero at very oblique incidences. The decrease of the intensity of the superficial wave with increasing normal distance from the surface is very rapid; when the incidence is not much greater than the critical angle, say about 50°, the luminosity is perceptible when the edge is within a wavelength or so from the surface. For larger angles of incidence, 60° or more, the decrease is far more rapid, and the luminosity is perceptible practically only when the razor-edge is in actual contact with the surface of the prism. When the incident light is unpolarised, the luminosity of the edge, observed as nearly as possible parallel to the surface, is found to be strongly polarised with the strong component of the electric vector perpendicular to the edge.

## 8. Direct measurement of phase-changes

It has already been mentioned that Bevan, by observations on Lloyd's fringes formed by internal reflection, verified the fact that the phase-differences $\delta_p$ and $\delta_s$ between the incident and totally-reflected waves amount to $\pi$ at grazing incidence. In an investigation on the colours of mixed plates*, it was shown by the present writer that part of the light falling upon the curved edges of the air-bubbles contained in these plates is totally reflected at various angles between grazing and critical incidence and interferes with another part which also emerges after two refractions through the curved edges. The positions of the maxima and

*C V Raman and K Seshagiri Rao, *Philos. Mag.* **42** 1921 (679).

minima in these interferences depend on the phase of the totally-reflected light, and the good agreement found between the observed and calculated values is a confirmation of the correctness of the formula used for calculating the phase-difference. The method also enabled the difference $\delta_p - \delta_s$ to be directly observed and measured, and this was found to be an *acute* angle. It would appear quite practicable to arrange a simple interference experiment in which the phase-advances $\delta_p$ and $\delta_s$ may be measured separately and compared with each other.

## 9. Conclusion

The method of the knife-edge described above was shown to be practicable in observations made at the author's suggestion by Mr D P Acharya and was later more fully tested by Mr S C Sirkar. Further observations by this method (using stronger illumination), and an experimental study of the state of polarisation of light scattered by small particles in the vicinity of a totally reflecting surface, would appear to be called for.

In conclusion, the author has much pleasure in referring to the valuable assistance received by him from Mr K S Krishnan in the preparation of the paper.

## Discussion

Mr T Smith: It is gratifying to find that the total reflection problem for a harmonic wave-train can be treated so thoroughly on the simple Huygenian principle. It would be still more satisfying to have a corresponding discussion in which events according to some of the modern quantum theories of light are considered; since these theories are still in their infancy we shall probably have to wait some time for an investigation of such special problems on these bases. Unlike the important question of phase, which relates essentially to the waves and is thus common to all theories, the question whether the real light energy, that is the light quantum, penetrates the second medium does not appear to be yet answered. It is obvious that there must be something in the nature of a superficial wave in the second medium—one can hardly imagine that conditions just on one side of a surface, while a train of waves is being reflected from the other side, are no different from those holding when no disturbance reaches the surface—but the energy of these waves is not necessarily identical with the energy of the light.

Mr J W Perry: The boundary condition here assumes the natural confines of the first and second media to be a common boundary surface. But it is found necessary to postulate a transition layer in order to explain the residual elliptic polarisation on reflection at the boundary. It would be of interest if from the experimental study of the results obtained some indication were found serving to corroborate the existence of the transition layer.

Mr E T Hanson: Prof. Raman is to be congratulated upon his ingenious and interesting method of attacking the problem of total reflection, which, strange to say, has been a stumbling-block to so many celebrated physicists and mathematicians. I am not, however, in entire agreement with him as to the advisability of using Huygens' principle in the way he has done. Kirchhoff's formula, which is derived by purely mathematical reasoning, might have been applied. But Huygens' principle is a concrete physical conception, which somewhat loses its meaning when reduced to the limiting equation (6) of Prof. Raman's paper, for, in this limiting case, no energy can be said to be radiated into space from the elementary secondary wavelets.

Drude's very careful treatment of the problem has always appeared to me to be excellent in every essential respect, and it is satisfactory to have Prof. Raman's alternative confirmation of its correctness, even though his method may be open to criticism.

Drude's attempt to explain how the energy passes from the first medium into the second medium and back into the first in the case of total reflection appears to me to be incorrect, but the matter is perhaps of small importance. One of the most interesting phenomena at, or near, total reflection, is that of the diffraction of a beam of light by a slit when the latter is placed inside the first medium and not too close to the surface at which total reflection takes place. The explanation of the observed phenomena by the use of Huygens' principle and Fresnel's formulae is very instructive.

With regard to the superficial wave itself, apart from diffraction, the following remarks may be of interest. For angles of incidence equal to and greater than the critical angle it is possible to combine the incident and reflected wave systems into a single plane wave of variable amplitude. In fact, in the incident space, there is a plane wave travelling parallel to the interface, the amplitude varying harmonically along the wave-front. In the refracted space there is a plane wave also travelling parallel to the interface, the amplitude decreasing exponentially from the interface. These two wave systems, derived by theory, satisfy all the conditions of the mathematical problem. Now it can be shown that at critical incidence no energy is transmitted to and from across the interface. The physical explanation appears to be, therefore, that the two systems of waves in the first medium adjust themselves to the velocity of a possible plane wave in the second medium, so that there is no reaction to the propagation of the electromagnetic displacements along the boundary.

Prof. Raman (communicated): Replying to Mr Perry's remark, it may be said that transition layers have but a small influence on the total reflection of light (Drude, *Wied. Ann.* **43** 1891 (126) and Maclaurin, "Theory of Light," pp. 62 and 82). They may, however, be important in the total reflection of X-rays, where the wavelength is much smaller. The effect of the transition layers in total reflection is experimentally evident in observations on the scattering of light by liquid surfaces (*Proc. R. Soc. London* **A109** 1925 (150)).

In reply to Mr Hanson, I may say that the simplest expression of Huygens' principle is employed, as it answers the purpose sufficiently and avoids unnecessary mathematical difficulties. It must be remembered in this connection that Kirchhoff's principle is only one way of formulating the propagation of waves in an uninterrupted medium and, even as such, is not unique. In the present case we are concerned with the effects occurring at the boundary between two media, and not with a single uninterrupted medium.

With regard to the question of energy it is clear that the elementary wavelets entering into the second medium attenuate each other's effects by interference so completely that the actual energy conveyed by them is an infinitesimal quantity. I would interpret formula (6) as being just a mathematical expression of this physically intelligible result.

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# The diffraction of light by metallic screens

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## 1. Introduction

Gouy* discovered that when a metallic screen with a sharp and highly-polished edge is held in the path of a pencil of light, its boundary appears as luminous line diffracting light through large angles, both into the region of shadow (interior diffraction) and into the region of light (exterior diffraction). He noticed further that this diffracted light is strongly polarised, but in perpendicular planes in the two regions mentioned; the colour of the diffracted light and its state of polarisation depend in a remarkable manner on the material of the screen and on the extent to which its edge is rounded off in the process of polishing. When the edge is viewed through a double-image-prism from within the shadow, only that image appears coloured which is more intense and is polarised with the magnetic vector parallel to the edge. The second image which is fainter and is polarised with the electric vector parallel to the edge, appears perfectly white. When the incident light is polarised in any arbitrary azimuth, the diffracted light is found to exhibit elliptic polarisation. These and other results have been confirmed by later observers.†

Gouy's experimental results were discussed by Poincaré on the basis of the electromagnetic theory of light in two memoirs published in the "*Acta Math*‡". The special case of an ideal screen (plane or wedge-shaped), supposed perfectly-reflecting and having a sharp edge, is amenable to complete theoretical treatment, and was dealt with by Poincaré himself, and later in a rigorous manner by Sommerfeld,§ and following him by numerous other mathematicians. The behaviour of actual metallic screens, however, differs considerably from that found theoretically for this ideal case. Though attempts have been made by Poincaré himself in the memoirs quoted, and later also by Epstein‖, to take the

**Ann. Chim. Phys.* **8**, p. 145 (1886).

†W Wien, *Wied. Ann.*, **28**, p. 117 (1886).

‡*Acta Math.*, **16**, p. 297 (1892), and **20**, p. 313 (1896).

§*Math. Annalen*, **47**, p. 317 (1896).

‖*Diss. Munich*; also *Encykl. Math. Wissensch.*, **5**, p. 491.

nature of the screen and the rounding of its edge into account, it cannot be said that Gouy's observations have so far received a complete or satisfactory explanation. We propose in this paper to discuss more particularly the influence of the material of the screen on the diffraction by a sharp edge, and to show how it may be explained in a very simple manner. The case of rounded edges is reserved for discussion in a separate paper.

In the fifth section of his first memoir, Poincaré discussed the electromagnetic boundary conditions at the surface of an imperfectly conducting screen, and made the important remark that the extreme smallness of the depth to which an optical disturbance penetrates into any actual metal, should considerably simplify the theory. In his actual attempt, however, to discuss the problem of diffraction by an imperfectly conducting screen, he made no use of the elegant mathematical methods and results contained in the earlier parts of his memoir, and contented himself with a qualitative discussion on the basis of the Kirchhoff formulation of Huygens' Principle. The treatment given does not, as was indeed remarked by Poincaré himself, appear capable of leading to quantitative results. In the course of our paper, we shall show how it is possible to apply the Fresnel–Huygens' principle with success to the problem of diffraction by imperfectly-conducting screens. It is more convenient, however, to base our treatment in the first instance on a modification of the known exact solutions for the case of perfectly-reflecting screens or wedges.

## 2. Theory

Sommerfeld's solution of the wave-equation in cylindrical co-ordinates for the case of a semi-infinite screen which is a perfect reflector and lies in the $xz$-plane, with its edge along the $z$-axis, is

$$u = F(\rho, \phi, \phi_0) \mp F(\rho, \phi, -\phi_0). \tag{1}$$

The upper (minus) sign refers to the case in which the plane of polarisation and the plane of incidence are parallel to each other; (we shall refer to this as the $\|^l$ case), and $u$ then denotes the electric force parallel to the edge. The lower (plus) sign refers to the case in which the plane of polarisation of the incident light is perpendicular to the plane of incidence (we shall refer to this as the $\perp^r$ case), $u$ then denoting the magnetic force parallel to the edge.

$$F(\rho, \phi, \phi_0) = \left(\frac{i}{\pi}\right)^{1/2} \exp\left(i\frac{2\pi t}{T}\right) \exp\left[ik\rho\cos(\phi - \phi_0)\right] \int_{-\infty}^{\tau} \exp(-i\lambda^2)\mathrm{d}\lambda, \tag{2}$$

where

$$\tau = \sqrt{2k\rho} \cdot \cos\tfrac{1}{2}(\phi - \phi_0).$$

The expression (2) has the property that when $\tau$ is positive and sufficiently

large, which is the case when $\pi + \phi_0 > \phi > 0$, the function tends to the limiting value $\exp[i(2\pi t/T)] \exp[ik\rho\cos(\phi - \phi_0)]$, which represents a train of plane waves incident on the screen in a direction making an angle $\phi_0$ with its plane; when $\tau$ is negative and sufficiently large, which is the case when $\pi + \phi_0 < \phi < 2\pi$, the function tends to the limit zero. $F(\rho, \phi, -\phi_0)$ is obtained by writing $-\phi_0$ for $\phi_0$. When $\pi - \phi_0 > \phi > 0$ it tends to the limit $\exp[i(2\pi t/T)] \exp[ik\rho \cos(\phi + \phi_0)]$, which represents a plane train of waves reflected from the screen. When $\pi - \phi_0 < \phi < 2\pi$, $F(\rho, \phi, -\phi_0)$ tends to the limit zero.

Now the solutions (1) satisfy the conditions $u = 0$ and $(\partial u/\partial\phi) = 0$ respectively, on both faces of the screen, supposed to be infinitely thin and perfectly reflecting, that is, when $\phi = 0$ and also when $\phi = 2\pi$. Now any actual screen to be opaque must be of finite thickness, hence a solution of the form (1) or any simple modification of it cannot be expected to represent the behaviour of such screens *completely*. Nevertheless, as already mentioned above, a metallic screen of a thickness which is only a small fraction of a wavelength, is practically opaque, and this makes it possible to represent its behaviour with a high degree of accuracy by a comparatively simple modification of (1). Consider expressions of the form

$$u = F(\rho, \phi, \phi_0) - (C_s + iD_s)\cdot F(\rho, \phi, -\phi_0) \tag{3}$$

and

$$u = F(\rho, \phi, \phi_0) + (C_p + iD_p)\cdot F(\rho, \phi, -\phi_0) \tag{4}$$

in which the (numerical) factors $C_s + iD_s$ and $C_p + iD_p$ are so chosen that they represent the amplitude of the wave reflected at the illuminated face of the particular screen used, for the particular angle of incidence under consideration. Equation (3) refers to the $\|^l$ case, and (4) to the $\perp^r$ case. Since $C_s + iD_s$ and $C_p + iD_p$ are functions only of the angle of incidence $\phi_0$, (3) and (4) continue to satisfy the wave-equation and represent distributions of light and shadow of the same general character as those indicated by (1), with this difference, however, that the disturbance on the illuminated face of the screen expressed by (3) and (4) will have the actual values corresponding to the screen used, while (1) corresponds to a screen with properties which cannot be physically realised. We are therefore justified in expecting that (3) and (4) would represent the disturbance throughout the whole field in the actual problem much more accurately than the Sommerfeld formulae.

To understand the physical significance of the formulae, it is best to use the asymptotic expressions for the functions for large values of $\rho$. We have, when $\cos\frac{1}{2}(\phi - \phi_0)$ is positive,

$$F(\rho, \phi, \phi_0) \sim \exp\left(i\frac{2\pi t}{T}\right)\exp\left[ik\rho\cos(\phi-\phi_0)\right] + \frac{i^{3/2}\exp\left(i\frac{2\pi t}{T}\right)\exp(-ik\rho)}{2\sqrt{2\pi k\rho}\cdot\cos\frac{1}{2}(\phi-\phi_0)}, \tag{5}$$

while when $\cos\frac{1}{2}(\phi-\phi_0)$ is negative, there is a similar expression in which the first term is missing. Similarly when $\cos\frac{1}{2}(\phi+\phi_0)$ is positive, we have

$$F(\rho, \phi, -\phi_0) \sim \exp\left(i\frac{2\pi t}{T}\right)\exp\left[ik\rho\cos(\phi+\phi_0)\right] + \frac{i^{3/2}\exp\left(i\frac{2\pi t}{T}\right)\exp(-ik\rho)}{2\sqrt{2\pi k\rho}\cdot\cos\frac{1}{2}(\phi+\phi_0)}, \tag{6}$$

and a similar expression in which the first term is missing, if $\cos\frac{1}{2}(\phi+\phi_0)$ is negative. The general expression for the light diffracted by the edge is either

$$E_z = \frac{i^{3/2}\exp\left(i\frac{2\pi t}{T}\right)\exp(-ik\rho)}{2\sqrt{2\pi k\rho}}\left[\frac{1}{\cos\frac{1}{2}(\phi-\phi_0)} - \frac{C_s+iD_s}{\cos\frac{1}{2}(\phi+\phi_0)}\right] \tag{7}$$

or

$$H_z = \frac{i^{3/2}\exp\left(i\frac{2\pi t}{T}\right)\exp(-ik\rho)}{2\sqrt{2\pi k\rho}}\left[\frac{1}{\cos\frac{1}{2}(\phi-\phi_0)} + \frac{C_p+iD_p}{\cos\frac{1}{2}(\phi+\phi_0)}\right]. \tag{8}$$

## 3. An alternative treatment

The formulae (7) and (8) may also be derived in the following manner based on the Fresnel–Huygens' principle. It is readily shown that the wave-equation in cylindrical co-ordinates,

$$\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0 \tag{9}$$

is exactly satisfied by putting

$$u = \exp\left(i\frac{2\pi t}{T}\right)\frac{\exp(-ik\rho)}{2\sqrt{2\pi k\rho}}\cos\tfrac{1}{2}(\phi+\varepsilon), \tag{10}$$

which represents a cylindrical wave of Poisson's type diverging from the $z$-axis. A

plane wave incident on the $xz$-plane in the direction $\phi_0$ is transmitted through the part of the plane to the left of the edge and is reflected from the part on the right. To find the disturbance diverging from the edge of the screen, at any point very distant from it, we divide the area of the $xz$-plane adjacent to the edge into half-period strips parallel to it on either side, and show in the usual way that the effect of the transmitted wave reduces to one-half of the first half-period strip on one side of the edge, and that the effect of the reflected wave reduces to that of a similar strip on the other side. Assuming that each of these strips is the origin of a cylindrical wave of the type appearing in (10), we may write the total disturbance at $(\rho, \phi)$ diverging from the origin, in the form

$$u = \exp\left(i\frac{2\pi t}{T}\right)\frac{\exp(-ik\rho)}{2\sqrt{2\pi k\rho}}[A_0 \cos\tfrac{1}{2}(\phi + \alpha) + B_0 \cos\tfrac{1}{2}(\phi + \beta)], \tag{11}$$

where $A_0$, $B_0$, $\alpha$ and $\beta$ have to be so chosen as to give the amplitudes and phases of the divergent waves correctly. Now the width of either half-period strip is easily shown to be

$$\frac{\lambda}{4\cos\frac{1}{2}(\phi - \phi_0)\cos\frac{1}{2}(\phi + \phi_0)},$$

and we may assume, as is usual in the elementary diffraction theory, that the amplitudes $A_0$ and $B_0$ are proportional to this width. It is necessary that the term in (11) proportional to $A_0$, contributed by the transmitted wave, remains finite when $\phi = \pi - \phi_0$, and that proportional to $B_0$, contributed by the reflected wave, remains finite when $\phi = \pi + \phi_0$, as these directions do not coincide with the respective directions of travel of these waves. We are thus obliged to assume that $\alpha = \phi_0$ and that $\beta = -\phi_0$. The equation (11) then reduces to

$$u = \exp\left(i\frac{2\pi t}{T}\right)\frac{\exp(-ik\rho)}{2\sqrt{2\pi k\rho}}\left[\frac{A}{\cos\frac{1}{2}(\phi - \phi_0)} + \frac{B}{\cos\frac{1}{2}(\phi + \phi_0)}\right], \tag{12}$$

where $A$ and $B$ are suitably chosen constants. As in the elementary diffraction theory we write

$$A = \exp\left(-i\frac{\pi}{4}\right) = i^{3/2},$$

which expresses the difference of path of $\lambda/8$ between a parent plane wave and the divergent cylindrical wave from a laminar strip cut out of it. The value of $B$ relatively to $A$ evidently depends on the change of amplitude and phase occurring in reflection at the surface of the screen. We write therefore

$$B/A = -(C_s + iD_s) \quad \text{or} \quad +(C_p + iD_p),$$

according to the state of polarisation of the incident beam. The formulae (7) and (8) are then reproduced.

## 4. Explanation of ellipticity of the diffracted light

From expressions (7) and (8) the ellipticity of the light diffracted through large angles, when the incident light is polarised in any arbitrary azimuth, follows as an immediate consequence. We shall consider first the case of normal incidence on a plane screen. We have then

$$C_s + iD_s = C_p + iD_p = \frac{n(1 - i\kappa) - 1}{n(1 - i\kappa) + 1}.$$

Taking for the case of a steel edge and for $\lambda = 5{\cdot}80 \times 10^{-5}$ cm, $n\kappa = 3{\cdot}24$ and $n = 2{\cdot}46$, we have $C_s + iD_s = C_p + iD_p = 0{\cdot}69 - i \times 0{\cdot}29$. With these numerical values and writing the expressions in the square brackets on the right-hand side of (7) and (8) in the form $F_s + iG_s$ and $F_p + iG_p$ respectively, the values of $F_s^2 + G_s^2$ and of $F_p^2 + G_p^2$ for various angles of observation, and the phase differences between these two components are shown in table 1.

Gouy noticed that with a sharp steel edge, the light diffracted into the region of shadow shows no sensible ellipticity when the deviation is less than 45°. For larger deviations, it becomes sensible, the $\|^l$ component being in advance of the $\perp^r$ component, the difference of path being, however, always numerically less than $\lambda/4$. It will be seen that this is in general agreement with the figures for the difference of path shown in the fifth column of table 1. The table also indicates the interesting result that in the region of exterior diffraction, the path difference

**Table 1.** Diffraction by steel edge: normal incidence. $\|^l$ and $\perp^r$ indicate plane of polarisation parallel and perpendicular respectively to the plane of incidence. $\lambda = 5{\cdot}80 \times 10^{-5}$ cm.

| Region of observation | Direction of diffracted ray $\phi$ | Intensity of $\|^l$ component $F_s^2 + G_s^2$ | Intensity of $\perp^r$ component $F_p^2 + G_p^2$ | Difference of path between the components $(\delta_p - \delta_s)/\lambda$ | Difference of path according to Sommerfeld formulae |
|---|---|---|---|---|---|
| | ° | | | | |
| Boundary of reflection | 90 | — | — | 0·50 | 0·50 |
| Exterior diffraction | 120 | 15·0 | 3·9 | 0·45 | 0·50 |
| (region of illumination) | 150 | 6·7 | 0·38 | 0·35 | 0·50 |
| | 180 | 5·9 | 0·36 | 0·15 | — |
| | 210 | 8·0 | 1·55 | 0·06 | 0 |
| | 240 | 21·1 | 10·0 | 0·03 | 0 |
| Boundary of shadow | 270 | — | — | 0 | 0 |
| Interior diffraction | 285 | 49 | 70 | −0·01 | 0 |
| (region of shadow) | 300 | 10·0 | 21·1 | −0·03 | 0 |
| | 315 | 3·6 | 11·4 | −0·04 | 0 |
| | 330 | 1·55 | 8·0 | −0·06 | 0 |
| | 345 | 0·72 | 6·4 | −0·09 | 0 |
| | 360 | 0·36 | 5·9 | −0·15 | — |

changes sign and increases in a continuous manner up to the boundary of reflection, when it becomes half a wavelength, while according to the Sommerfeld formulae there is a sudden reversal of phase when the diffracted ray lies in the continuation of the plane of the screen.

## 5. Effect of oblique incidence

When the light is incident on the screen obliquely at an angle $\theta$ (measured as usual from the normal) we have

$$C_s + iD_s = -\frac{\cos\theta - \sqrt{n^2(1-i\kappa)^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2(1-i\kappa)^2 - \sin^2\theta}}, \tag{13}$$

and

$$C_p + iD_p = \frac{n^2(1-i\kappa)^2\cos\theta - \sqrt{n^2(1-i\kappa)^2 - \sin^2\theta}}{n^2(1-i\kappa)^2\cos\theta + \sqrt{n^2(1-i\kappa)^2 - \sin^2\theta}}. \tag{14}$$

At normal incidence (13) and (14) are equal, and at grazing incidence they are again equal but opposite in sign. At the principal incidence (which lies between 70° and 80° for most metals) the difference of path between the $\|^l$ and $\perp^r$ components of the reflected wave amounts to $\lambda/4$ and then rapidly diminishes to 0 as grazing incidence is approached. With the help of the formulae (7), (8), (13) and (14) the intensity of the two components of the diffracted light and their phase-difference can be calculated for any angle of incidence and of observation. Let us first consider moderately oblique incidence. Two cases have naturally to be distinguished, viz. when the incidence is from the screen-side of the normal, and when it is from the farther side. Tables 2 and 3 give the values for a steel screen when $\phi_0 = 135°$ and 45°, respectively, for yellow light. It can be seen that the effects observed in interior and exterior diffraction are no longer similar to each other.

In the case considered in table 2, the region of shadow is of much smaller angular width, and the degree of polarisation and also the ellipticity increase rapidly as we approach the plane of the screen ($\phi = 360°$). On the other hand, in table 3 the region of shadow is much wider, and the polarisation and ellipticity of the diffracted light increase less rapidly with increasing deviation of the diffracted ray. In exterior diffraction these effects are reversed.

## 6. Diminished intensity of diffracted light

If we compare the figures shown in tables 2 and 3 with those calculated from Sommerfeld's formulae for a perfectly reflecting screen, we find that the effect of

**Table 2.** Diffraction by a steel edge $\phi_0 = 135°$.

| Region of observation | Direction of diffracted ray $\phi$ | Intensity of $\|^r$ component $F_s^2 + G_s^2$ | Intensity of $\perp^r$ component $F_p^2 + G_p^2$ | Difference of path between the components $(\delta_p - \delta_s)/\lambda$ . |
|---|---|---|---|---|
| Exterior diffraction | 135 | 4·6 | 0·30 | 0·23 |
| | 180 | 3·8 | 0·35 | 0·14 |
| | 225 | 4·9 | 0·8 | 0·09 |
| | 270 | 12 | 4·0 | 0·04 |
| | 285 | 24 | 11 | 0·03 |
| | 300 | 75 | 48 | 0·02 |
| Boundary of shadow | 315 | — | — | 0 |
| Region of shadow | 330 | 41 | 75 | −0·02 |
| | 345 | 6·0 | 27 | −0·05 |
| | 360 | 0·7 | 18 | −0·17 |

**Table 3.** Diffraction by a steel edge $\phi_0 = 45°$.

| Region of observation | Direction of diffracted ray $\phi$ | Intensity of $\|^r$ component $F_s^2 + G_s^2$ | Intensity of $\perp^r$ component $F_p^2 + G_p^2$ | Difference of path between the components $(\delta_p - \delta_s)/\lambda$ |
|---|---|---|---|---|
| | ° | | | |
| Exterior diffraction | 135 | — | — | 0·45 |
| | 165 | 28 | 2·3 | 0·32 |
| | 195 | 31 | 8·4 | 0·06 |
| Boundary of shadow | 225 | — | — | 0 |
| Region of shadow | 255 | 9·6 | 22 | −0·03 |
| | 285 | 1·5 | 7·0 | −0·06 |
| | 315 | 0·5 | 4·2 | −0·09 |
| | 345 | 0·2 | 3·3 | −0·14 |
| | 360 | 0·1 | 3·1 | −0·17 |

imperfect conductivity of the screen is not only to introduce elliptic polarisation, but also to diminish the total intensity of the diffracted light and the ratio of the components of vibration for specified angles of incidence and diffraction. In fact, these effects are all closely related to one another, and become the more striking when the incidence is very oblique.

In table 4 the case of oblique incidence on a steel edge has been worked out and shown. It is assumed that $\phi_0 = 170°$, that is, only 10° short of grazing incidence.

The figures for the steel edge and for a perfectly reflecting screen are shown side by side for comparison, and it will be seen that the intensity of the $\perp^r$ component is diminished to one-fifth of its value, on the surface of the screen, by reason of the imperfect conductivity, while that of the $\|^l$ component is very slightly increased. Nevertheless, the ratio of the $\perp^r$ and $\|^l$ components remains large, showing that even at such incidences the polarisation remains large. We have to approach grazing incidence still more closely before the diminution of the $\perp^r$ component is such as to produce a striking diminution of the completeness of polarisation. From formulae (7), (8), (13) and (14) it follows that, at grazing incidence, the diffracted light should be unpolarised at all angles.

**Table 4.** Diffraction by a steel edge $\phi_0 = 170°$ region of shadow.

| Direction of diffracted ray $\phi$ | Intensity of $\|^l$ component for a *perfect conductor* | Intensity of $\|^l$ component for *steel* $F_s^2 + G_s^2$ | Intensity of $\perp^r$ component for *steel* $F_p^2 + G_p^2$ | Intensity of $\perp^r$ component for a *perfect conductor* | Difference of path between the components for *steel* $(\delta_p - \delta_s)/\lambda$ |
|---|---|---|---|---|---|
| ° | | | | | |
| 350 | — | — | — | — | 0 |
| 352 | 2590 | 2600 | 3100 | 4060 | −0·01 |
| 354 | 460 | 480 | 750 | 1280 | −0·02 |
| 356 | 119 | 128 | 313 | 746 | −0·04 |
| 358 | 23 | 28 | 170 | 571 | −0·07 |
| 360 | 0 | 0·9 | 109 | 527 | −0·23 |

The foregoing considerations help to explain, at least in part, the interesting observation of Gouy that for given directions of the incident and diffracted rays, the intensity of the diffracted light is a *maximum* when the plane of the screen bisects the angle between these two directions. According to Sommerfeld's formulae the intensity should be a *minimum* for this position of the screen. Owing, however, to the imperfect conductivity of actual screens, as we have seen, the intensity falls off in approaching the extreme cases in which the light is incident grazingly on the screen from either direction. As we shall see later, other circumstances, as, for instance, the finite angle formed by the faces of the screen at the edge, or the actual rounding off of the latter, would also operate in the direction of diminishing the intensity of the diffracted light in the two extreme positions of the screen. Hence the intermediate position for the screen actually gives the maximum instead of the minimum intensity for the diffracted rays in the particular direction.

## 7. Explanation of diffraction colours

The wavelength enters in the expression for the intensity of the diffracted light in two distinct ways. Referring to (7) and (8) it will be seen that the intensity is inversely proportional to $k$, that is, proportional to the wavelength. The longer wavelengths would thus tend to be more prominent in the light diffracted by the edge than in the incident light. This effect would operate both on the parallel and perpendicular components of vibration in the diffracted light. The colour of the diffracted light is also influenced, and in an entirely different way, by the factors $C_s + iD_s$ and $C_p + iD_p$ appearing in (7) and (8), which are, in general, functions of the wavelength of the incident light. If we confine ourselves to the case of normal incidence, $C_s + iD_s$ and $C_p + iD_p$ are identical in magnitude. But the former appears with a negative sign in (7) and the latter with a positive sign in (8). Hence, if a particular wavelength appears with a strengthened amplitude in (8) it will appear with a weakened amplitude in (7), and *vice versa*. This, taken together with the proportionality to $\lambda$ already mentioned, furnishes an explanation of the difference in the colour of the parallel and perpendicular components of the light diffracted into the region of shadow, which was discovered by Gouy for metals such as copper and gold. It can easily be seen that in the region of shadow, the longer wavelengths which are strongly reflected by the metal would be much enhanced in the perpendicular component, while the corresponding weakening in the parallel component would be almost insensible. In the region of exterior diffraction, these features are interchanged.

**Table 5.** Gold screen $\phi_0 = 90°$ (normal incidence).

| Direction of diffracted ray $\phi$ | $(F_s^2 + G_s^2)\lambda \times 10^5$, for $\lambda \times 10^5 =$ | | | $(F_p^2 + G_p^2)\lambda \times 10^5$, for $\lambda \times 10^5 =$ | | |
|---|---|---|---|---|---|---|
| | 4·20 | 5·80 | 7·00 | 4·20 | 5·80 | 7·00 |
| ° | | | | | | |
| 270 | — | — | — | — | — | — |
| 285 | 217 | 282 | 326 | 279 | 408 | 508 |
| 300 | 48·3 | 59·0 | 64·4 | 80·4 | 125 | 159 |
| 315 | 19·1 | 22·1 | 22·1 | 41·8 | 68·3 | 88·7 |
| 330 | 9·56 | 10·7 | 9·50 | 28·1 | 48·5 | 63·9 |
| 345 | 5·44 | 6·36 | 4·87 | 22·1 | 40·2 | 53·6 |
| 360 | 3·41 | 4·86 | 3·48 | 19·5 | 37·6 | 50·7 |

In table 5 the intensity of the diffracted light has been calculated for the case of a gold screen using the following data:

$$\lambda = 7{\cdot}00 \times 10^{-5}\,\text{cm.};\quad n = 0{\cdot}280,\quad n\kappa = 3{\cdot}800;$$
$$\lambda = 5{\cdot}80 \times 10^{-5}\,\text{cm.};\quad n = 0{\cdot}415,\quad n\kappa = 2{\cdot}750;$$
$$\lambda = 4{\cdot}20 \times 10^{-5}\,\text{cm.};\quad n = 1{\cdot}570,\quad n\kappa = 1{\cdot}800.$$

From the table it will be seen that in the region of shadow $(F_s^2 + G_s^2)\lambda$ has practically the same value for different wavelengths, while $(F_p^2 + G_p^2)\lambda$ increases in value as we proceed towards the red end of the spectrum. The increase with wavelength is much more marked for large angles of diffraction than for small angles. Further, $F_p^2 + G_p^2$ is always greater than $F_s^2 + G_s^2$, the ratio between the two increasing with the angle of diffraction. From these facts, it follows that when the region of shadow is examined, the $\|^l$ component of the diffracted light will be perfectly white, while the $\perp^r$ component, which is in fact much stronger than the other, will exhibit an orange-yellow tint, the colouration being the more marked, the further we go into the region of shadow. The same colour effects will be noticeable also in the region of exterior diffraction, the $\|^l$ and $\perp^r$ components now, however, exchanging places.

## 8. Intensified colours at oblique incidences

While the variation with wavelength of the intensity of the $\perp^r$ component shown in table 5 is marked enough, it is not exceptionally large, being in fact of the same order of magnitude as the variation of the reflecting power of the metal with wavelength. This is in agreement with observation, for Gouy found that the sharpest metallic edges do not show particularly vivid colours by diffraction. When, however, the incidence on the screen is made oblique, the colours of the $\perp^r$ component should become more lively. To understand why this should be the case, we have only to refer to section 6 above, in which it was shown that the imperfect reflectivity of the metal results in a diminution of the intensity of the diffracted light in comparison with the theoretical value for a perfectly reflecting screen, and that this diminution becomes the more marked as the incidence of the light on the screen becomes more oblique. Those wavelengths, however, for which the reflecting power of the metal approaches unity, persist in nearly full strength in the $\perp^r$ component of the diffracted light, and hence determine its colour in increasing measure as the obliquity of the screen is increased. It is to be noted also that the colour should appear at smaller deviations of the ray in interior diffraction, and at larger deviations in exterior diffraction, or *vice versa*, according to the position of the screen.

In table 6 the case of a gold screen, for a position of the screen 10° short of grazing incidence, has been worked out and the intensities of the $\|^l$ and $\perp^r$ components are shown for six different wavelengths, the direction of observation considered being along the surface of the screen in the region of shadow. The normal reflecting power of the metal for the same wavelengths is also shown for comparison.

It will be noticed that the intensity of the $\|^l$ component varies but little with wavelength, while the $\perp^r$ component shows large intensities in the orange and red regions in the spectrum. The effect in the latter case is of a highly selective

**Table 6.** Gold edge $\phi_0 = 170°$, $\phi = 360°$.

| $\lambda \times 10^5$ (cm) | Intensity of $\|^l$ component $(F_s^2 + G_s^2)\lambda \times 10^5$ | Intensity of $\perp^r$ component $(F_p^2 + G_p^2)\lambda \times 10^5$ | The reflection coefficient |
|---|---|---|---|
| 4·00 | 9·9 | 200 | 0·360 |
| 4·60 | 12·5 | 210 | 0·358 |
| 5·20 | 17·5 | 240 | 0·608 |
| 5·80 | 10·5 | 470 | 0·827 |
| 6·20 | 8·8 | 650 | 0·889 |
| 7·00 | 7·1 | 1000 | 0·930 |

character, becoming pronounced only for the wavelengths for which the reflecting power of the metal approaches unity.

When the figures shown in table 6 are computed after the manner employed by the late Lord Rayleigh* for discussion of the colours of thin plates, and plotted in Maxwell's colour-triangle, it is found that the $\|^l$ component is perfectly white, while the $\perp^r$ component is of a rich orange-yellow colour.

The case of other metals may be worked out in a similar manner. In table 7 are given the reflection-coefficients and the colour of the diffracted light as observed by Gouy and Wien, for a number of metals. The general relationship between them is fairly clear from the figures. In the case of the whiter metals, of course, the colour is largely determined by the factor $\lambda$ appearing in the expression for the intensity of the diffracted light.

**Table 7.** Reflection coefficients and diffraction colours.

| For $\lambda \times 10^5 =$ | 4·20 | 4·50 | 5·00 | 5·50 | 6·00 | 6·50 | 7·00 | Colour of $\perp^r$ component of diffracted light |
|---|---|---|---|---|---|---|---|---|
| Silver | 0·866 | 0·905 | 0·913 | 0·927 | 0·926 | 0·935 | 0·946 | Pale yellow |
| Copper | 0·33 | 0·37 | 0·44 | 0·48 | 0·72 | 0·80 | 0·83 | Red |
| Steel | 0·52 | 0·54 | 0·55 | 0·55 | 0·55 | 0·56 | 0·58 | Reddish white |
| Platinum | 0·518 | 0·547 | 0·584 | 0·611 | 0·642 | 0·663 | 0·69 | Yellow |
| Zinc | 0·803 | 0·806 | 0·805 | 0·789 | 0·774 | 0·771 | 0·770 | Colour insensible |
| Tin | — | 0·605 | 0·670 | 0·686 | 0·706 | 0·713 | 0·716 | Greenish yellow |

* Lord Rayleigh, *Scientific Papers*, **2**, 498.

## 9. Diffraction by metallic wedges

Poincaré considered the case of a perfectly reflecting wedge in his first memoir, and showed that if the surfaces of the wedge are given by the angles $\phi = 0$ and $\phi = \chi$, the rays diffracted from its edge have an amplitude proportional to

$$\left[\frac{1}{\cos\dfrac{\pi^2}{\chi} - \cos\dfrac{\pi}{\chi}(\phi - \phi_0)} \mp \frac{1}{\cos\dfrac{\pi^2}{\chi} - \cos\dfrac{\pi}{\chi}(\phi + \phi_0)}\right]. \tag{15}$$

As the result of a more elaborate analysis, Wiegrefe* found for the cylindrical wave diverging from a wedge-shaped edge the identical expression given by (15) with the multiplying factor

$$-i^{3/2}\exp\left(i\frac{2\pi t}{T}\right)\exp(-ik\rho)\sqrt{\frac{\lambda}{\rho}}\cdot\frac{\sin\dfrac{\pi^2}{\chi}}{2\chi}. \tag{16}$$

On putting $\chi = 2\pi$, the formulae (15) and (16) reduce to those for the case of a perfectly reflecting plane screen. In the case of an imperfectly conducting wedge, we modify expression (15) and write it in the form

$$\left[\frac{1}{\cos\dfrac{\pi^2}{\chi} - \cos\dfrac{\pi}{\chi}(\phi - \phi_0)} - \frac{C_s + iD_s}{\cos\dfrac{\pi^2}{\chi} - \cos\dfrac{\pi}{\chi}(\phi + \phi_0)}\right], \tag{17}$$

or

$$\left[\frac{1}{\cos\dfrac{\pi^2}{\chi} - \cos\dfrac{\pi}{\chi}(\phi - \phi_0)} + \frac{C_p + iD_p}{\cos\dfrac{\pi^2}{\chi} - \cos\dfrac{\pi}{\chi}(\phi + \phi_0)}\right], \tag{18}$$

where $C_s$, $D_s$, $C_p$, $D_p$ have the same significance as previously, and are functions of the angle of incidence of the light on the illuminated side of the wedge.

From formula (15) it appears that along the two surfaces of the wedge $\phi = 0$ and $\phi = \chi$, the diffracted light should be completely polarised with the intensity of the $\|^l$ component zero, and that of the $\perp^r$ component finite. As the rear surface of the wedge limits the region of shadow, it follows as a consequence that the polarisation-effects should usually appear at smaller deviations of the diffracted ray in the case of a wedge than for a plane screen. When the imperfect reflecting power of the metal is taken into account, as in formulae (17) and (18), it would also follow, for the reasons stated, that the colours of the diffracted light should also be observable at smaller deviations and be generally more striking than for a plane screen.

*A Wiegrefe, *Ann. Phys.* **39**, 449 (1912).

## 10. Summary

1. The paper contains a discussion of the observations of Gouy on the intensity, colour and polarisation of the light diffracted through large angles by metallic screens and wedges with polished edges. The well-known expressions due to Poincaré and Sommerfeld referring to diffraction by perfectly-reflecting screens and wedges, are modified so as to take into account the changes of phase and amplitude which occur when light is reflected at the surface of a metal. The modified formulae are then discussed.

2. The formulae show that when the incident light is plane-polarised in any arbitrary azimuth, the light diffracted through large angles is elliptically polarised, the sign of the ellipticity being different in interior and exterior diffraction.

3. In interior diffraction, the component polarised in the plane of incidence is white, while the perpendicular component is coloured, the colour depending on the nature of the metal. In exterior diffraction, these effects are reversed.

4. The effect of imperfect conductivity is to make the intensity of the diffracted light less and less as the incidence becomes more and more oblique. This diminution is least for the wavelength for which the reflection-coefficient is largest. The colour-effects arise in this way and therefore become more prominent at oblique incidences.

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# Diffraction of light by a transparent lamina

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ABSTRACT

The present paper embodies an attempt to consider the problem of diffraction of light by a *very thin* plate of transparent material, bounded by a straight edge, with greater exactness than is attained in the usual elementary treatment on the Fresnel–Huygen principle. The method adopted, though not completely rigorous, bases itself on the electromagnetic theory of light, and seeks to express the disturbance in the field in the form of functions which are solutions of the equations of wave-propagation. The formulae obtained indicate that the light diffracted by the edge should exhibit colour and polarization effects varying in a remarkable manner with the thickness of the plate and the direction of observation. Effects having the general character of those indicated by the theory have actually been observed in experiment. The theory, however, requires modification in the case of thicker laminae, where further complications arise which are not here taken account of.

## 1. Introduction

The problem of diffraction by a plane transparent lamina arises in considering the theory of such phenomena as the colours exhibited by the "striae" or laminar boundaries in mica,* and the colours of mixed plates.† It has been observed that the edges of thin laminae diffract light through large angles, and that the light thus diffracted exhibits colour and polarization effects which are in some respects analogous to those discovered by Gouy‡ with metallic edges. The elementary treatment of laminar diffraction usually given is thus inadequate. In the present paper an attempt is made to place the theory of diffraction by laminar boundaries on a more satisfactory basis. The case of a thick lamina is too complicated to offer hope of an exact solution. In the case of a thin lamina, however, certain simplifications are possible, as we shall see presently, which enable the problem to be dealt with successfully.

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*C V Raman and P N Ghosh, *Nature* (*London*) **102**, 205 (1918).
†C V Raman and B N Banerji, *Philos. Mag.*, **41**, 338 (1921).
‡G Gouy, *Ann. Chim. Phys.* **8**, 145 (1886).

## 2. Theory

We shall confine ourselves to the case in which the light is incident on the lamina in a plane perpendicular to its edge. The region round the lamina can then be divided into three parts: The first consists of the region in which the light transmitted through the lamina alone appears. The second is the region in which only the incident waves appear. The third is the region in which the incident and reflected waves are superposed.

The middle surface of the lamina may be taken as the plane $\varphi = 0$, and its edge as the $Z$-axis in a system of cylindrical co-ordinates. We may represent the incident wave by the expression

$$\exp(i \cdot 2\pi t/T) \exp[i\kappa\rho \cos(\varphi - \varphi_0)], \tag{1}$$

$\varphi_0$ being the angle between the plane of the lamina and the incident rays. The train of waves reflected from the lamina may be represented by the expression

$$-\exp(i \cdot 2\pi t/T)(A_s + iB_s) \exp[i\kappa\rho \cos(\varphi + \varphi_0)] \tag{2}$$

or by

$$\exp(i \cdot 2\pi t/T)(A_p + iB_p) \exp[i\kappa\rho \cos(\varphi + \varphi_0)] \tag{3}$$

and the train of waves transmitted through the lamina by the expression

$$\exp(i \cdot 2\pi t/T)(C_s + iD_s) \exp[i\kappa\rho \cos(\varphi - \varphi_0)] \tag{4}$$

or by

$$\exp(i \cdot 2\pi t/T)(C_p + iD_p) \exp[i\kappa\rho \cos(\varphi + \varphi_0)] \tag{5}$$

The expressions (2) and (4) refer to the case (which we shall refer to as the $\|^l$ case), in which the planes of incidence and polarization are coincident. The expressions (3) and (5) refer to the case (which we shall refer to as the $\perp^r$ case) in which they are mutually perpendicular. The multiplying factors $(A_s - iB_s)$, etc., are those given by the well known theory of the colours of thin plates,* and may be deduced directly from the electromagnetic theory so as to satisfy the boundary conditions on either face of the lamina.

Now the disturbances represented by expressions (1) to (5) do not extend throughout the whole field, but are confined to the particular regions of the field already indicated. To find the solution of the diffraction problem, we seek a function which satisfies the equations of wave-propagation, and which, while being valid throughout the whole field outside the substance of the lamina, represents a disturbance approaching asymptotically to the values given by those expressions at a sufficient distance from the origin in the respective parts of the field to which they refer.

*Drude's *Theory of Optics*, English Translation, p. 302.

Such an expression is

$$u = F(\rho, \varphi, \varphi_0) - (A_s + iB_s)F(\rho, \varphi, -\varphi_0) + (C_s + iD_s)F(\rho, \varphi, \varphi_0 - 2\pi) \tag{6}$$

or

$$u = F(\rho, \varphi, \varphi_0) + (A_p - iB_p)F(\rho, \varphi, -\varphi_0) + (C_p - iD_p)F(\rho, \varphi, \varphi_0 - 2\pi) \tag{7}$$

in which $F(\rho, \varphi, \varphi_0)$ is the well known solution of the wave equation due to Sommerfeld.

$$F(\rho, \varphi, \varphi_0) = \exp(i \cdot 2\pi t/T)\left(\frac{i}{\pi}\right)^{1/2} \exp[i\kappa\rho\cos(\varphi - \varphi_0)] \int_{-\infty}^{T} \exp(-i\lambda^2)\mathrm{d}\lambda \tag{8}$$

where

$$T = \sqrt{2k\rho}\cos\tfrac{1}{2}(\varphi - \varphi_0). \tag{9}$$

$F(\rho, \varphi, -\varphi_0)$ and $F(\rho, \varphi, \varphi_0 - 2\pi)$ are obtained by writing $-\varphi_0$ and $(\varphi_0 - 2\pi)$ respectively for $\varphi_0$ in (8) and (9).

In the $\|^l$ case $u$ represents the electric force parallel to the edge of the screen and in the $\perp^r$ case it represents the magnetic force in the same direction. It can readily be verified that the asymptotic values of (6) and (7) are those given by (1), (2), (3), (4) and (5), in the parts of the field to which they refer. When (9) is positive, the asymptotic expansion of (8) is

$$F(\rho, \varphi, \varphi_0) \sim \exp(i \cdot 2\pi t/T)\left[\exp[i\kappa\rho\cos(\varphi - \varphi_0)] + \frac{i^{3/2}\exp(-i\kappa\rho)}{4\pi\sqrt{\rho/\lambda}\cos\frac{1}{2}(\varphi - \varphi_0)}\right] \tag{10}$$

while, if (9) be negative, there is a similar expansion in which the first term is left out. The light diffracted by the edge may accordingly be written thus

$$E_z = \exp(i \cdot 2\pi t/T) \cdot \frac{i^{3/2}\exp(-i\kappa\rho)}{4\pi\sqrt{\rho/\lambda}}\left[\frac{1 - (C_s + iD_s)}{\cos\frac{1}{2}(\varphi - \varphi_0)} - \frac{A_s + iB_s}{\cos\frac{1}{2}(\varphi + \varphi_0)}\right] \tag{11}$$

$$H_z = \exp(i \cdot 2\pi t/T) \cdot \frac{i^{3/2}\exp(-i\kappa\rho)}{4\pi\sqrt{\rho/\lambda}}\left[\frac{1 - (C_p + iD_p)}{\cos\frac{1}{2}(\varphi - \varphi_0)} + \frac{A_p + iB_p}{\cos\frac{1}{2}(\varphi + \varphi_0)}\right]. \tag{12}$$

We may write the expression within the square brackets in (11) and (12) respectively in the form $(F_s + iG_s)$ and $(F_p + iG_p)$. $(F_s^2 + G_s^2)$ and $(F_p^2 + G_p^2)$ are measures of the intensity of the components in the radiation diffracted from the edge, and their ratio indicates its state of polarisation when the incident light is unpolarised. $\delta_s = G_s/F_s$ and $\delta_p = G_p/F_p$ give the phases of the components, and $(\delta_p - \delta_s)/\lambda$ is a measure of the ellipticity of the diffracted radiation when the incident light is polarized in any arbitrary azimuth.

## 3. Normal incidence: Very thin lamina

In the case of normal incidence, we have $\varphi_0 = \pi/2$, and further

$$A_s + iB_s = A_p + iB_p = \frac{i(\mu^2 - 1)\sin p}{i(\mu^2 + 1)\sin p + 2\mu\cos p} \tag{13}$$

$$C_s + iD_s = C_p + iD_p = \frac{2\mu(\cos q + i\sin q)}{i(\mu^2 + 1)\sin p + 2\mu\cos p}, \tag{14}$$

where $p = 2\pi\mu d/\lambda$ and $q = 2\pi d/\lambda$, $d$ being the thickness of the lamina and $\mu$ its refractive index. We have only to substitute (13) and (14) in equations (11) and (12) and evaluate them numerically to find the intensity and state of polarization of the diffracted light in any direction.

Consider first a lamina so thin that we may put $\sin p = p$, $\sin q = q$, $\cos p = \cos q = 1$. Since $p = \mu q$, we find readily on making these substitutions, that

$$1 - (C_s + iD_s) = 1 - (C_p + iD_p) = A_s + iB_s = A_p + iB_p$$

and that

$$F_s + iG_s = \frac{\pi(\mu^2 - 1)d}{\lambda}\left[\frac{1}{\cos\frac{\varphi - \varphi_0}{2}} - \frac{1}{\cos\frac{\varphi + \varphi_0}{2}}\right] \tag{15}$$

$$F_p + iG_p = \frac{\pi(\mu^2 - 1)d}{\lambda}\left[\frac{1}{\cos\frac{\varphi - \varphi_0}{2}} + \frac{1}{\cos\frac{\varphi + \varphi_0}{2}}\right]. \tag{16}$$

From (15) and (16), it follows at once that along the surface of the screen ($\varphi = 0$ and $\varphi = 2\pi$), the $\|^l$ component of the diffracted light would be zero, and the $\perp^r$ component would be finite, thus giving complete polarization. In the opposite direction ($\varphi = \pi$), however, the $\|^l$ component of the diffracted light would be finite, and the $\perp^r$ component would be zero, thus again giving complete polarization but in a perpendicular plane. The intensity of the diffracted light would, for such thin laminae, be proportional to the square of the thickness.

## 4. Thicker laminae: Colour and elliptic polarization

If we assume that formulae (11) and (12) remain valid for greater thicknesses of the lamina, at least as an approximation, it is evident from these expressions taken together with (13) and (14) that the intensity and state of polarization of the diffracted light would be functions of the wavelength, the thickness of the lamina, and the angle of diffraction. Hence, in white light, the diffracted radiation from the edge would exhibit colour.

If we assume that the incident light is unpolarized, the diffracted light from the edge would be partially polarized to an extent depending essentially on the contribution proportional to $(A_s + iB_s)$ or $(A_p + iB_p)$ which arises from the reflected wave and appears with a negative sign in (11) and with a positive sign in (12). It follows that the degree of polarization should vary with $\sin p$ in a periodic manner, being zero when the thickness of the lamina is such that the reflected light vanishes and maximum for intermediate thicknesses. The intensity of the diffracted light, however, depends chiefly on the resultant of the contributions from the incident and transmitted waves which are proportional to $(1 - C_s - iD_s)$ or $(1 - C_p - iD_p)$. It is easily shown that this part depends on $\sin(p - q)$ and that its intensity is a maximum when the relative retardation on the two sides of the boundary is half a wavelength, and a minimum when the relative retardation is a complete wavelength, and so on. Two sets of periodic variations in intensity thus occur giving rise to corresponding colour effects in the diffracted light. One set of variations affects both components of the vibration in the same way, while the other set affects the two components in opposite senses. Hence, the colour of the diffracted light would, to some extent, depend on the azimuth of polarization, and the two images of the diffracting edge seen through a double-image prism may in favourable cases actually exhibit complementary colours.

Figure 1 represents the values of $F_s^2 + G_s^2$ and $F_p^2 + G_p^2$ corresponding to an angle of diffraction of 90° ($\varphi = 180°$) for different thicknesses of the lamina, the refractive index being assumed to be 1·5. The periodic variations of the $\|^l$ and $\perp^r$ components of the intensity of the diffracted light are clearly seen. The

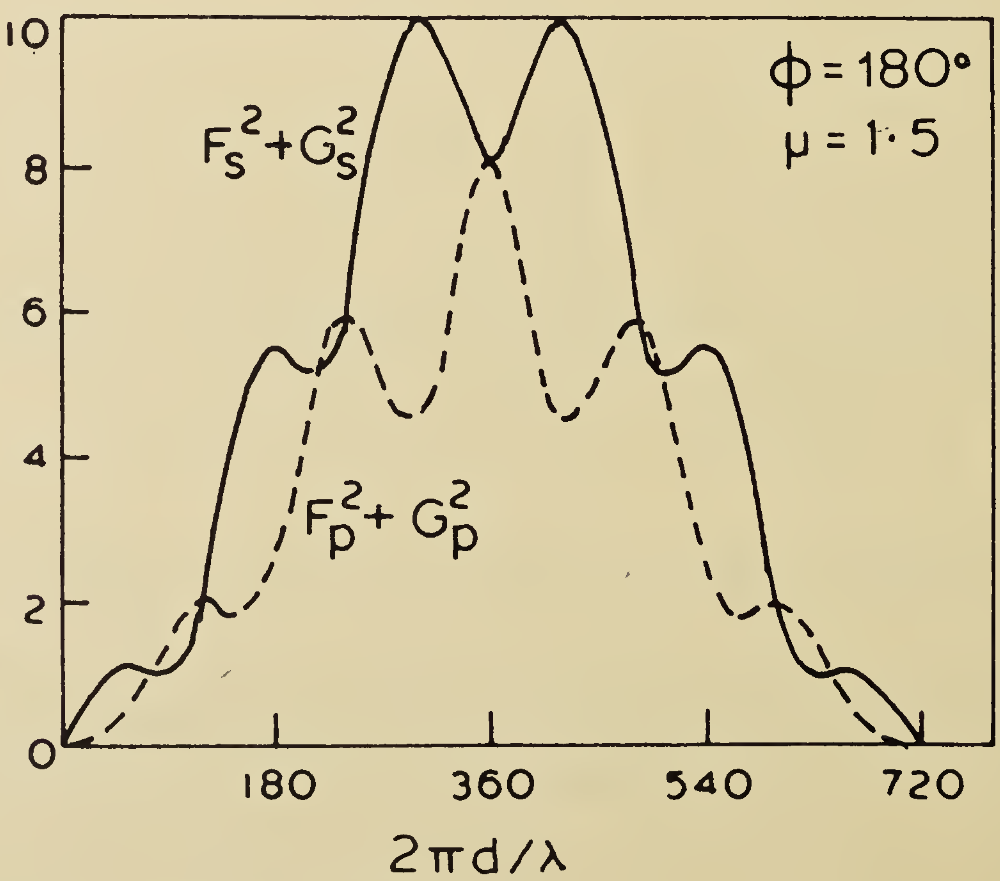


**Figure 1.** Intensities of components of diffracted light as functions of thickness.

polarization, which is given by the ratio of the two components, is almost complete for small thicknesses of the lamina; it is zero for thicknesses given by $2\pi d/\lambda = 78°$, 120°, 218°, 240°, and 360°. For intermediate thicknesses, the polarization fluctuates.

From (11), (12), (13) and (14) it is obvious that, when the incident light is plane-polarized in any azimuth, the diffracted light would in general be elliptically polarized. The phases of the $\perp^r$ and $\|^l$ components as calculated from the values of $\tan^{-1}(G_p/F_p)$ and $\tan^{-1}(G_s/F_s)$ respectively are shown in figure 2 for the same cases as those considered in figure 1. It is evident from the figure that, in certain cases, particularly for small thicknesses of the lamina, the phase differences are large and should easily be observed.

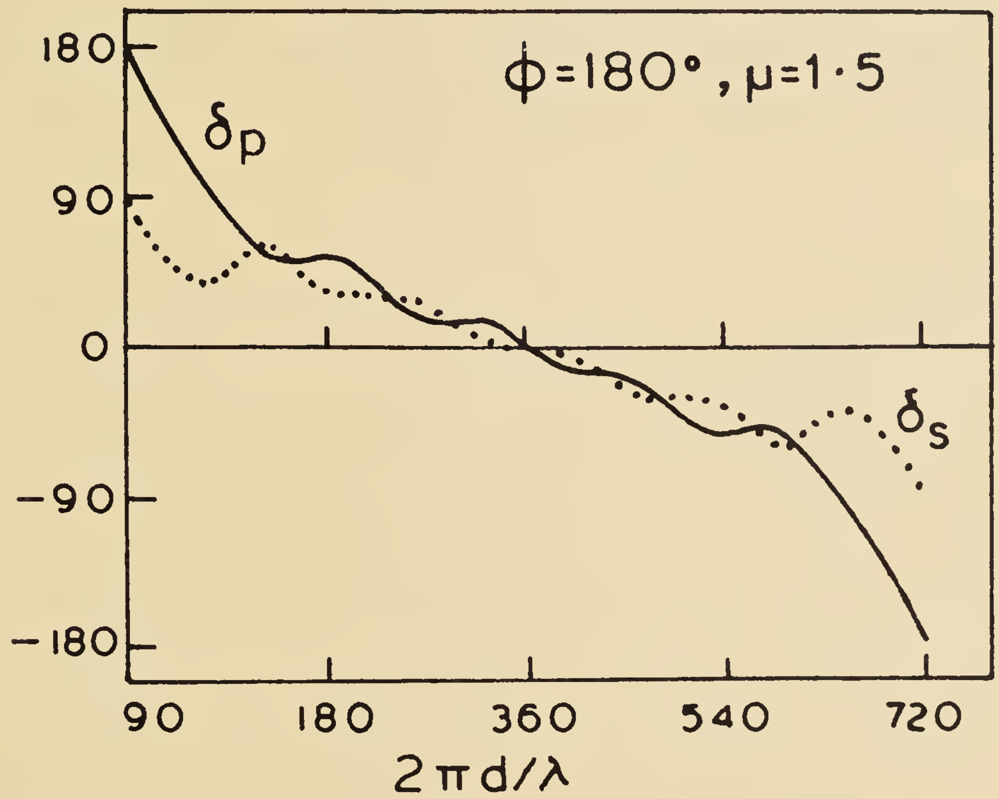


**Figure 2.** Phases of components of diffracted light as functions of thickness.

## 5. Supplementary remarks

The theory set out above is an approximation to the truth which is strictly applicable only to the thinnest laminae. It succeeds in explaining the polarization effects which would otherwise be unintelligible. With laminae of greater thickness, however, complications arise which are not here taken account of. In reality, we have not one edge, but two edges to deal with, namely, those relating to the front and rear surfaces of the lamina, at which the waves passing on either side of the boundary are diffracted and, diverging with a difference of path, interfere with each other.

It is readily seen that the path-difference between the interfering rays from the two edges would alter continually with increasing deviation of the diffracted ray, but in different ways in the two portions of the field to the right and the left of the direction of the incident pencil. Since the conditions on the two sides of the boundary between the incident and transmitted waves are thus dissimilar, the diffraction effects observed should be asymmetrical in intensity with reference to this boundary. Experimentally this is actually observed to be the case. Further, owing to the interference of the diffracted waves from the front and rear edges occurring under varying difference of path in different directions, the colour and intensity of the diffracted light should vary periodically with increasing deviation of the diffracted ray. This again is actually observed in experiment. It is not unlikely that a modified treatment which takes account of the complications referred to here may be successfully worked out. To enter into them more fully, however, or to make detailed comparisons between theory and observation would lie beyond the scope of the present paper.

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# The diffraction of light by high frequency sound waves: Part I

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## 1. Introduction

As is well known, Langevin showed that high frequency sound-waves of great intensity can be generated in fluids by the use of piezoelectric oscillators of quartz. Recently, Debye and Sears* in America and Lucas and Biquard† in France have described very beautiful experiments illustrating the diffraction of light by such high-frequency sound-waves in a liquid. Amongst the experimenters in this new field, may be specially mentioned R Bär‡ of Zürich who has carried out a thorough investigation and has published some beautiful photographs of the effect. The arrangement may be described briefly as follows. A plane beam of monochromatic light emerging from a distant slit and a collimating lens is incident normally on a cell of rectangular cross-section and after passing through the medium emerges from the opposite side. Under these conditions, the incident beam will be undeviated if the medium be homogeneous and isotropic. If, however, the medium be traversed by high-frequency sound-waves generated by introducing a quartz oscillator at the top of the cell, the medium becomes stratified into parallel layers of varying refractive index. Considering the case in which the incident beam is parallel to the plane of the sound-waves, the emerging light from the medium will now consist of various beams travelling in different directions. If the inclination of a beam with the incident light be denoted by $\theta$, it has been found experimentally that the formula

$$\sin\theta = \pm\frac{n\lambda}{\lambda^*}, \quad n(\text{an integer}) \geqslant 0 \tag{1}$$

*P Debye and F W Sears, *Proc. Natl. Acad. Sci.* (*Washington*), **18**, 409, 1932.
†R Lucas and P Biquard *J. Phys. Rad.*, **3**, 464, 1932.
‡R Bär, *Helv. Phys. Acta*, **6**, 570, 1933.

is in satisfactory agreement with the observed results, where $\lambda$ and $\lambda^*$ are the wavelengths of the incident light and the sound wave in the medium respectively. With sound waves of sufficient intensity, numerous orders of these diffraction spectra have been obtained; a wandering of the intensity amongst these orders has also been noticed by Bär* when the experimental conditions are varied.

Various theories of the phenomena have been put forward by Debye and Sears,[†] by Brillouin,[‡] and by Lucas and Biquard.[§] The former have not presented quantitative results and it is hard to understand from their theory as to why there should be so many orders and why the intensity should wander between the various orders under varying experimental conditions. In Brillouin's theory, the phenomenon is attributed to the reflection of light from striations of the medium caused by the sound waves. We know, however, from the work of Rayleigh that the reflection of light by a medium of varying refractive index is negligible if the variation is gradual compared with the wavelength of light. Under extreme conditions, we might perhaps obtain the Brillouin phenomenon, but the components of reflection should be very weak in intensity compared to the transmitted ones. As one can see later on in this paper, the whole phenomenon including the positions of the diffracted beams and their intensities can be explained by a simple consideration of the transmission of the light beam in the medium. Lucas and Biquard attribute the phenomenon to an effect of mirage of light waves in the medium. In what way the relation (1) enters in their theory is not clear. The wandering of the intensities of the various components observed by Bär has not found explanation in any of the above theories.

We propose in this paper a theory of the phenomenon on the simple consideration of the regular transmission of light in the medium and the phase changes accompanying it. The treatment is limited to the case of normal incidence. The formula (1) has been established in our theory. Also, a formula for the intensities of the various components has been derived. It is found that the above results are in conformity with the experimental results of Bär.*

## 2. Diffraction of light from a corrugated wave-front

The following theory bears a very close analogy to the theory of the diffraction of a plane wave (optical or acoustical) incident normally on a periodically corrugated surface, developed by the late Lord Rayleigh. He showed therein that a diffraction phenomenon would ensue in which the positions of the various components are given by a formula similar to (1) and their relative intensities are given by a formula similar to the one we have found.

*R Bär, *Helv. Phys. Acta*, **6** 570, 1933.

[†]P Debye and F W Sears, *Proc. Nat. Acad. Sci.* (*Washington*), **18**, 409, 1932.

[‡]L Brillouin, "La Diffraction de la Lumiere par des Ultra sons", *Acta Sci. et. Ind.* **59**, 1933.

[§]R Lucas and P Biquard, *J. Phys. et Rad.*, **3**, 464, 1932.

[‖]Lord Rayleigh, *Theory of Sound* **2**, page 89.

Consider a beam of light with a plane wave-front emerging from a rectangular slit and falling normally on a plane face of a medium with a rectangular cross-section and emerging from the opposite face parallel to the former. If the medium has the same refractive index at all its points, the incident beam will emerge from the opposite face with its direction unchanged. Suppose we now create layers of varying refractive index in the medium, say by suitably placing a quartz oscillator in the fluid. If the distance between the two faces be small, the incident light could be regarded as arriving at the opposite face with variations in the phase at its different parts corresponding to the refractive index at different parts of the medium. The change in the phase of the emerging light at any of its parts could be simply calculated from the optical lengths found by multiplying the distance between the faces and the refractive index of the medium in that region. This step is justified for $\int \mu(x, y, z)\mathrm{d}s$ taken over the actual path is minimum, i.e. it differs from the one taken over a slightly varied hypothetical path by a differential of the second order. So, the incident wave-front becomes a periodic corrugated wave-front when it traverses a medium which has a periodic variation in its refractive index. The origin of the axes of reference is chosen at the centre of the incident beam projected on the emerging face, the boundaries of the incident beam being assumed to be parallel to the boundaries of the face. The $X$-axis is perpendicular to the sound-waves and the $Z$-axis is along the direction of the incident beam of light. If the incident wave is given by

$$Ae^{2\pi i v t}$$

it will be

$$Ae^{2\pi i v}\{t - L\mu(x)/c\}$$

when it arrives at the other face where $L$ is the distance between the two faces and $\mu(x)$ the refractive index of the medium at a height $x$ from the origin. It is assumed that the radii of curvature of the corrugated wave-front are large compared with the distance between the two faces of the cell. If $\mu_0$ be the refractive index of the whole medium in its undisturbed state, we can write $\mu(x)$ as given by the equation

$$\mu(x) = \mu_0 - \mu \sin \frac{2\pi x}{\lambda^*}$$

ignoring its time variation, $\mu$ being the *maximum variation* of the refractive index from $\mu_0$.

The amplitude due to the corrugated wave at a point on a distant screen parallel to the face of the medium from which light is emerging whose join with the origin has its $x$-direction-cosine $l$, depends on the evaluation of the diffraction integral

$$\int_{-p/2}^{p/2} e^{2\pi i\{lx + \mu L \sin(2\pi x/\lambda^*)\}/\lambda} \mathrm{d}x$$

where $p$ is the length of the beam along the $X$-axis. The real and the imaginary parts of the integral are

$$\int_{-p/2}^{p/2} \{\cos ulx \cos(v \sin bx) - \sin ulx \sin(v \sin bx)\} dx$$

and

$$\int_{-p/2}^{p/2} \{\sin ulx \cos(v \sin bx) + \cos ulx \sin(v \sin bx)\} dx$$

where $u = 2\pi/\lambda, b = 2\pi/\lambda^*$ and $v = u\mu L = 2\pi\mu L/\lambda$.

We need the well-known expansions

$$\cos(v \sin bx) = 2\sum_0^{\infty}{}' J_{2r} \cos 2rbx$$

$$\sin(v \sin bx) = 2\sum_0^{\infty} J_{2r+1} \sin \overline{2r+1}\, bx$$

to evaluate the integrals, where $J_n[= J_n(v)]$ is the Bessel function of the $n$th order and a dash over the summation sign indicates that the coefficient of $J_0$ is half that of the others. The real part of the integral is then

$$2\sum_0^{\infty}{}' J_{2r} \int_{-p/2}^{p/2} \cos ulx \cos 2rbx\, dx - 2\sum_0^{\infty} J_{2r+1} \int_{-p/2}^{p/2} \sin ulx \sin \overline{2r+1}\, bx\, dx$$

or

$$\sum_0^{\infty}{}' J_{2r} \int_{-p/2}^{p/2} \{\cos(ul + 2rb)x + \cos(ul - 2rb)x\} dx$$

$$+ \sum_0^{\infty} J_{2r+1} \int_{-p/2}^{p/2} \{\cos(ul + \overline{2r+1}\, b)x$$

$$- \cos(ul - \overline{2r+1}\, b)x\} dx.$$

Integrating the above, we obtain

$$p\sum_0^{\infty}{}' J_{2r} \left\{ \frac{\sin(ul + 2rb)p/2}{(ul + 2rb)p/2} + \frac{\sin(ul - 2rb)p/2}{(ul - 2rb)p/2} \right\}$$

$$+ p\sum_0^{\infty} J_{2r+1} \left\{ \frac{\sin(ul + \overline{2r+1}\, b)p/2}{(ul + \overline{2r+1}\, b)p/2} - \frac{\sin(ul - \overline{2r+1}\, b)p/2}{(ul - \overline{2r+1}\, b)p/2} \right\}. \qquad (2)$$

The integral corresponding to the imaginary part of the diffraction integral is zero. One can see that the magnitude of each individual term of (2) attains its highest maximum (the other maxima being negligibly small compared to the highest) when its denominator vanishes. Also, it can be seen that when any one of the terms is maximum, all the others have negligible values as the numerator of

each cannot exceed unity and the denominator is some integral non-vanishing multiple of $b$ which is sufficiently large. So the maxima of the magnitude of (2) correspond to the maxima of the magnitudes of the individual terms. Hence the maxima occur when

$$ul \pm nb = 0 \quad n(\text{an integer}) \geqslant 0 \tag{3}$$

where $n$ is any even or odd positive integer. The equation (3) gives the directions in which the magnitude of the amplitude is maximum which correspond also to the maximum of the intensity. If $\theta$ denotes the angle between such a direction in the XZ-plane along which the intensity is maximum and the direction of the incident light, (3) can be written as

$$\sin\theta = \pm\frac{n\lambda}{\lambda^*} \tag{4}$$

remembering that $u = 2\pi/\lambda$ and $b = 2\pi/\lambda^*$. This formula is identical with the formula (1) given in the first section. The magnitudes of the various components in the directions given by (4) can be calculated if we know,

$$J_n \text{ or } J_n(v) \text{ or } J_n(2\pi\mu L/\lambda).$$

Thus the relative intensity of the $m$th component to the $n$th component is given by

$$\frac{J_m^2(v)}{J_n^2(v)} \quad \text{where } v = 2\pi\mu L/\lambda.$$

In the undisturbed state of the medium there is no variation of the refractive index, i.e. $\mu = 0$. In this case all the components vanish except the zero component for

$$J_m(0) = 0 \text{ for all } m \neq 0 \text{ and } J_0(0) = 1.$$

In the disturbed state, the relative intensities depend on the quantity $v$ or $2\pi\mu L/\lambda$ where $\lambda$ is the wavelength of the incident light, $\mu$ is the maximum variation of the refractive index and $L$ is the path traversed by light in the medium. We have calculated the relative intensities of the various components which are observable for values of $v$ lying between 0 and 8 at different steps (figure 1).

Figure 1 shows that the number of observable components increases as the value of $v$ increases. When $v = 0$, we have only the central component. As $v$ increases from 0, the first orders begin to appear. As $v$ increases still more, the intensity of the central component decreases steadily and the first orders increase steadily in their intensity till they attain maximum intensity when the zero order will nearly vanish and the second orders will have just appeared. As $v$ increases still more, the zero order is reborn and increases in its intensity, the first orders fall in their intensity giving up their former exalted places to the second orders, while the third orders will have just appeared and so on.

Our theory shows that the intensity relations of the various components

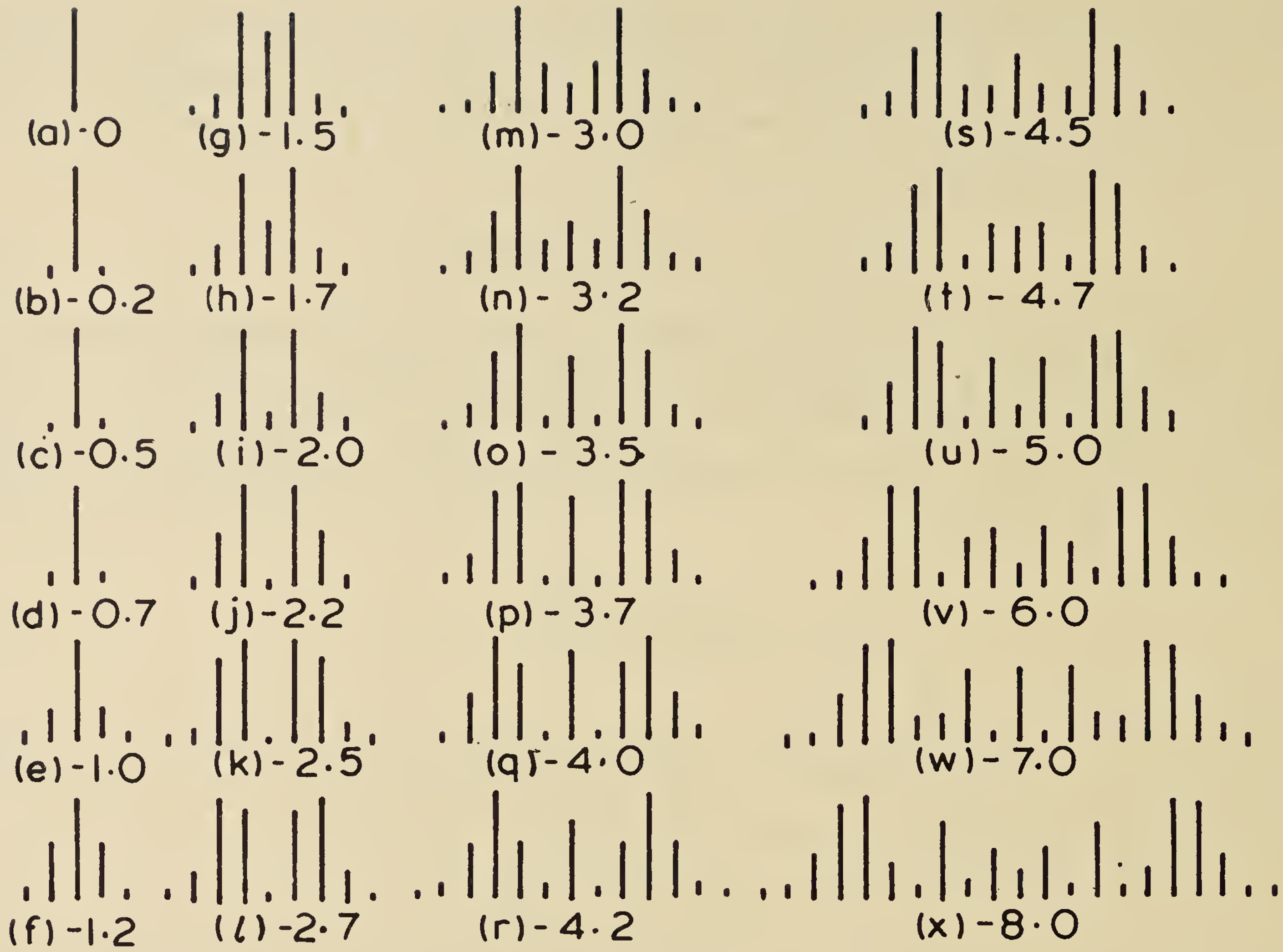


**Figure 1**

Relative intensities of the various components in the diffraction spectra. (For tables, see Watson's *Bessel Functions and Report of the British Association*, 1915).

depend on the quantity $v$ or $2\pi\mu L/\lambda$. Thus an *increase* of $\mu$ (i.e. an increase of the supersonic intensity which creates a greater variation in the refractive index of the medium) or an *increase* of $L$ or a *decrease* of $\lambda$ should give similar effects *except* in the last case where the directions of the various beams will be altered in accordance with (4).

## 3. Interpretation of Bär's experimental results

(*a*) *Dependence of the effect on the supersonic intensity*: Bär has observed that only the zero order (strong) and the first orders (faint) are present when the supersonic intensity is not too great. He found that more orders appear as the supersonic intensity is increased but that the intensity of the zero order decreases while the first orders gain in their intensity. Increasing the supersonic intensity more, he found that the first order would become very faint while the second and third orders will have about the same intensity. The figures 1*a* of his paper may very well be compared with our figures 1(c), 1(h) and 1(k). Thus, we are able to explain the appearance of more and more components and the wandering of the intensity

amongst them as the supersonic intensity is increased, in a satisfactory manner.

(*b*) *Dependence of the effect on the wavelength of the incident light*: We have already pointed out that the effects due to an increase of $\mu$ caused by an increase of supersonic intensity are similar due to those with a decrease of $\lambda$ except for the fact that the positions of the components of the emerging light alter in accordance with (4). Bär has obtained two patterns of the phenomenon by using light with wavelengths 4750Å and 3650Å. He obtained, using the former seven components and using the latter eleven components in all. He also observed great variations in the intensities of the components. Not only is the increase in the number of components an immediate consequence of our theory, but we can also find the pattern with 3650Å if we assume the pattern with 4750Å. The pattern with the latter in Bär's paper shows a strong resemblance to our figure 1(p) for which $2\pi\mu L/\lambda$ is 3·7. Thus we can calculate $2\pi\mu L/\lambda$ when $\lambda$ is 3650Å. It comes to about 4·8. Actually our figure for which $2\pi\mu L/\lambda$ is 4·8 closely corresponds to Bär's pattern with 3650Å.

(*c*) *Dependence of the effect on the length of the medium which the light traverses*: It is clear from our theory that an increase of $L$ corresponds to an increase of $v$ and that the effects due to this variation would be similar to those with an increase of the supersonic intensity. But the basis of our theory does not actually cover any large change in $L$. However, we should find more components and the wandering of the intensity amongst the various components.

## 4. Summary


(*a*) A theory of the phenomenon of the diffraction of light by soundwaves of high frequency in a medium, discovered by Debye and Sears and Lucas and Biquard, is developed.

(*b*) The formula

$$\sin\theta = \pm\frac{n\lambda}{\lambda^*} \quad n(\text{an integer}) \geqslant 0$$

which gives the directions of the diffracted beams from the direction of the incident beam and where $\lambda$ and $\lambda^*$ are the wavelengths of the incident light and the sound wave in the medium, is established. It has been found that the relative intensity of the $m$th component to the $n$th component is given by

$$J_m^2(2\pi\mu L/\lambda)/J_n^2(2\pi\mu L/\lambda)$$

where the functions are the Bessel functions of the $m$th order and the $n$th order, $\mu$ is the maximum variation of the refractive index and $L$ is the path traversed by light. These theoretical results interpret the experimental results of Bär in a very gratifying manner.

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# The diffraction of light by sound waves of high frequency: Part II

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## 1. Introduction

In the first* of this series of papers, we were concerned with the explanation of the diffraction effects observed when a beam of light traverses a medium filled by sound waves of high frequency. For simplicity, we confined our attention to the case in which a plane beam of light is normally incident on a cell of the medium with rectangular cross-section and travels in a direction strictly perpendicular to the direction along which the sound waves are propagated in the medium. By taking into account the corrugated form of the wave-front on emergence from the cell, the resulting diffraction-effects were evaluated. This treatment will be extended in the present paper to the case in which the light waves travel in a direction inclined at a definite angle to the direction of the propagation of the sound waves. The extension is simple, but it succeeds in a remarkable way in explaining the very striking observations of Debye and Sears† who found a characteristic variation of the intensity of the higher orders of the diffraction spectrum when the angle between the incident beam of light and the plane of the sound waves was gradually altered.

We shall first set out a simple geometrical argument by which the changes in the diffraction phenomenon which occur with increasing obliquity can be inferred from the results already given for the case of the normal incidence. An analytical treatment then follows which confirms the results obtained geometrically.

*C V Raman and N S Nagendra Nath, *Proc. Indian Acad. Sci.* **2**, 406–412 1935.
†P Debye and F W Sears, *Proc. Natl. Acad. Sci.* (*Washington*), **18**, 409, 1932.

## 2. Elementary geometrical treatment

The following diagrams illustrate the manner in which the amplitude of the corrugation in the emerging wave-front alters as the incidence of light on the planes of the sound waves is gradually changed. In the diagrams, the planes of maximum and minimum density caused by the sound waves at any instant of time are indicated by thick and thin lines (e.g. $AB$ and $CD$) respectively. The paths of the light rays are represented by dotted lines in figures 1(b), (c) and (d). As we are mainly interested in the calculation of the phase-changes which the incident wave undergoes before it emerges from the cell, the bending of the light rays within the medium may, in virtue of Fermat's well-known principle, be ignored without a sensible error, *provided* the total depth of the cell is not excessive.

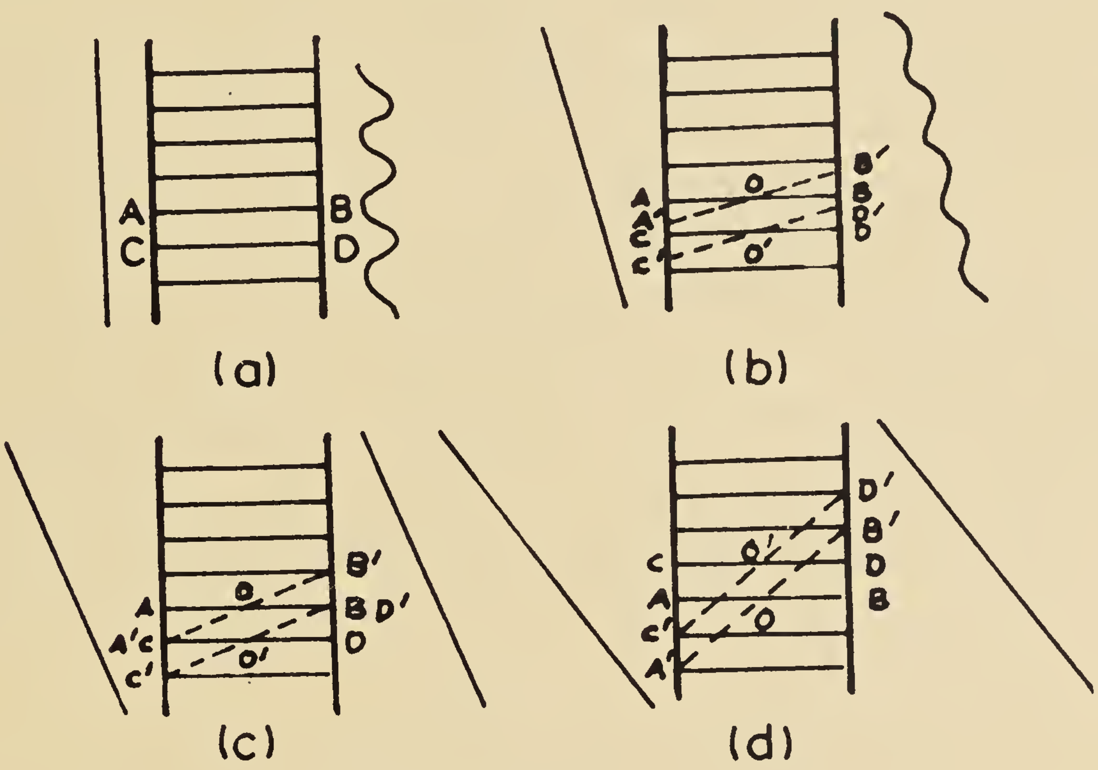


**Figure 1**

Considering the variation in the refractive index to be simply periodic, the neighbouring light-paths with maximum and minimum optical lengths $AB$ and $CD$ respectively, in the case of normal incidence, are shown in figure 1(a). The lines $AB$ and $CD$ are separated by $\lambda^*/2$ where $\lambda^*$ is the wavelength of the sound waves. The difference between the maximum and the minimum optical lengths gives a measure of the corrugation of the wave-front on emergence. Considering now a case in which the light rays make an angle $\phi$ with the planes of the sound waves, we may denote the maximum and the minimum optical lengths by $A'B'$ and $C'D'$ respectively. These would be symmetrically situated with respect to $AB$ and $CD$ and would tend to coincide with them as $\phi$ is decreased. The optical length of $A'B'$ is *less* than that of $AB$, for the refractive index at any point except at $O$ is less than the constant maximum refractive index along $AB$, $\phi$ being small. On the

other hand, the optical length of $C'D'$ is *greater* than that of $CD$, for the refractive index is minimum along $CD$. A simple consideration of the above shows that the difference between the optical lengths of $A'B'$ and $C'D'$ is less than that between those of $AB$ and $CD$. As this difference gives twice the amplitude of the corrugation of the emerging wave-front, it follows, in the case shown in figure 1(b), that the amplitude of the corrugation of the emerging wave-front is less than that in the case of figure 1(a).

Figure 1(c) illustrates a case when the maximum optical length is just equal to the minimum optical length. This occurs when the direction of the incident beam is inclined to the planes of the sound-wave-fronts at an angle $\alpha_1$ given by $\tan^{-1}(B'B/OB) = \tan^{-1}(\lambda^*/2)/(L/2) = \tan^{-1}(\lambda^*/L)$. That the optical lengths of $A'B'$ and $C'D'$ in figure 1(c) are equal follows by a very simple geometrical consideration. Thus, when light rays are incident on the sound waves at an angle $\tan^{-1}(\lambda^*/L)$, the amplitude of the corrugation of the emerging wave-front vanishes, i.e. a plane incident beam of light remains so when it emerges from the medium. This result would also be true whenever $\alpha_n = \tan^{-1}(n\lambda^*/L)$, $N \neq 0$. The case when $n = 2$ is illustrated in figure 1(d). In all these cases the diffraction effects disappear. As the corrugation vanishes when $\phi$ is $\alpha_{n+1}$ or $\alpha_n$, there is an intermediate direction which makes an angle $\beta_n$ with the sound waves giving the maximum corrugation if light travels along that direction. We can take $\beta_0(=0)$ to represent the case when the incident beam of light is parallel to the sound waves.

Thus, we have deduced that the corrugation of the emerging wave-front is maximum when the direction of light is parallel to the sound waves $[\beta_0(=0)]$, decreases steadily to zero as the inclination $\phi$ between the incident light and the sound waves is increased to $\alpha_1$, increases to a smaller maximum as $\phi$ increases from $\alpha_1$ to $\beta_1$, decreases to zero as $\phi$ increases from $\beta_1$ to $\alpha_2$, increases to a still smaller maximum as $\phi$ increases from $\alpha_2$ to $\beta_2$, and so on.

As the variation of the refractive index is simply periodic along the direction normal to the sound-wave-fronts, it follows that the optical length of the light path is also simply periodic along the same direction when the incident light rays are parallel to the sound waves. This means that the corrugation of the emerging wave-front is also simply periodic. When the incident light rays are incident at an angle $\phi$ to the sound waves, the optical length of the light path would be simply periodic in a direction perpendicular to the light rays. This means that the emerging wave-front would be tilted by the angle $\phi$ about the line of the propagation of the sound waves and that its corrugation would be simply periodic along the same line.

We have shown in our previous paper that a simply periodic corrugated wave is equivalent to a number of waves travelling in directions which make angles, denoted by $\theta$, with the direction of the incident beam given by

$$\sin\theta = \pm\frac{n\lambda}{\lambda^*} \quad n(\text{an integer}) \geqslant 0 \tag{1}$$

where $\lambda$ is the wavelength of the incident light. In view of the results obtained in the previous paragraph, the formula (1) would also hold good when the incident light is a small angle with the sound waves.

The relative intensities of the various diffraction spectra which depend on the amplitude of the corrugation should obey a law similar to the one in the case of the normal incidence.

Thus, we find that the results in the case of an oblique incidence would be similar to those of the normal incidence with the amplitude of the corrugation modified. Hence, we deduce , in virtue of the statement I, the following results, assuming the results, in the case of normal incidence, obtained in our earlier paper.

The diffraction spectrum will be most prominent when $\phi = 0$. The intensity of the various components wander when $\phi$ is increased. When $\phi$ increases from zero to $\alpha_1$, the number of the observable orders in practice decreases and when $\phi = \alpha_1$ all the components disappear except the central one which will attain maximum intensity. This does not mean that the intensities of all the orders except the central one decreases to zero monotonically as $\phi$ varies from zero to $\alpha_1$, but some of them may attain maxima and minima in their intensities before they attain the zero intensity when $\phi = \alpha_1$. This is obvious in virtue of the property that the intensity of the $n$th component depends on the square of the Bessel function $J_n$. As $\phi$ increases from $\alpha_1$ to $\beta_1$ the intensity of the central component falls and the other orders are reborn one by one. As $\phi$ increases from $\beta_1$ to $\alpha_2$, the number of observable orders decreases and when $\phi = \alpha_2$ all the orders vanish except the central one which will attain the maximum intensity and so on.

## 3. Analytical treatment

In the following, we employ the same notation as in our earlier paper. The optical length of a path in the medium parallel to the direction of the incident light making an angle $\phi$ with the sound waves may be easily calculated. It is

$$\int_0^{L\sec\phi} \mu(s)\,\mathrm{d}s$$

or

$$\mu_0 L\sec\phi - \mu \int_0^{L\sec\phi} \sin b(x - s\sin\phi)\mathrm{d}s.$$

Integrating we obtain the integral as

$$\mu_0 L\sec\phi - \frac{\mu}{b\sin\phi}\{\sin(bL\tan\phi)\sin bx + [\cos(bL\tan\phi) - 1]\cos bx\}.$$

The last term can be written as

$$-A\sin bx + B\cos bx$$

where

$$A = \frac{\mu}{b\sin\phi}\sin(bL\tan\phi)$$

$$B = -\frac{\mu}{b\sin\phi}[\cos(bL\tan\phi) - 1].$$

Thus the optical length of the path can be written as

$$\mu_0 L\sec\phi - \sqrt{(A^2+B^2)}\sin b\left(x - \tan^{-1}\frac{B}{A}\right).$$

Ignoring the constant phase factor, the optical length is

$$\mu_0 L\sec\phi - \frac{2\mu}{b\sin\phi}\sin\left(\frac{bL\tan\phi}{2}\right)\sin \mathrm{bx}.$$

If the incident light is

$$\exp\left[2\pi i\nu\left(t - \frac{x\sin\phi}{c}\right)\right]$$

when it arrives at the face of the cell, it will be

$$\exp\left[\frac{2\pi i}{\lambda}\left(ct - x\sin\phi - \int_0^{L\sec\phi}\mu(s)\mathrm{d}s\right)\right]$$

when it arrives at the face from which it emerges.

The amplitude of the corrugated wave at a point on the screen whose join with the origin has its $x$-direction-cosine $l$, depends on the evaluation of the diffraction integral

$$\int_{-p/2}^{p/2}\exp\left[\frac{2\pi i}{\lambda}\left\{(l-\sin\phi)x + \frac{2\mu}{b\sin\phi}\sin\left(\frac{bL\tan\phi}{2}\right)\sin bx\right\}\right]\mathrm{d}x.$$

The evaluation of the integral and the discussion of its behaviour with respect to $l$ may be effected in the same way as in our earlier paper. Maxima of the intensity due to the corrugated wave occur in directions making angles, denoted by $\theta$, with the direction of the incident beam when

$$\sin(\theta+\phi) - \sin\phi = \pm\frac{n\lambda}{\lambda^*} \quad n(\text{an integer}) \geqslant 0. \tag{1}$$

The relative intensity of the $m$th order to the $n$th order is given by

$$\frac{J_m^2(v)}{J_n^2(v)} \tag{2}$$

where

$$v = \frac{2\pi}{\lambda} \cdot \frac{2\mu}{b \sin \phi} \sin\left(\frac{bL\tan\phi}{2}\right)$$

$$= \frac{2\pi\mu L}{\lambda} \sec\phi \frac{\sin t}{t} \text{ where } t = \frac{bL\tan\phi}{2} = \frac{\pi L \tan\phi}{\lambda^*}.$$

The expression for the relative intensities in our earlier paper can be obtained from (2) by making $\phi \to 0$ when $v \to (2\pi\mu L/\lambda) = v_0$. So the expression for the relative intensities

$$J_m^2(v_0)/J_n^2(v_0) \tag{3}$$

in the case of normal incidence will change to

$$J_m^2(v)/J_n^2(v)$$

where

$$v = v_0 \sec\phi \frac{\sin t}{t} \tag{4}$$

and

$$t = \frac{\pi L \tan\phi}{\lambda^*}.$$

Even if $\phi$ be small so that $\sin\phi \approx \tan\phi \approx \phi$, it is *not* justifiable to write $\sin t \approx t$ *unless* $\pi L\phi/\lambda^*$ is also small to admit the approximation. As $\pi L/\lambda^*$ is sufficiently large we should expect great changes in the diffraction phenomenon even if $\phi$ be a fraction of a degree. $v$ vanishes when

$$t = n\pi \quad n(\text{an integer}) > 0,$$

that is, when $L\tan\phi = n\lambda^*$,

or

$$\phi = \tan^{-1}\frac{n\lambda^*}{L}, \; n(\text{an integer}) > 0,$$

confirming the same result obtained geometrically. Whenever $v$ vanishes, it can be seen that the amplitude of the corrugation of the wave-front also vanishes. The statement I in section 2 and the consequences with regard to the behaviour of the intensity among the various orders can all be confirmed by the expression (3).

In the numerical case when $L = 1$ cm, and $\lambda^* = 0{\cdot}01$ cm, the amplitude of the corrugation vanishes $\tan\alpha_1 = 0{\cdot}01$ or $\alpha_1 = 0° 34'$. This means that as $\phi$ varies from 0° to 0° 34′, the relative intensities of the various orders wander according to (2) till when $\phi = 0° 34'$, all the orders disappear except the central one which

attains maximum intensity. This does not mean that the intensities of all the orders except the central one decrease monotonically to zero but they *may possess* several maxima and minima before they become zero. The intensity of the $n$th order depends on the behaviour of $J_n^2\,[v_0 \sec\phi\,(\sin(\pi L \tan\phi/\lambda^*)/(\pi L \tan\phi/\lambda^*))]$ under the above numerical conditions as $\phi$ varies from 0° to 0° 34′. As $\phi$ just exceeds 0° 34′, all the orders are reborn one by one till a definite value of $\phi$ after which they again fall one by one and when $\phi = 1°\ 8'$, all the orders disappear except the central one.

The numerical example in the above paragraph shows the delicacy of the diffraction phenomenon. If the wavelength is quite small, the diffraction phenomenon will be present in the case of the strictly normal *incidence* as the relative intensity expression (3) does not depend on $\lambda^*$ but will soon considerably change even for slight variations of $\phi$ as the relative intensity expression (4) depends on $\lambda^*$. One should be very careful in carrying out the intensity measurements in the case of normal incidence, for even an error of a few minutes of arc in the incidence will affect the intensities of the various orders.

## 4. Comparison with the experimental results of Debye and Sears

Debye and Sears make the following statement in their paper: "Fixing the attention on one of the spectra *preferably of higher order*, one can observe that it attains its maximum intensity if the trough is turned through a small angle such that the primary rays are no longer parallel to the planes of the supersonic waves. Different settings are required to obtain highest intensities in different orders. If the trough is turned continuously in one direction, starting from a position which gave the highest intensity to one of the orders, the intensity decreases steadily, goes through zero, increases to a value much smaller than the first maximum, decreases to zero a second time and goes up and down again through a still smaller maximum." This statement very aptly describes the behaviour of the function

$$J_n^2\left[v_0 \sec\phi \frac{\sin(\pi L \tan\phi/\lambda^*)}{(\pi L \tan\phi/\lambda^*)}\right]$$

as $\phi$ alters under the conditions imposed in the above statement. The zeroes and the maxima of the intensity of the $n$th order, as a function of $\phi$, correspond to the zeroes and the maxima of the above function.

## 5. Summary

The theory of the diffraction of light by sound waves of high frequency developed in our earlier paper is extended to the case when the light beam is incident at an

angle to the sound wave-fronts, both from a geometrical point of view and an analytical one. It is found that the maxima of intensity of the diffracted light occur in directions which make definite angles, denoted by $\theta$, with the direction of the incident light given by

$$\sin(\theta+\phi)-\sin\phi = \pm\frac{n\lambda}{\lambda^*}, \quad n(\text{an integer}) \geqslant 0$$

where $\lambda$ and $\lambda^*$ are the wavelengths of the incident light and the sound waves in the medium. The relative intensity of the $m$th order to the $n$th order is given by

$$J_m^2\left(v_0 \sec\phi \frac{\sin t}{t}\right) \Big/ J_n^2\left(v_0 \sec\phi \frac{\sin t}{t}\right)$$

where $v_0 = (2\pi\mu L/\lambda)$, $t = (\pi L \tan\phi/\lambda^*)$, $\phi$ is the inclination of the incident beam of light to the sound waves, $\mu$ is the maximum variation of the refractive index in the medium when the sound waves are present and $L \sec\phi$ is the distance of the light path in the medium. These results explain the variations of the intensity among the various orders noticed by Debye and Sears for variations of $\phi$ in a very gratifying manner.

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# The diffraction of light by high frequency sound waves: Part III

## Doppler effect and coherence phenomena

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## 1. Introduction

In part I[1] of this series of papers, a theory of the diffraction of light by high frequency sound waves was developed starting from the simple basic idea that the incident plane waves of light, after transmission through the medium traversed by the sound waves assume a corrugated form, owing to the fluctuations in the density and consequently also in the refractive index of the medium. The Fourier analysis of the emerging corrugated wave-front automatically gives the diffraction effects observed when the emergent waves are brought to focus by the lens of the observing telescope. The results deduced from the theory gave a gratifyingly satisfactory explanation of the observations of Bär[3] regarding the changes in the diffraction pattern when the supersonic intensity, the wavelength of the incident light and the length of the cell are varied.

In part II[2], we extended the theory to the case of the oblique incidence of the light on the sound waves and were successful in explaining the variations of the diffraction effects reported by Debye and Sears[4] as the angle of obliquity is varied.

In parts I and II, we deliberately ignored the variation of the refractive index with time in order to bring out the essential features of the theory without unnecessary complications. In this the third part of the paper, we proceed to take this factor also into consideration. It will be shown that light diffracted by progressive sound waves exhibits Doppler shifts of a very simple type. In the case, however, of the diffraction of light by *standing sound waves* in a medium, we get the much more interesting result that *in any even order, radiations with frequencies* $\nu \pm 2r\nu^*$ *would be present* where $\nu$ is the frequency of the incident light, $\nu^*$ is the frequency of sound in the medium and $r$ is any integer and that *in an odd order, radiations with frequencies* $\nu \pm \overline{2r+1}\,\nu^*$ *would be present*. This implies that *any*

*pair of even orders or odd orders can partly cohere* and that *an even order and an odd one are incoherent.* This latter result has already been arrived at by Bär[5] purely by his experimental investigations. The remarkable results of Bär in the field of supersonic research thus find a natural explanation in terms of our theory.

It should be, however, noted that the theory developed in the following is subject to the same limitations as those in the previous parts, viz., that the depth of the cell is not too great to permit the form of the emerging wave-front to be deduced in the simple manner indicated in part I. A more general consideration of the problem will be presented in a later communication.

## 2. Doppler effects due to a progressive sound wave

Let us suppose that the progressive sound wave travels in a direction parallel to the $X$-axis perpendicular to two faces of a rectangular vessel containing some homogeneous and isotropic medium. We use the same notation and the axes of reference as in our earlier paper. When the sound wave travels in the medium, the density of the medium and its refractive index undergo periodic fluctuations. If the sound wave is a simple one, we could assume that the variation of the refractive index at a point in the medium is given by

$$\mu(x,t) - \mu_0 = \mu \sin 2\pi(\nu^* t - x/\lambda^*) \tag{1}$$

where $\mu(x,t)$ is the refractive index of the medium at a height $x$ from the origin at time $t$, $\mu_0$ is the refractive index of the medium in its undisturbed state, $\mu$ is the *maximum variation* of the refractive index from $\mu_0$ and $\nu^*$ and $\lambda^*$ refer to the frequency and the wavelength of the sound wave in the medium.

Let the light wave be incident along the Z-axis perpendicular to two faces of the medium and the direction of the propagation of the sound wave. If the incident light wave is given by

$$\exp[2\pi i\nu t]$$

it will be

$$\exp[2\pi i\nu\{t - L\mu(x,t)/c\}]$$

when it arrives at the other face where $L$ is the distance between the two faces.

The amplitude of the corrugated wave at a point on a distant screen parallel to the face of the medium from which light is emerging, whose join with the origin has its $x$-direction-cosine $l$ depends on the evaluation of the diffraction integral

$$\int_{-p/2}^{p/2} \exp[2\pi i\{lx - \mu L \sin 2\pi(\nu^* t - x/\lambda^*)\}/\lambda] \mathrm{d}x \tag{2}$$

where $p$ is the length of the beam along the $X$-axis. The real and the imaginary

parts of the diffraction integral (2) are

$$\int_{-p/2}^{p/2} \{\cos ulx \cos (v \sin \overline{bx - \varepsilon}) - \sin ulx \sin (v \sin \overline{bx - \varepsilon})\} dx \tag{3}$$

and

$$\int_{-p/2}^{p/2} \{\sin ulx \cos (v \sin \overline{bx - \varepsilon}) + \cos ulx \sin (v \sin \overline{bx - \varepsilon})\} dx$$

where

$$u = 2\pi/\lambda, \quad b = 2\pi/\lambda^*, \quad v = 2\pi\mu L/\lambda \quad \text{and} \quad \varepsilon = 2\pi\nu^* t.$$

Putting $bx - \varepsilon$ as $x'$ we could write the integrals* (3) as

$$\frac{2}{b}\sum_0^\infty{}' J_{2r}(v) \int_{-bp/2-\varepsilon}^{bp/2-\varepsilon} \cos\left(ul\frac{x'+\varepsilon}{b}\right) \cos 2rx' \, dx'$$
$$-\frac{2}{b}\sum_0^\infty J_{2r+1}(v) \int_{-bp/2-\varepsilon}^{bp/2-\varepsilon} \sin\left(ul\frac{x'+\varepsilon}{b}\right) \times \sin \overline{2r+1}\, x' \, dx'$$

and

$$\frac{2}{b}\sum_0^\infty{}' J_{2r}(v) \int_{-bp/2-\varepsilon}^{bp/2-\varepsilon} \sin\left(ul\frac{x'+\varepsilon}{b}\right) \cos 2rx' \, dx'$$
$$+\frac{2}{b}\sum_0^\infty J_{2r+1}(v) \int_{-bp/2-\varepsilon}^{bp/2-\varepsilon} \cos\left(ul\frac{x'+\varepsilon}{b}\right) \times \sin \overline{2r+1}\, x' \, dx'$$

or

$$\frac{1}{b}\sum_0^\infty{}' J_{2r}(v) \int_{-bp/2-\varepsilon}^{bp/2-\varepsilon} \left\{\cos\left(\overline{\frac{ul}{b} + 2r}\, x' + \frac{ul\varepsilon}{b}\right)\right.$$
$$\left. + \cos\left(\overline{\frac{ul}{b} - 2r}\, x' + \frac{ul\varepsilon}{b}\right)\right\} dx'$$
$$+\frac{1}{b}\sum_0^\infty J_{2r+1}(v) \int_{-bp/2-\varepsilon}^{bp/2-\varepsilon} \left\{\cos\left(\overline{\frac{ul}{b} + \overline{2r+1}}\, x' + \frac{ul\varepsilon}{b}\right)\right.$$
$$\left. - \cos\left(\overline{\frac{ul}{b} - \overline{2r+1}}\, x' + \frac{ul\varepsilon}{b}\right)\right\} dx' \tag{4a}$$

*The dash over the summation sign indicates that the coefficient of the first term has to be multiplied by half.

and

$$\frac{1}{b}\sum_0^{\infty}{}' J_{2r}(v)\int_{-bp/2-\varepsilon}^{bp/2-\varepsilon}\left\{\sin\left(\frac{ul}{b}+\overline{2r}\,x'+\frac{ul\varepsilon}{b}\right)+\sin\left(\frac{ul}{b}-\overline{2r}\,x'+\frac{ul\varepsilon}{b}\right)\right\}\mathrm{d}x'$$

$$+\frac{1}{b}\sum_0^{\infty} J_{2r+1}(v)\int_{-bp/2-\varepsilon}^{bp/2-\varepsilon}\left\{\sin\left(\frac{ul}{b}+\overline{2r+1}\,x'+\frac{ul\varepsilon}{b}\right)\right.$$

$$\left.-\sin\left(\frac{ul}{b}-\overline{2r+1}\,x'+\frac{ul\varepsilon}{b}\right)\right\}\mathrm{d}x'. \tag{4b}$$

Integrating and combining the real and the imaginary parts (4*a*) and (4*b*) we find that the amplitude depends on

$$p\sum_0^{\infty}{}' J_{2r}(v)\left\{\frac{\sin\{(ul+2rb)p/2\}}{(ul+2rb)p/2}e^{-i2r\varepsilon}+\frac{\sin\{(ul-2rb)p/2\}}{(ul-2rb)p/2}e^{i2r\varepsilon}\right\}$$

$$+p\sum_0^{\infty} J_{2r+1}(v)\left\{\frac{\sin\{(ul+\overline{2r+1}\,b)p/2\}}{(ul+\overline{2r+1}\,b)p/2}e^{-i\overline{2r+1}\varepsilon}\right.$$

$$\left.-\frac{\sin\{(ul-\overline{2r+1}\,b)p/2\}}{(ul-\overline{2r+1}\,b)p/2}e^{i\overline{2r+1}\varepsilon}\right\} \tag{5}$$

where $\varepsilon = 2\pi\nu^*t$. We should remember that the amplitude function has the other time factor $e^{2\pi i\nu t}$ which has been taken out as a constant from the integrand of the diffraction integral. One can see that the magnitude of each individual term of (5) attains its highest maximum when its denominator vanishes. Also, it can be seen that when any one of the terms is maximum, all the others have negligible values as the numerator of each cannot exceed unity and the denominator is some integral non-vanishing multiple of $b$ which is sufficiently large. When

$$ul + nb = 0$$

$$\sin\theta = -\frac{n\lambda}{\lambda^*} \tag{6}$$

where $n$ is a positive or a negative integer and $\theta$ is the angle between the direction whose $x$-direction-cosine is $l$ and the $Z$-axis.

The wave travelling in the direction whose inclination with the incident light beam is $\sin^{-1}(-n\lambda/\lambda^*)$ is determined by

$$J_n(v)e^{2\pi i(\nu-n\nu^*)t} \tag{7}$$

having the frequency $\nu - n\nu^*$, $n$ being a positive or negative integer; when $n$ is negative the direction of propagation of that order has positive direction-cosines with respect to the directions of the propagation of the sound and light waves. Consequently the radiations in the different orders will be incoherent with each other. (See figure 1.)

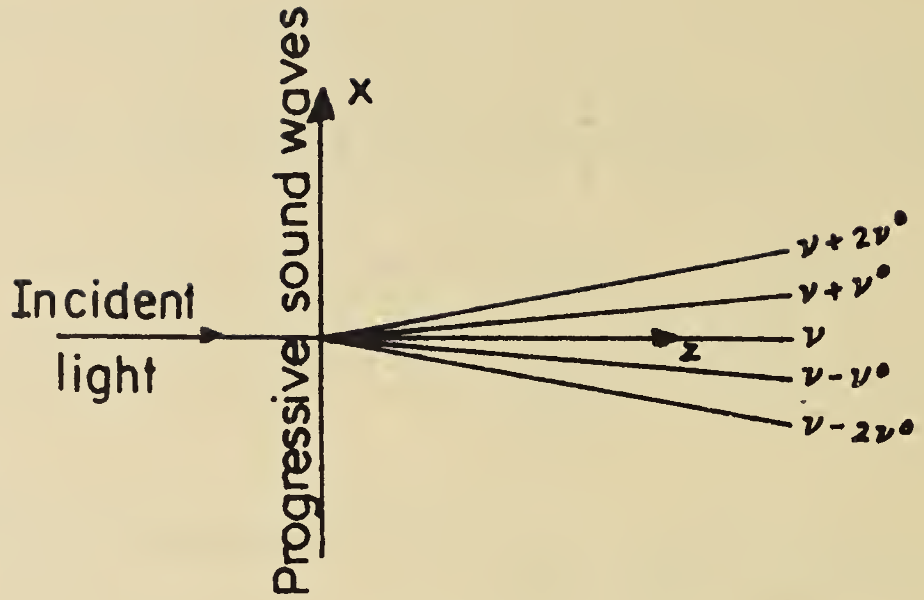


Figure 1

The relative intensity of the $m$th order to the $n$th order is given by the expression

$$J_m^2\left(\frac{2\pi\mu L}{\lambda}\right)\Big/J_n^2\left(\frac{2\pi\mu L}{\lambda}\right)$$

identical with the one given in part I.

## 3. Doppler effects due to a standing sound wave

In the case of a standing wave produced by the interference of two simple waves travelling in opposite directions parallel to the $X$-axis, we could assume that the variations of the refractive index at a point in the medium is given by

$$\mu(x,t)-\mu_0=-\mu\sin 2\pi\nu^*t\sin(2\pi x/\lambda^*) \tag{8}$$

with the same notation as in the previous section. Under the same restrictions as in Part I, we find that the emerging wave-front is given by

$$\exp[2\pi i\nu\{t-L\mu(x,t)/c\}] \tag{9}$$

The diffraction integral is then

$$\int_{-p/2}^{p/2}\exp[2\pi i\{lx+\mu L\sin\varepsilon\sin(2\pi x/\lambda^*)\}/\lambda]\,dx \tag{10}$$

where $\varepsilon=2\pi\nu^*t$.

The real and the imaginary parts of the integral (10) are

$$\int_{-p/2}^{p/2}\{\cos ulx\cos(v'\sin bx)-\sin ulx\sin(v'\sin bx)\}\,dx$$

and

$$\int_{-p/2}^{p/2} \{\sin ulx \cos (v' \sin bx) + \cos ulx \sin (v' \sin bx)\}\, dx$$

where

$$u = 2\pi/\lambda, \quad b = 2\pi/\lambda^*, \quad v' = v \sin \varepsilon = (2\pi\mu L \sin \varepsilon)/\lambda.$$

Following the same procedure as in our earlier paper, we find that the real part of the diffraction integral (10) is

$$p\sum_0^\infty{}' J_{2r}(v \sin 2\pi v^* t)\left\{\frac{\sin [(ul+2rb)p/2]}{(ul+2rb)p/2} + \frac{\sin [(ul-2rb)p/2]}{(ul-2rb)p/2}\right\}$$
$$+ p\sum_0^\infty J_{2r+1}(v \sin 2\pi v^* t)\left\{\frac{\sin [(ul+\overline{2r+1}\, b)p/2]}{(ul+\overline{2r+1}\, b)p/2}\right.$$
$$\left. - \frac{\sin [(ul-\overline{2r+1}\, b)p/2]}{(ul-\overline{2r+1}\, b)p/2}\right\}.$$

The integral corresponding to the imaginary part of the diffraction integral is zero.

Following similar arguments as in part I or in the previous section we can show that the wave travelling in the direction given by

$$ul + nb = 0$$

or

$$\sin \theta = -\frac{n\lambda}{\lambda^*}$$

is

$$\pm J_n(v \sin 2\pi v^* t)\, e^{2\pi i v t} \tag{11}$$

multiplied by a constant usually taken out from the diffraction integral. The wave given by (11) is not a simple one but is a superposition of a number of waves given by the Fourier analysis of $J_n(v \sin 2\pi v^* t)$ and multiplied by $e^{2\pi i v t}$.

*Fourier analysis of* $J_n$ $(v \sin \varepsilon)$: The well-known Neumann's addition theorem

$$J_0(\tilde{\omega}) = 2\sum_0^\infty{}' J_m(Z) J_m(z) \cos m\phi$$

where

$$\tilde{\omega} = \sqrt{(Z^2 + z^2 - 2Zz \cos \phi)}$$

has been generalised by Graf[6] as

$$J_n(\tilde{\omega})\left\{\frac{Z - ze^{-i\phi}}{Z - ze^{i\phi}}\right\}^{n/2} = \sum_{-\infty}^{+\infty} J_{n+m}(Z) J_m(z) e^{im\phi}$$

provided $|ze \pm i\phi| < Z$. If $n$ is an integer, the inequality need not be in force. Putting

$$Z = z = v/2$$
$$\text{and } \phi = 2\varepsilon$$

we get

$$J_n(v\sin\varepsilon)e^{-in\varepsilon}(-1)^{n/2} = \sum_{-\infty}^{+\infty} J_{n+m}(v/2)J_m(v/2)e^{i2m\varepsilon}.$$

From this, changing $n$ to $2n$ we deduce that

$$J_{2n}(v\sin\varepsilon) = (-1)^n \sum_{-\infty}^{+\infty} J_{m+2n}(v/2)J_m(v/2)e^{-i(2m+2n)\varepsilon}.$$

Putting $m = -n + r$ and after a little simplification, we get

$$J_{2n}(v\sin\varepsilon) = (-1)^n J_{-n}(v/2)J_n(v/2) + 2\sum_1^{\infty} J_{-n+r}(v/2)J_{n+r}(v/2)\cos 2r\varepsilon$$

$$= (-1)^n 2\sum_0^{\infty}{}' J_{-n+r}(v/2)J_{n+r}(v/2)\cos 2r\varepsilon$$

$$= 2\sum_0^{\infty}{}'(-1)^r J_{n-r}(v/2)J_{n+r}(v/2)\cos 2r\varepsilon.$$

Similarly we can deduce that

$$J_{2n+1}(v\sin\varepsilon) = 2\sum_0^{\infty}(-1)^r J_{n-r}(v/2)J_{n+r+1}(v/2).\sin 2r+1\varepsilon$$

$$J_{2n}(v\cos\varepsilon) = 2\sum_0^{\infty}{}' J_{n-r}(v/2)J_{n+r}(v/2)\cos 2r\varepsilon$$

$$J_{2n+1}(v\cos\varepsilon) = 2\sum_0^{\infty} J_{n-r}(v/2)J_{n+r+1}(v/2)\cos\overline{2r+1}\,\varepsilon.$$

Returning now to the Fourier analysis of the diffraction components, the diffracted waves can be resolved into a number of simple waves, for

$$J_{2n}(v\sin 2\pi v^*t)e^{2\pi ivt}$$

$$= e^{2\pi ivt}2\sum_0^{\infty}{}'(-1)^r J_{n-r}(v/2)J_{n+r}(v/2)\cos(2r\cdot 2\pi v^*t)$$

$$= \sum_0^{\infty}{}'(-1)^r J_{n-r}(v/2)J_{n+r}(v/2)\{e^{2\pi i(v+2rv^*)t} + e^{2\pi i(v-2rv^*)t}\}$$

and

$$J_{2n+1}(v \sin 2\pi\nu^* t)e^{2\pi i\nu t}$$

$$=\frac{1}{i}\sum_{0}^{\infty}(-1)^r J_{n-r}(v/2)J_{n+r+1}(v/2)\{e^{2\pi i(\nu+\overline{2r+1}\,\nu^*)t}-e^{2\pi i(\nu-\overline{2r+1}\,\nu^*)t}\}.$$

Thus in all even orders radiation frequencies

$$\nu \pm 2r\nu^*, \quad r \text{ a positive integer,}$$

are present. The relative intensity of the $\nu \pm 2r\nu^*$ sub-component in the $2n$th order is given by

$$J^2_{n-r}(v/2)J^2_{n+r}(v/2).$$

In all odd orders radiation frequencies

$$\nu \pm \overline{2r+1}\,\nu^*, \; r \text{ a positive integer,}$$

are present (see figure 2). The relative intensity of the $\nu \pm \overline{2r+1}\,\nu^*$ sub-component in $2n+1$th order is given by

$$J^2_{n-r}(v/2)J^2_{n+r+1}(v/2).$$

We can conclude from the above analysis that *an even order and an odd one are incoherent while any two even or any two odd orders can partly cohere.* Any two orders symmetrically situated to the 0th order are completely coherent. We have calculated the relative intensities of the various Doppler sub-components of the various orders as $v$ ranges from 0 to 5 in steps of unity and represented them in figure 3.

We may also note that the intensity of each of the sub-components of each order depends on the amplitude of the supersonic vibration, the length of the cell and the wavelength of the incident light.

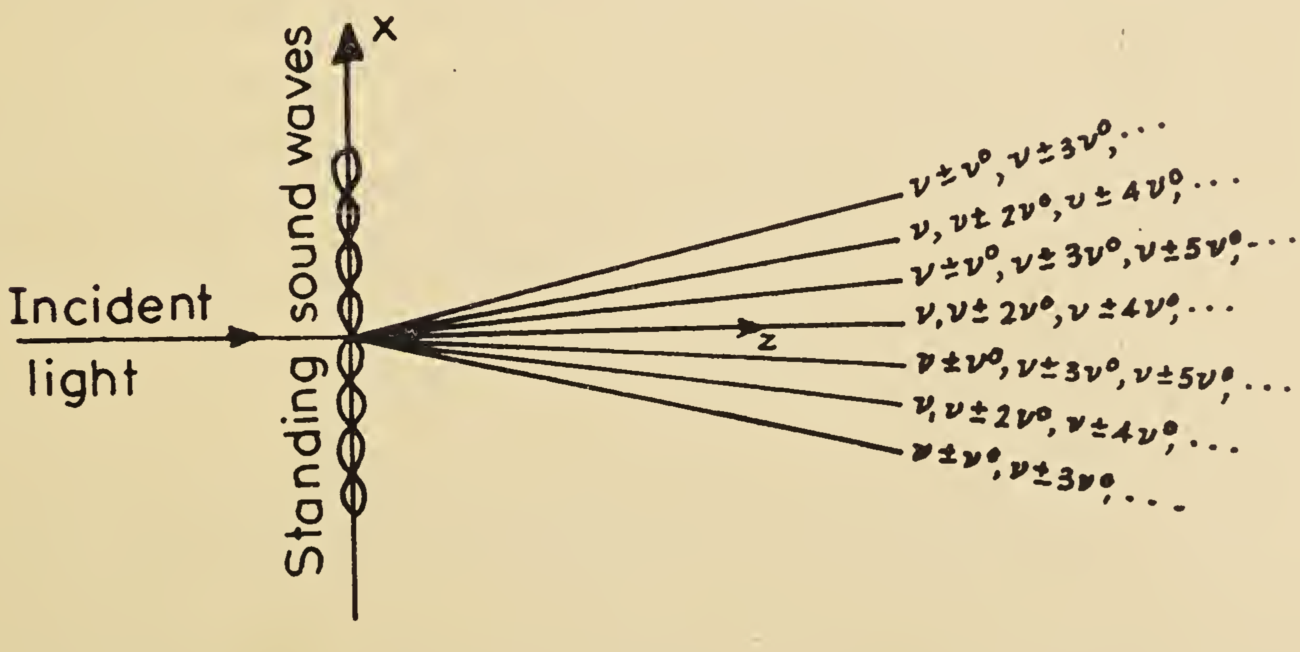


Figure 2

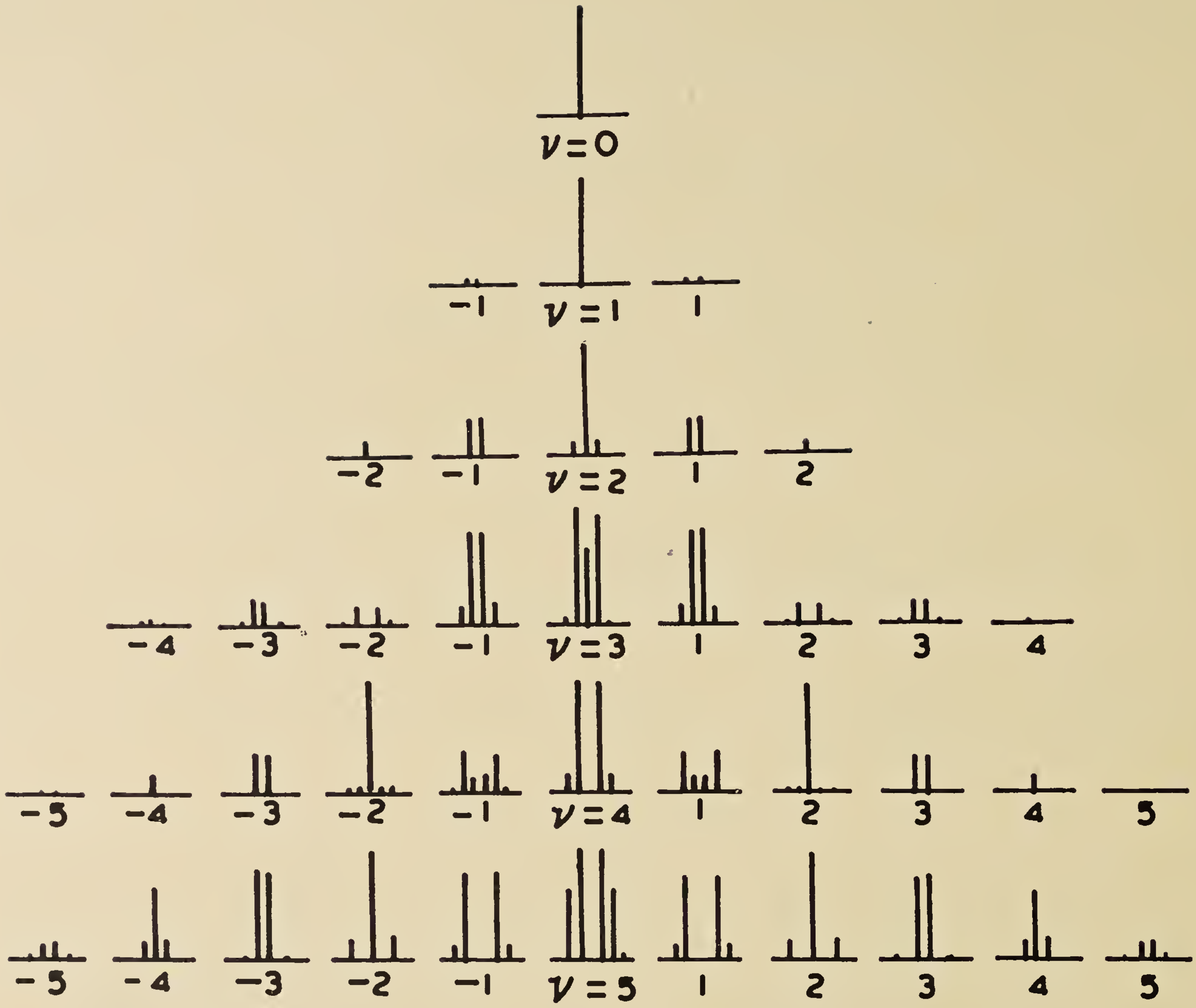


**Figure 3.** Relative intensities of the various sub-components of observable orders; the sub-components of an odd order standing on a base correpond to ---, $v - 2r + 1\ v^*$, ---, $v - v^*$, $v + v^*$, ---, $v + 2r + 1\ v^*$, --- and those of an even order standing on a base correspond to ---, $v - 2rv^*$, ---, $v$, —, $v + 2rv^*$, ---. In the figure $v = 5$, some lower orders are missing as their relative intensities are negligibly small. negligibly small.

If we ignore the spectral character of each order, then the relative intensity of the $m$th order to the $n$th order is

$$\frac{\int_0^{2\pi} J_m^2(v \sin\theta)\mathrm{d}\theta}{\int_0^{2\pi} J_n^2(v \sin\theta)\mathrm{d}\theta} \quad \text{where } v = \frac{2\pi\mu L}{\lambda}$$

which follows from Parseval's theorm.

## 4. Interpretation of Bär's experimental results

Bär[5] has recently investigated by an interference method the coherence of the diffraction components of light produced by a standing supersonic wave. He has

found that the various orders could be classed into two groups, one comprising the even orders and the other comprising the odd orders and that any two orders of a group cohere partly while two orders from different groups are completely incoherent. These results are readily understood when we notice that an even order contains radiations with frequencies $v \pm 2rv^*$ while an odd order contains radiations with frequencies $v \pm \overline{2r+1}v^*$. The experimental results of Bär are thus fully explicable in terms of the theory we have developed in the previous section. Bär has himself remarked that the observed coherence indicates the presence of a series of frequency components in each of the diffraction spectra. It will be noticed that, according to our theory, even the zero-order spectrum includes such a series of frequency components.

## 5. Summary

The theory developed in part I of this series of papers has been developed in this paper to find the Doppler effects in the diffraction components of light produced by the passage of light through a medium containing (1) a progressive supersonic wave and (2) a standing supersonic wave.

(1) In the case of the former the theory shows that the $n$th order which is inclined at an angle $\sin^{-1}(-n\lambda/\lambda^*)$ to the direction of the propagation of the incident light has the frequency $v - nv^*$ where $v$ is the frequency of light, $v^*$ is the frequency of sound and $n$ is a positive or negative integer and that the $n$th order has the relative intensity $J_n^2(2\pi\mu L/\lambda)$ where $\mu$ is the maximum variation of the refractive index, $L$ is the distance between the faces of the cell of incidence and emergence and $\lambda$ is the wavelength of light.

(2) In the case of a standing supersonic wave, the diffraction orders could be classed into two groups, one containing the even orders and the other odd orders; any even order, say $2n$, contains radiations with frequencies $v \pm 2rv^*$ where $r$ is an integer including zero, the relative intensity of the $v \pm 2rv^*$ sub-component being $J_{n-r}^2(\pi\mu L/\lambda)\, J_{n+r}^2(\pi\mu L/\lambda)$; and odd order, say $2n+1$, contains radiations with frequencies $v \pm \overline{2r+1}v^*$, the relative intensity of the $v \pm \overline{2r+1}v^*$ sub-component being $J_{n-r}^2(\pi\mu L/\lambda)\, J_{n+r+1}^2(\pi\mu L/\lambda)$. These results satisfactorily interpret the recent results of Bär that any two odd orders or even ones partly cohere while an odd one and an even one are incoherent.

## References

1. C V Raman and N S Nagendra Nath, *Proc. Indian Acad. Sci.* (A), 1935, **2**, 406.
2. C V Raman and N S Nagendra Nath, *Proc. Indian Acad. Sci.* (A), 1935, **2**, 413.
3. R Bär, *Helv. Phys. Acta* 1933, **6**, 570.
4. P Debye and F W Sears, *Proc. Nat. Acad. Sci.* (*Washington*), 1932, **18**, 409.
5. R Bär, *Helv. Phys. Acta* 1933, **8**, 591.
6. *A Treatise on the Theory of Bessel Functions* by G N Watson, 1922, p. 359.

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# The diffraction of light by high frequency sound waves: Part IV

## Generalised theory

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## 1. Introduction

In part III[†] of this series of papers, we considered the Doppler effects and coherence phenomena among the diffracted components of light emerging from a rectangular cell of a medium traversed by supersonic waves perpendicular to the direction of the propagation of the incident plane wave of light. We showed, in the case of a progressive supersonic wave, that the $n$th order diffraction component which is inclined at an angle $\sin^{-1}(-n\lambda/\lambda^*)$ to the direction of propagation of the incident light has the frequency $\nu - n\nu^*$, where $\nu$ and $\lambda$ denote the frequency and the wavelength of the incident light while $\nu^*$ and $\lambda^*$ correspond to those of the sound wave. In the case of the diffraction of light by a standing sound wave, we got the interesting result that in any even order, radiations with frequencies $\nu \pm 2r\nu^*$, $(r = 0, 1, 2, \ldots)$, would be present while in any odd order, radiations with frequencies $\nu \pm \overline{2r+1}\,\nu^*$, $(r = 0, 1, 2, \ldots)$, would be present. These results give a satisfactory interpretation of the coherence phenomena among the diffraction components observed by Bär[‡]. In the following, we show that our previous results remain valid even if we consider a *general* periodic supersonic wave and that they can be derived in a simple and direct fashion. We have also presented in the following, some general considerations of the problem on hand.

[†]C V Raman and N S Nagendra Nath, *Proc. Indian Acad. Sci.* (*A*), 1936, **3**, 75.
[‡]R Bär, *Helv. Phys. Acta*, 1935, **8**, 591.

## 2. Doppler effect and coherence phenomena

The partial differential equation governing the propagation of light in a medium with time-variation and space-variation in its refractive index is

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=\left[\frac{\mu(x,y,z,t)}{c}\right]^2\frac{\partial^2\psi}{\partial t^2}$$

if the frequency of the time-variation of $\mu(x,y,z,t)$ is very slow compared to the time-variation of the wave-function of light. This would be so in the case of the propagation of light in a medium filled with sound waves for the frequency of the variation of $\mu(x,y,z,t)$ corresponds to the frequency of the sound waves present in the medium, which is negligible compared to the frequency of light.

If we choose our axes of reference such that the $X$-axis points to the direction of the propagation of the plane sound waves and the $Z$-axis points to the direction of the propagation of the incident plane wave of light, we could ignore the dependence of $\psi$ on $y$ and write the differential equation as

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}=\left[\frac{\mu(x,t)}{c}\right]^2\frac{\partial^2\psi}{\partial t^2}.$$

If $\mu(x,t)$ did not depend on time, $\psi$ would have had the only time factor $\exp(2\pi i\nu t)$ where $\nu$ is the frequency of the incident light. If we consider the time variation of $\mu(x,t)$, we can write $\psi$ as given by

$$\psi=\exp[2\pi i\nu t]\phi(x,z,t)$$

where $\phi(x,z,t)$ varies slowly in time compared to $\exp[2\pi i\nu t]$. On the consideration that $\nu^*\ll\nu$, we can show that

$$\left|4\pi\nu\frac{\partial\phi}{\partial t}\right|\ll\left|4\pi^2\nu^2\phi\right| \quad\text{and}\quad \left|\frac{\partial^2\phi}{\partial t^2}\right|\ll\left|4\pi^2\nu^2\phi\right|.$$

With these considerations, we can consider the differential equation

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=-\frac{4\pi^2}{\lambda^2}\{\mu(x,t)\}^2\phi$$

and obtain $\psi$ by the equation

$$\psi=\exp[2\pi i\nu t]\phi(x,z,t).$$

As the sound waves which travel along the $X$-axis are periodic in space and time, we can regard $\mu(x,t)$ to be also periodic in $x$ and $t$ with the same periods in space and time. It should be noticed that we do not restrict $\mu(x,t)$ to be simply periodic in $x$ and $t$ but it may be a general periodic function of $x$ and $t$, amenable to Fourier analysis. Thus

$$\mu(x+p\lambda^*,t)=\mu(x,t)$$

and

$$\mu(x, t + p/v^*) = \mu(x, t)$$

where $p$ is any integer.

If we consider the differential equation in which $\mu(x, t)$ has the above properties, we see that $\phi(x, z, t)$ should also be periodic in $x$ and $t$ with the same periods in the case we are considering. That is,

$$\phi(x + p\lambda^*, z, t) = \phi(x, z, t)$$

and

$$\phi(x, z, t + p/v^*) = \phi(x, z, t).$$

Hence we can write the double-Fourier expansion of $\phi(x, z, t)$ as

$$\sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} f_{rs}(z) \exp(2\pi irx/\lambda^*) \exp(2\pi isv^*t).$$

*Progressive sound waves:* In the case of the progressive waves travelling along the positive direction of the $X$-axis, we have the property that

$$\mu(x + \rho\lambda^*, t) = \mu(x, t - \rho/v^*)$$

where $\rho$ is any number. Thus

$$\phi(x + \rho\lambda^*, z, t) = \phi(x, z, t - \rho/v^*) \tag{1}$$

Using the double-Fourier expansion, we can write (1) as

$$\begin{aligned} &\sum\sum f_{rs}(z) \exp(2\pi irx/\lambda^*) \exp(2\pi isv^*t) \exp(2\pi ir\rho) \\ &\quad = \sum\sum f_{rs}(z) \exp(2\pi irx/\lambda^*) \exp(2\pi isv^*t) \exp(-2\pi is\rho). \end{aligned} \tag{2}$$

Comparing the Fourier coefficients on each side of (2), we get

$$f_{rs}(z) \exp(2\pi ir\rho) = f_{rs}(z) \exp(-2\pi is\rho).$$

This could be true only if

$$f_{rs}(z) = 0 \quad \text{when } r \neq -s. \tag{3}$$

The condition (3) restricts the number of terms in the Fourier expansion of $\phi$, so that

$$\phi(x, z, t) = \sum_{-\infty}^{\infty} f_{rs}(z) \exp(2\pi irx/\lambda^*) \exp(-2\pi irv^*t).$$

Thus

$$\psi(x, z, t) = \sum_{-\infty}^{\infty} f_{rs}(z) \exp(2\pi irx/\lambda^*) \exp(2\pi i(v - rv^*)t). \tag{4}$$

If one considers the diffraction effects of $\psi(x, z, t)$ given by (4), it is fairly obvious that the $n$th order diffraction component will be inclined at an angle

$\sin^{-1}(-n\lambda/\lambda^*)$ with the incident beam of light and will have the frequency $\nu - n\nu^*$ and the relative intensity expression $|f_n(z)|^2$.

*Standing sound waves:* In the case of standing waves, we have the property that

$$\mu\left(x + \frac{p\lambda^*}{2}, t\right) = \mu\left(x, t \pm \frac{p}{2\nu^*}\right), \quad p \text{ an integer,}$$

so that

$$\phi\left(x + \frac{p\lambda^*}{2}, z, t\right) = \phi\left(x, z, t \pm \frac{p}{2\nu^*}\right). \tag{5}$$

If we use (5) in the double Fourier expansion of $\phi$ we get

$$\begin{aligned}&\sum\sum f_{rs}(z)\exp(2\pi irx/\lambda^*)\exp(2\pi is\nu^*t)\exp(\pi irp)\\ &= \sum\sum f_{rs}(z)\exp(2\pi irx/\lambda^*)\exp(2\pi is\nu^*t)\exp(\pi isp).\end{aligned} \tag{6}$$

Comparing the Fourier coefficients in (6), we get

$$f_{rs}(z)\exp(\pi irp) = f_{rs}(z)\exp(\pi isp).$$

This means that $f_{rs}(z)$ is zero unless $r$ and $s$ are both even integers or odd integers. Returning now to the Fourier expansion of $\phi$, we could write it as

$$\begin{aligned}\phi(x, z, t) = &\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} f_{2r,2s}(z)\exp(2\pi i2rx/\lambda^*)\exp(2\pi i2s\nu^*t)\\ &+ \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} f_{2r+1,2s+1}(z)\exp(2\pi i\overline{2r+1}x/\lambda^*)\exp(2\pi i\overline{2s+1}\nu^*t).\end{aligned}$$

Thus

$$\begin{aligned}\psi(x, z, t) = &\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} f_{2r,2s}(z)\exp(2\pi i2rx/\lambda^*)\exp(2\pi i(\nu + 2s\nu^*)t\\ &+ \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} f_{2r+1,2s+1}(z)\exp(2\pi i\overline{2r+1}x/\lambda^*)\\ &\times \exp(2\pi i(\nu + \overline{2s+1}\nu^*)t.\end{aligned} \tag{7}$$

If one considers the diffraction effects of $\psi(x, z, t)$ given by (7), it will be quite easy to see that the diffraction orders could be classed into two groups, one containing the even ones and the other odd ones; any even order contains radiations with frequencies, $\nu, \nu \pm 2\nu^*, \ldots, \nu \pm 2r\nu^*, \ldots$, and any odd order contains radiations with frequencies, $\nu + \nu^*$, $\nu \pm 3\nu^*, \ldots, \nu \pm \overline{2r+1}\nu^*, \ldots$.

## 3. The case when the disturbance in the medium is simple harmonic

If we suppose that the variation in the refractive index of the medium is simple harmonic along the $X$-axis, it can be represented as

$$\mu(x,t)=\mu_0+\mu\sin 2\pi(\nu^* t-x/\lambda^*)$$

in the case of a progressive wave, while it will be of the form

$$\mu(x,t)=\mu_0-\mu\sin(2\pi x/\lambda^*)\sin(2\pi\nu^* t)$$

in the case of a standing wave, where $\mu(x,t)$ is the refractive index of the medium at height $x$ and at time $t$, $\mu_0$ is the constant refractive index of the medium when there is no sound wave and $\mu$ is the *maximum variation* of the refractive index from $\mu_0$.

*Progressive wave*: To obtain the wave function for the emerging wavefront of light, we have to solve the differential equation

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=-\frac{4\pi^2}{\lambda^2}\{\mu(x,t)\}^2\phi$$

$$=\left[A+\frac{B}{2i}\{\exp(i(bx-\varepsilon))-\exp(-i(bx-\varepsilon))\}\right]\phi \tag{8}$$

where $b=2\pi/\lambda^*, \varepsilon=2\pi\nu^* t, A=-4\pi^2\mu_0^2/\lambda^2$ and $B=8\pi^2\mu_0\mu/\lambda^2$ omitting the second order term with coefficient $\mu^2$.

We have shown in the previous section that $\phi$ can be developed as a Fourier series in $x$ and $t$ as

$$\sum_{-\infty}^{\infty} f_r(z)\exp(2\pi irx/\lambda^*)\exp(-2\pi ir\nu^* t)$$

or

$$\sum_{-\infty}^{\infty} f_r(z)\exp(irbx)\exp(-ir\varepsilon). \tag{9}$$

Substituting the Fourier series (9) in the differential equation (8) and comparing the Fourier coefficients we obtain the equation

$$\frac{d^2 f_n}{dz^2}-(A+b^2n^2)f_n=\frac{B}{2i}(f_{n-1}-f_{n+1}).$$

Putting $f_n(z)=\exp(-iu\mu_0 z)\phi_n(z)$ where $u=2\pi/\lambda$ we obtain

$$\frac{d^2\phi_n}{dz^2}-2iu\mu_0\frac{d\phi_n}{dz}-b^2n^2\phi_n=-\frac{Bi}{2}(\phi_{n-1}-\phi_{n+1}).$$

Putting $z=(2\pi\mu)^{-1}\lambda\xi$, we obtain

$$\mu^2\frac{d^2\phi_n}{d\xi^2}-2i\mu_0\mu\frac{d\phi_n}{d\xi}-\frac{n^2\lambda^2}{\lambda^{*2}}\phi_n=-\mu_0\mu i(\phi_{n-1}-\phi_{n+1}).$$

As $\mu_0$, being the refractive index of the medium, is in the neighbourhood of unity and $\mu$ is in the neighbourhood of $10^{-5}$, we can omit the first term on the left hand side and consider the differential equation

$$2\frac{d\phi_n}{d\xi}-(\phi_{n-1}-\phi_{n+1})=\frac{in^2\lambda^2}{\mu_0\mu\lambda^{*2}}\phi_n.$$

If there were no term on the right hand side, $\phi_n$, would be the Bessel function $J_n(\xi)$ or $J_n\,(2\pi\mu z/\lambda)$ satisfying the required boundary conditions. This follows as a consequence of Sonine's* theorem which gives that if

$$2\frac{d\phi_n}{d\xi}-(\phi_{n-1}-\phi_{n+1})=0,$$

then $\phi_n$ could be developed as a series in Bessel functions as

$$\phi_n(\xi)=\phi_n(0)J_0(\xi)+\sum_1^\infty[\phi_{n-s}(0)(-)^s\phi_{n+s}(0)]J_s(\xi).$$

Setting the boundary conditions that

$$\phi_0(0)=1 \quad\text{and}\quad \phi_s(0)=0,\quad s\neq 0$$

we get

$$\phi_n(\xi)=J_n(\xi).$$

If $n$ is not too great and $\lambda^2/\lambda^{*2}\mu$ is small, we can approximate

$$\phi_n(\xi)\approx J_n(\xi)=J_n\left(\frac{2\pi\mu z}{\lambda}\right).$$

If the cell is bound by $z=L$ at the emerging face, it will be easy to see that the relative intensity of the $n$th order diffraction component would be $J_n^2(2\pi\mu L/\lambda)$.

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*N Nielsen, *Handbuch der theorie der Cylinderfunktionen*, p. 286 (1904 edition).

*The case of the standing wave:* In this case we have to write $\phi(x,z,t)$ as given by

$$\phi(x,z,t) = \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} f_{2r,2s}\exp(2\pi i 2rx/\lambda^*)\exp(2\pi i 2sv^* t)$$

$$+\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} f_{2r+1,2s+1}\exp(2\pi i\overline{2r+1}\,x/\lambda^*\exp(2\pi i\overline{2s+1}\,v^* t)$$

$$= \sum_{-\infty}^{\infty} g_r(z,t)\exp(2\pi irx/\lambda^*). \tag{10}$$

Substituting (10) the differential equation for $\phi$ and comparing the coefficients, we obtain

$$\frac{\partial^2 g_n}{\partial z^2} - \frac{4\pi i\mu_0}{\lambda}\frac{\partial g_n}{\partial z} - \frac{4\pi^2 n^2}{\lambda^{*2}} g_n = \frac{4\pi^2\mu_0\mu\sin\varepsilon}{\lambda^2 i}(g_{n-1} - g_{n+1}).$$

Putting $z = (2\pi\mu)^{-1}\lambda\xi$ we obtain

$$\mu^2\frac{\partial^2 g_n}{\partial\xi^2} - 2i\mu_0\mu\frac{\partial g_n}{\partial\xi} - \frac{n^2\lambda^2}{\lambda^{*2}} g_n = -\mu_0\mu i\sin\varepsilon(g_{n-1} - g_{n+1}).$$

Under the same considerations as in the previous paragraph, we will have to solve the equation

$$2\frac{\partial g_n}{\partial\xi} - \sin\varepsilon(g_{n-1} - g_{n+1}) = \frac{in^2\lambda^2}{\mu_0\mu\lambda^{*2}} g_n.$$

If $n$ is not too great and $\lambda^2/\lambda^{*2}\mu$ is small we can approximate

$$g_n(\xi,\varepsilon) \approx J_n(\xi\sin\varepsilon) = J_n\left(\frac{2\pi\mu z}{\lambda}\sin\overline{2\pi v^* t}\right).$$

But we have shown in part III[1], that

$$J_{2n}(v\sin\varepsilon) = \sum_{-\infty}^{\infty}(-)^r J_{n-r}(v/2)J_{n+r}(v/2)\exp(i2r\varepsilon)$$

$$J_{2n+1}(v\sin\varepsilon) = -i\sum_{-\infty}^{\infty}(-)^r J_{n-r}(v/2)J_{n+r+1}(v/2)\exp(i\overline{2r+1}\,\varepsilon).$$

Hence,

$$\psi(x,z,t) \approx \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}(-)^r J_{r-s}(v/2)J_{r+s}(v/2)\exp(2\pi i 2rx/\lambda^*)\exp(2\pi i(v + 2sv^*))t$$

$$-i\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}(-)^r J_{r-s}(v/2)J_{r+s+1}(v/2)\exp(2\pi i\overline{2r+1}\,x/\lambda^*)\exp(2\pi i(v + \overline{2s+1}\,v^*))t.$$

If one considers now the diffraction effects due to this emerging wave-front at $z = L$, it can be seen that an even order, say $2n$, contains radiations with

frequencies $\nu \pm 2r\nu^*$, $(r = 0, 1, 2, \ldots.)$, the relative intensity of the $\nu \pm 2r\nu^*$ sub-component being $J^2_{n-r}(\pi\mu L/\lambda)\ J^2_{n+r}(\pi\mu L/\lambda)$ and an odd order, say $2n+1$, contains radiations with frequencies $\nu \pm \overline{2r+1}\,\nu^*$, $(r = 0, 1, 2, \ldots.)$, the relative intensity of the $\nu \pm \overline{2r+1}\,\nu^*$ sub-component being $J^2_{n-r}(\pi\mu L/\lambda) J^2_{n+r+1}(\pi\mu L/\lambda)$.

## 4. Summary

The essential idea that the phenomenon of the diffraction of light by high frequency sound waves depends on the corrugated nature of the transmitted wave-front of light, pointed out by the authors in their first paper, has been developed on general considerations in this paper. The results in this paper can be summarised as follows:

(1) If progressive sound-waves travel in a rectangular medium normal to two faces and the direction of propagation of a plane beam of incident light, the incident light will be diffracted at the angles given by $\sin^{-1}(-n\lambda/\lambda^*)$ and the light belonging to the $n$th order will have the frequency $\nu - n\nu^*$.

(2) If the sound waves are stationary, the incident light will be diffracted at the angles given by $\sin^{-1}(-n\lambda/\lambda^*)$, and even order would contain radiations with frequencies, $\nu$, $\nu \pm 2\nu^*$, $\nu \pm 4\nu^*, \ldots.$, $\nu \pm 2r\nu^*, \ldots.$, and an odd order would contain radiations with frequencies $\nu \pm \nu^*$, $\nu \pm 3\nu^*$, $\nu \pm 5\nu^*, \ldots.$, $\nu \pm \overline{2r+1}\,\nu^*, \ldots..$

(3) A differential-difference equation has been obtained for the amplitude function of the diffracted orders whose approximate solution is satisfied by the Bessel Functions already obtained by the authors in their previous papers.

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# The diffraction of light by high frequency sound waves: Part V

## General considerations—oblique incidence and amplitude changes

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## 1. Introduction

The essential idea that the phenomenon of the diffraction of light by high frequency sound waves depends on the corrugated form of the wave-front of the transmitted light has been pointed out by us in part I of this series of papers[1]. Therein, we considered that the corrugated wave-front of light could be simply obtained by considering the phase changes accompanying the traversing beam which was assumed to undergo no amplitude changes at its various points. This course was adopted by us to bring out the essential features of the theory without unnecessary complications. By a close study of the problem, one can however easily see that the consideration of the phase changes is far more important than the amplitude changes if we desire to understand the essential features of the phenomenon. Indeed, this fact holds if we consider the case when the sound wave field is small and the wavelength of sound is large. This has been experimentally confirmed quite recently by Bär[2].

In part IV of this series of papers, we proposed the method of obtaining the wave function of light by considering the partial differential equation governing the propagation of light in a medium filled with sound waves. *Such a procedure would naturally take account of both the amplitude changes and the phase changes accompanying the beam.* These changes should be however periodic in character. On these considerations we found that, in the case of a progressive sound wave, the $n$th order diffraction component will be inclined at an angle $\sin^{-1}(-n\lambda/\lambda^*)$ to the direction of propagation of the incident light and will have the frequency $\nu - n\nu^*$ where $\nu$ and $\lambda$ denote the frequency and the wavelength of the incident light while $\nu^*$ and $\lambda^*$ correspond to those of the sound wave. We also showed that when the disturbance in the medium is simple harmonic, the relative intensity of

the $n$th order is given by $|\phi_n|^2$ where $\phi_n$ is the solution of the equation

$$\mu^2\frac{d^2\phi_n}{d\xi^2} - 2i\mu_0\mu\frac{d\phi_n}{d\xi} - \frac{n^2\lambda^2}{\lambda^{*2}}\phi_n = -\mu_0\mu i(\phi_{n-1} - \phi_{n+1}) \tag{1}$$

where $\xi = 2\pi\mu z/\lambda, \mu_0$ is the refractive index of the undisturbed medium, $\mu$ is the amplitude of the variation of the refractive index and the $z$-axis is along the direction of propagation of the incident light. As $\mu$ is of the order $10^{-5}$ and $\mu$ is of the order of unity, we could consider the equation for $\phi_n$ as given by

$$\boxed{2\frac{d\phi_n}{d\xi} - (\phi_{n-1} - \phi_{n+1}) = \frac{in^2\lambda^2}{\mu_0\mu\lambda^{*2}}\phi_n.} \tag{2}$$

In the case of a stationary sound wave, we obtained the result that, in any even order, radiations with frequencies $\nu \pm 2r\nu^*$ would be present, while in any odd order, radiations with frequencies $\nu \pm \overline{2r+1}\,\nu^*$ would be present. These results interpret the experimental results of Bär[2] regarding the coherence phenomena among the diffracted orders. If the disturbance in the medium is simple harmonic, we obtained the result that the amplitudes of the various components of the $n$th order are given by the Fourier analysis of $g_n(\xi, t)$ which satisfies the equation

$$\boxed{2\frac{\partial g_n}{\partial \xi} - \sin\varepsilon(g_{n-1} - g_{n+1}) = \frac{in^2\lambda^2}{\mu_0\mu\lambda^{*2}}g_n} \tag{3}$$

where $\varepsilon = 2\pi\nu^* t$ and the term containing the second derivative of $g_n$ is omitted as its coefficient is very small. *If one however ignores the spectral character of each order*, then the relative intensity of the $n$th order is given by

$$\int_0^{2\pi} |g_n(\xi, \varepsilon)|^2\, d\varepsilon. \tag{4}$$

These results pertain to the case of the incident light falling normally on the sound waves. One of the purposes of this paper is to extend the above considerations to the case of the oblique incidence of light to the sound waves. We have found that, in the case of oblique incidence, the intensity of the $n$th order need not be equal to that of the $-n$th order, thus explaining the results of Debye and Sears[3], Lucas and Biqard[4], Bär and Parthasarathy.[5] We have also investigated the amplitude changes accompanying the traversing wave-front explaining the results of Hiedemann,[7] Bär[2] and Lucas[6].

## 2. The diffraction of light when it is incident obliquely to the sound waves

We choose the axes of reference such that the $x$-axis points to the direction of propagation of the sound waves and the $Z$-axis lies in the plane contained by the

directions of propagation of the sound and the incident light waves. With the same considerations as in part IV, the wave function of the light traversing the medium is given by

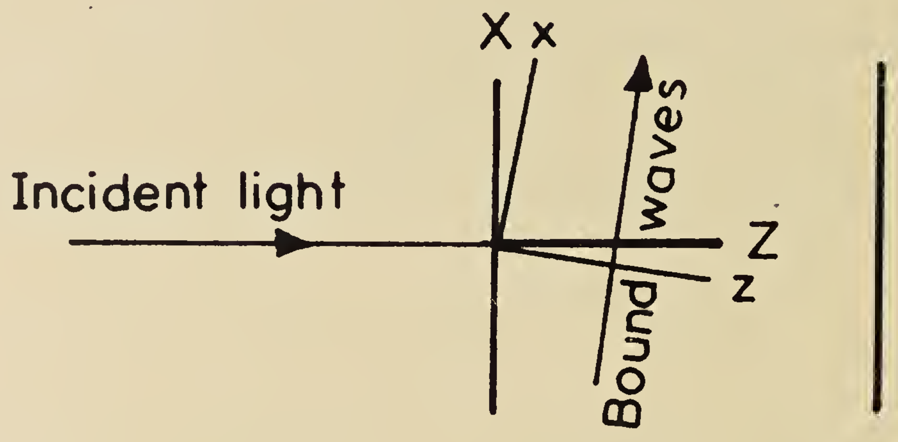


**Figure 1**

$$\psi = \exp(2\pi i\nu t)\,\Phi(x, z, t), \tag{5}$$

where $\Phi$ satisfies the equation

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = -\frac{4\pi^2}{\lambda^2}\{\mu(x, t)\}^2\Phi. \tag{6}$$

Let $\cos\phi$ and $\sin\phi$ be the $z$- and $x$-direction cosines of the direction of propagation of the incident light. The transmitted wave travelling in the medium will suffer periodic fluctuations in its phase and amplitude with the period $\lambda^*$ $\sec\phi$ along the line in the incident plane of light and the $xz$ plane. Thus,

$$\Phi(x, z, t) = \Phi(x + p\lambda^*, z - p\lambda^* \tan\phi, t). \tag{7}$$

So, the wave travelling in the medium is given by

$$\exp(2\pi i\nu t) \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} f_{rs}(x\sin\phi + z\cos\phi)\exp(2\pi i r)(x\cos\phi - z\sin\phi)/$$

$$\lambda^* \sec\phi \exp(2\pi i s\nu^* t). \tag{8}$$

We choose a new axis of reference defined by

$$\begin{aligned} X &= x\cos\phi - z\sin\phi \\ Z &= x\sin\phi + z\cos\phi. \end{aligned} \tag{9}$$

The new $Z$-axis is along the direction of propagation of the incident light. In the new system of reference, the wave function has to be written as

$$\exp(2\pi i\nu t) \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} f_{rs}(Z)\exp(2\pi i r)\,X\cos\phi/\lambda^* \exp(2\pi i s\nu^* t). \tag{10}$$

Then

$$\Phi = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} f_{rs}(Z) \exp(2\pi i r) X \cos\phi/\lambda^* \exp(2\pi i s v^* t). \tag{11}$$

In the case of a progressive sound wave

$$\Phi(X + \rho\lambda^* \sec\phi, Z, t) = \Phi(X, Z, t - \rho/v^*). \tag{12}$$

This condition restricts the number of terms in the above expansion (11) so that

$$\Phi = \sum_{-\infty}^{\infty} f_r(Z) \exp(2\pi i r) X \cos\phi/\lambda^* \exp(-2\pi i r v^* t). \tag{13}$$

Thus

$$\psi = \sum_{-\infty}^{\infty} f_r(Z) \exp(2\pi i r) X \cos\phi/\lambda^* \exp(2\pi i)(v - r v^*) t. \tag{14}$$

If one considers the diffraction effects of $\psi$ given by (14), it will be fairly obvious that the $n$th order will be inclined at an angle $\sin^{-1}(-n\lambda\cos\phi/\lambda^*)$ to the $Z$-axis and will have the frequency $v - nv^*$ with the relative intensity expression $|f_n(Z)|^2$.

## 3. The case when the disturbance in the medium is simple harmonic

If we suppose that the vibration in the refractive index of the medium is simple harmonic along the $x$-axis, it can be represented as

$$\begin{aligned}
\mu(x,t) - \mu_0 &= \mu \sin 2\pi(v^* t - x/\lambda^*) \\
&= -\frac{\mu}{2i}\{\exp(i(bx - \varepsilon)) - \exp(-i(bx - \varepsilon))\} \\
&= -\frac{\mu}{2i}\{\exp(i(\overline{bX\cos\phi + Z\sin\phi} - \varepsilon)) \\
&\quad - \exp(-i(\overline{bX\cos\phi + Z\sin\phi} - \varepsilon))\}
\end{aligned} \tag{15}$$

where $\varepsilon = 2\pi v^* t$ and $b = 2\pi/\lambda^*$.

We know from (6) that $\Phi$ satisfies the equation

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = -\frac{4\pi^2}{\lambda^2}\{\mu(x,t)\}^2\Phi$$

or

$$\frac{\partial^2\Phi}{\partial X^2} + \frac{\partial^2\Phi}{\partial Z^2} = -\frac{4\pi^2}{\lambda^2}\{\mu(X,Z,t)\}^2\Phi. \tag{16}$$

Substituting the Fourier series (13) and the expression (15) for $\mu(X,Z,t)$ in the

equation (16) and neglecting the second order term with the coefficient $\mu^2$, we get by comparing the coefficients

$$\frac{d^2 f_r}{dZ^2} - \frac{4\pi^2 r^2 \cos^2 \phi}{\lambda^{*2}} f_r - A f_r$$

$$= \frac{B}{2i}\{f_{r-1}\exp(ibZ)\sin\phi - f_{r+1}\exp(-ibZ)\sin\phi\} \tag{17}$$

where $A = -4\pi^2\mu_0^2/\lambda^2$ and $B = 8\pi^2\mu_0\mu/\lambda^2$.
Putting $f_r(Z) = \exp(-iu\mu_0 Z)\Phi_r(Z)$, where $u = 2\pi/\lambda$, we get

$$\frac{d^2\Phi_r}{dZ^2} - 2iu\mu_0\frac{d\Phi_r}{dZ} - \frac{4\pi^2 r^2\cos^2\phi}{\lambda^{*2}}\Phi_r$$

$$= -\frac{Bi}{2}\{\Phi_{r-1}\exp(ibZ)\sin\phi - \Phi_{r+1}\exp(-ibZ)\sin\Phi\}. \tag{18}$$

Putting $Z = (2\pi\mu)^{-1}\lambda\xi$ we obtain

$$\mu^2\frac{d^2\Phi_r}{d\xi^2} - 2i\mu_0\mu\frac{d\Phi_r}{d\xi} - \frac{r^2\lambda^2\cos^2\phi}{\lambda^{*2}}\Phi_r$$

$$= -\mu_0\mu i\{\Phi_{r-1}\exp(ia\xi)\sin\phi - \Phi_{r+1}\exp(-ia\xi)\sin\phi\}. \tag{19}$$

where $a = \lambda/\mu\lambda^*$.

As $\mu$ is of the order $10^{-5}$ and $\mu_0$ is of the order unity we may omit the first term and consider the equation

$$\boxed{2\frac{d\Phi_r}{d\xi} - (\Phi_{r-1}\exp(ia\xi)\sin\phi - \Phi_{r+1}\exp(-ia\xi)\sin\Phi) = \frac{ir^2\lambda^2\cos^2\phi}{\mu_0\mu\lambda^{*2}}\Phi_r.} \tag{20}$$

The relative intensity of the $r$th order is given by $|\Phi_r(\xi)|^2$. We may now show that, in general, $|\Phi_r(\xi)|^2 \neq |\Phi_{-r}(\xi)|^2$. We will prove the same by assuming the contradictory result. Suppose

$$\Phi_r(\xi) = \exp(i\rho r)\Phi_{-r}(\xi). \tag{21}$$

Then we get

$$2\frac{d\Phi_{-r}}{d\xi} = (\Phi_{-r+1}\exp(i(\rho_{r-1} - \rho r))\exp(ia\xi)\sin\phi - \Phi_{-r-1}$$

$$\times \exp(i(\rho_{r+1} - \rho_r))\exp(-ia\xi)\sin\phi)$$

$$= \left\{-2i\frac{d\rho_r}{d\xi} + \frac{ir^2\lambda^2\cos^2\phi}{\mu_0\mu\lambda^{*2}}\right\}\Phi_{-r}. \tag{22}$$

The actual equation for $\Phi_{-r}$ is

$$2\frac{d\Phi_{-r}}{d\xi}-(\Phi_{-r-1}\exp(ia\xi)\sin\phi-\Phi_{-r+1}\exp((-ia\xi)\sin\phi)$$
$$=\frac{ir^2\lambda^2\cos^2\phi}{\mu_0\mu\lambda^{*2}}\Phi_{-r}. \tag{23}$$

Comparing the equations, we obtain the result that they can never be identical unless $\phi=0$ when $\rho_r=r\pi$. Thus in the case of oblique incidence in which $\phi\neq 0$.

$$\Phi_r(\xi)\neq\exp(i\rho r)\Phi_{-r}(\xi) \tag{24}$$
$$\text{i.e.}\quad |\Phi_r(\xi)|^2\neq|\Phi_{-r}(\xi)|^2.$$

This means that the intensity of the $r$th order is not equal to the intensity of the $-r$th order. Similar results corresponding to the above could be easily derived in the case of the standing sound waves on the same lines.

In case the coefficient of the term on the right-hand side of the equation (20) has no appreciable influence in the wave-function and $\phi$ is small, it can be shown that $\Phi_r$ approximates to the wave-function given in part II of this series of papers. In this case the diffraction pattern will be very nearly symmetrical.

## 4. Amplitude changes on the emerging wave-front of light

According to the notation of part IV, the wave-function for a *general* periodic supersonic disturbance in the medium is given by

$$\psi=\exp(2\pi i\nu t)\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}f_{r,s}(z)\exp(2\pi irx/\lambda^*)\exp(2\pi is\nu^*t).$$

In the case of the normal incidence of the incident light to the sound waves. Therefore the intensity is given by

$$|\psi|^2=\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}A_{l,m}\exp(2\pi ilx/\lambda^*)\exp(2\pi im\nu^*t)$$

where

$$A_{l,m}=\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}f_{r,s}f_{r-l,s-m}{}^{\dagger}.$$

Thus, the intensity (or the amplitude) will be periodic in $x$ and $t$ on the wave-front which forms the basis of the explanation of the amplitude grating found by Bär[2] and Lucas[6]. This forms also the basis of the explanation of the observability

†Denoting the conjugate.

of the sound waves found by the investigators[7] at Köln. In the case of a standing sound wave, *the average intensity with respect to time* will be given by

$$I(x,z) = \sum B_s B_s^{\dagger}$$

where

$$B_s = \sum_{r=-\infty}^{\infty} f_{r,s} \exp(2\pi i r x/\lambda^*)$$

and $r$ and $s$ are both even integers or odd integers. It follows from the above that $I(x,z)$ is periodic in $\lambda^*/2$.

The intensity (or the amplitude) will be constant on the wave-front when all $A_{l,m}$ vanish except $A_{0,0}$ as will be so in the case governed by the restrictions of part I.

## 5. Summary

The essential idea that the phenomenon of the diffraction of light by high frequency sound waves depends on the corrugated nature of the transmitted wave-front of light has been developed on general considerations in this paper to apply for the case of the oblique incidence of the incident light to the sound waves. It is found that the intensity distribution will not be symmetrical in general thus explaining the results of Debye and Sears, Lucas and Biquard, Bär and Parthasarathy. The consideration of the amplitude changes of the traversing beam of light explains the results of Hiedemann, Bär and Lucas.

We are highly thankful to Prof. Dr R Bär of Zürich for having kindly sent us a copy of the proof of a paper by him describing experimental tests of our theory which is now in course of publication in the *Helv. Phys. Acta.*

## References

[1] C V Raman and N S Nagendra Nath, *Proc. Indian Acad. Sci.*, 1935, **2**, 406 and 413; 1936, **3**, 75 and 119.
[2] R Bär, *Helv. Phy. Acta*, 1935, **8**, 591; and a paper to be published shortly.
[3] P Debye and F W Sears, *Proc. Natl. Acad. Sci.*, 1932, **18**, 409.
[4] R Lucas and P Biquard, *J. Phys. et Rad.* 1932, **3**, 464.
[5] S Parthasarathy, *Proc. Indian Acad. Sci.* 1936, **3**, 442.
[6] E Hiedemann, *Erg. der Exakt. Naturw.*, 1935, **14**, 201.
[7] R Lucas, *Compt. Rend.* 1936, **202**, 1165.

*Nature* (*London*) **138** 616 (1936)

# Diffraction of light by ultrasonic waves

F H Sanders[1], in a recent note in these columns, has reported excellent agreement between our theory and his experimental results. His note, however, calls for a statement from us clarifying the theoretical position. As is well known, Debye and Sears in America and Lucas and Biquard in France discovered, in 1932, that a beam of light after passing through a supersonic field breaks up into a fan of diffraction spectra. Following this discovery, Prof. R Bär, of Zurich, carried out extensive investigations regarding the nature of the phenomenon; he obtained numerous beautiful results concerning the manner in which the relative intensities of the various diffraction spectra depend on the wavelength of light, the supersonic intensity and the thickness of the cell. He also discovered that the frequencies of light in the diffracted spectra are modulated by the sound field in a very peculiar manner depending on the order of the spectrum.

As has been remarked by many investigators, these results of Bär, and even the appearance of a large number of diffraction spectra, found no explanation in terms of the theory of Brillouin. Indeed, the existence of higher orders had been erroneously ascribed to the existence of overtones in the supersonic field. In the theory of Lucas and Biquard, which was mentioned by Sanders in his note, the laws of geometrical optics were applied to the problem, and it was assumed that the individual rays of the incident light follow paths independent of one another. This theory ignores the interference effects which are fundamental to the problem, and does not succeed in explaining the characteristic features observed in experiment.

The theory of the phenomenon initiated by us is set out in a series of papers[2]. At the outset, our purpose was to develop a theory of the simplest possible character which would satisfactorily account for Bär's experimental results. A simplification was affected by assuming that the wavelength of the sound is not too small and the thickness of the cell is not too large; in which circumstances, it can be shown theoretically from Fermat's principle that only the phase changes occurring in the passage of light through the cell need be considered. Indeed, Bär[3] reported later that the results in our papers I, II and III agreed qualitatively with most of the observed features of the phenomenon even in the general case, and in

[1] F H Sanders, *Nature* (*London*), **138**, 285 (1936).
[2] C V Raman and N S Nagendra Nath, *Proc. Ind. Acad. Sci.*, **2**, 406 and 413 (1935); **3**, 75, 119 and 459 (1936). N S Nagendra Nath, *Proc. Ind. Acad. Sci.*, **4**, 222 (1936).
[3] R Bär, *Helv. Phys. Acta*, **9**, 265 (1936).

a perfectly quantitative manner when the experimental restrictions postulated by us were actually satisfied. In our papers IV and V, the restrictions mentioned above were dispensed with and the theory of the phenomenon was developed quite rigorously on the basis of the electromagnetic wave-equations. This general theory has been fully worked out by one of us (NSN) and leads to a satisfactory explanation of some remarkable experimental results obtained by Dr S Parthasarathy[4] at this Institute. It is found that, when the light is incident obliquely to the sound waves and the latter are of sufficiently high frequency, the intensity of the diffraction spectra shows very marked asymmetry and that particular orders attain maximum intensity at characteristic angles of incidence given by a formula of the Bragg type. This is in agreement with the deductions from the theory.

Another aspect of the problem has been worked out by one of us (NSN) in a paper now under publication. It has been explained why the supersonic waves can be seen directly through a microscope focussed on a plane to the rear of the sound-wave cell. The theory predicts the interesting result that the grating-like pattern observed through the microscope repeats itself periodically as the focal plane of the microscope is moved away from the cell by integral multiples of a definite distance. This prediction has been confirmed quantitatively in a very recent (as yet unpublished) investigation made at this Institute by Dr Parthasarathy. Other peculiar features of the sound field as optically observed—for example, a doubling of the number of fringes in certain positions of the microscope, and a disappearance of the fringes at certain other positions—are also indicated by the theory and are beautifully confirmed by the experiments.

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9 September

[4]S Parthasarathy, *Proc. Indian Acad. Sci.*, 3, 594 (1936).

Proc. Indian Acad. Sci. A9 455–466 (1939)

# On the wave-like character of periodic precipitates

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## 1. Introduction

The similarity between wave patterns and periodic precipitates was remarked upon nearly thirty years ago by St. Leduc[1]. The suggestion of a possible physical basis for such similarity is conveyed in Wo. Ostwald's "diffusion-wave" theory of the formation of the Liesegang precipitates; in this theory, Ostwald[2] postulated the existence of three diffusion waves in the system corresponding to the external and internal electrolytes and the soluble product of the reaction. More recently the idea of a diffusion wave has been applied by Michaleff[3] and his co-workers in order to interpret an important feature of the observed pattern, namely, the widening of the space between the successive rings which corresponds with the slowing down of the diffusion occurring as we proceed outwards from the centre of the pattern. Christiansen[4] and Woulff have gone further and suggested a physical connection between the formation of the periodic precipitates and the De Broglie matter waves associated with the diffusing ion. Nikiforoff[5] and Kharamonenko find additional support for the wave idea in the observation that any point in the diffusion field can become the centre of a ring system; they have made experiments which show that the front of a ring system is 'refracted' in the optical sense when passing across the boundary between gels of different concentration.

When it is completely formed, a Liesegang pattern is a static structure and is therefore scarcely to be regarded as a wave phenomenon in the usual sense, as the latter involves a movement or periodicity in time. It may be permissible however to describe a periodic precipitate as a wave-like phenomenon, meaning thereby, that it presents some analogies in its spatial distribution to the configuration at a particular instant of a periodic train of waves. Even here a difficulty presents itself when we remember that in a periodic wave-train the disturbance may be either positive or negative whereas the density of a precipitate is of necessity a positive quantity. Indeed the essence of a wave is the fact just mentioned, namely, that its amplitude can have either sign, on which depends the possibility of the interference effects which occur when superposed trains of waves reinforce or

destroy each other. It is thus clear that the analogies between a wave and a periodic precipitate would be without physical content unless it can be shown by investigation that superposition effects can be observed in periodic precipitates analogous to interference and diffraction phenomena in acoustics and optics. Indeed, the existence of such superposition effects, if established, would enable the wave-like character of Liesegang precipitates to be regarded as an established fact besides giving it a real physical significance.

Three years ago, one of us (K Subba Ramaiah) undertook systematic studies on periodic precipitates with a view to ascertain whether the particles in these precipitates had any specific crystal orientation. Observations made in the course of these studies by us in collaboration made it clear that superposition effects of the kind referred to in the preceding paragraph are actually exhibited by Liesegang precipitates, thus establishing their wave-like character. A general account of the observed phenomena and of their interpretation on the wave-hypothesis, together with the photographs illustrating the present paper, were laid before the Indian Academy of Sciences at Bangalore in a lecture delivered by the senior author on the 12th of November 1936. The hopes which were entertained of developing a mathematical theory of periodic precipitates on a wave-basis led to the printing of the lecture being deferred. The results were however included in the thesis for the doctorate submitted by K Subba Ramaiah to the Madras University in October 1938. A preliminary note summarising them also appeared as a letter to *Nature* (*London*) in its issue of August 20, 1938. Owing to various circumstances, it has not as yet been possible so far for either of us to devote much further attention to the subject. It has therefore been considered desirable to publish our work of 1936 substantially as presented in the Academy lecture of that year.

## 2. Superposition of wave-trains of equal amplitude

In the theory of interference and diffraction phenomena, attention is usually paid only to the distribution of the intensity in the field, and not to the actual position of the wave-fronts in space at any given instant. This is natural, as the variation of intensity is the very essence of the theory of interference and in optical and acoustical phenomena is the feature accessible to direct observation. The configuration of the wave-fronts is indeed unobservable without special experimental aid, and the employment of stationary waves or alternatively of stroboscopic methods is necessary to reveal the position of the individual waves. In fact, the details of the wave-pattern in interference and diffraction phenomena are most readily grasped by examining stroboscopic photographs of ripples on water or of ultrasonic vibration within liquids. In relation to our present problem, the consideration of such features is even more important than that of intensity, for the reason that though the density of a precipitate may be

qualitatively estimated or even quantitatively measured, it is a simpler matter to observe the geometric configuration of the precipitate. We shall therefore proceed to consider a few typical cases of superposition which, as we shall see presently, have an application to observable effects in periodic precipitates.

(i) *Two intersecting trains of the same wavelength and of the same amplitude*— This is the typical case which shows in the clearest possible way how interference modifies the geometric configuration of a wave-pattern. If there were *no interference*, the wave-pattern at any instant would consist of two continuous and intersecting sets of parallel straight lines cutting each other obliquely and dividing the whole field into a net of similar rhombus-shaped figures, in the two pairs of parallel sides of which we recognise the two intersecting wave-trains. Interference completely alters this picture. We observe a pattern in which the two interfering wave-trains are no longer distinguishable, as these cohere with each other, forming wave-fronts which are everywhere parallel to the longer diagonal of the rhombus, while the lines of interference maxima and minima run parallel to the shorter diagonal, in other words perpendicularly to the direction of the resultant wave-fronts. It is important to notice that the wave-fronts are not continuous: the lines of maximum *positive* disturbance are to be found in each *alternate* rhombus, while the lines of maximum *negative* disturbance are to be found in the intervening ones, the two being separated by the *interference lines of zero amplitude.* In other words, if we consider only the waves of maximum positive displacement or only those of maximum negative displacement, *their fronts run discontinuously*, the interference lines of zero disturbance forming the terminations, and the wave-fronts on the other side of such interference line *being displaced by half a wavelength with respect to each other.* This is an effect very characteristic of interference, and taken together with the fact that the two separate sets of wave-fronts become indistinguishable is a complete demonstration of the wave-like nature of any phenomenon in which such features are noticed.

(ii) *Two intersecting trains of different wavelength and of equal amplitude*— This case is very similar to the preceding one, except that the superposed wave-trains would form a net of parallelograms with unequal sides instead of rhombuses. The wave-fronts and interference lines would therefore be inclined to each other instead of being perpendicular. The wave-fronts would exhibit discontinuities similar to those already referred to in the preceding case. Another new feature of the case now considered would be the secular movement of the interference lines in a direction parallel to themselves. This would give rise to the phenomenon of "beats" if we consider the variation of the disturbance at any point of space as a function of time. The wave-fronts as they move parallel to themselves and pass through any given point undergo periodic changes of amplitude, thus breaking up the disturbance into periodic groups. We are not however concerned with these secular aspects, if we confine attention to the form of the wave-pattern at a *particular instant.*

(iii) *Two parallel trains of unequal wavelength and of equal amplitude*—In this case again, the two wave-trains (supposed to be of harmonic type) completely cohere and become indistinguishable, being replaced by a single train of intermediate wavelength which however is broken up into groups of waves separated by interference lines of zero-disturbance. The group-length is twice the distance between such successive interference lines, there being a sudden reversal in the phase of the resultant waves as we pass a line of zero disturbance. Half-way between the lines of zero intensity, we have the group maxima in which the amplitude of the individual waves is the sum of the amplitudes of the two superposed disturbances.

## 3. Superposition of wave-trains of unequal amplitude

The results stated in the preceding section would be modified in a significant way if the superposed wave-trains are of unequal amplitudes. We shall consider first the typical case of two intersecting trains of the same wavelength. As the result of the inequality of the amplitudes, the interference minima will cease to be of zero amplitude, and the discontinuous changes of phase occurring at these minima must therefore disappear and be replaced by rapid but continuous changes of phase. The positions of the interference maxima and minima will however be unaltered as these are determined only by the relative phases of the two waves. We can easily indicate the changes to be expected in the forms of the wave-fronts by taking a case in which one of the interfering wave-trains is assumed to vanish and assuming its amplitude to be gradually increased to the full value, while that of the second wave-train is correspondingly diminished until it finally vanishes. It is obvious that in the two extreme cases when one or the other wave-train is non-existent, the resultant disturbance would have a continuous wave-front identical with that of the survivor. It follows that there would be continuity of wave-fronts also in the intermediate cases, such continuity being secured by a small rotation of the wave-fronts near the interference maxima accompanied by a marked curving round in the vicinity of the interference minima, so that they join up continuously with the neighbouring wave-fronts on the other side of the minima. It is clear that when one of the interfering waves has a greater amplitude than the other, the resultant disturbance would follow more closely the outline of the stronger wave. The direction of the rotation and curvature of the wave-fronts must therefore undergo reversal when the stronger wave becomes the weaker, and *vice versa*. In the limiting case, therefore, when the waves are of nearly equal amplitude, we should have a bifurcation or forking consisting of very steeply inclined lines which connect up the wave-front more or less symmetrically with its two nearest neighbours on the other side of the interference minimum. This forking is replaced by a discontinuity of the wave-fronts when their amplitudes are exactly equal.

The features referred to above and deduced from general geometrical reasoning are readily to be noticed in any good photograph of interference effects with capillary waves on water or with ultrasonic waves in liquids. Figure 1 is a drawing made from a ripple photograph and exhibits clearly the effects mentioned. They may be explained in full detail by considering the configuration of disturbance as has been done by G L Datta[6] from whose paper the accompanying drawing of a small area of the central part of an interference field (figure 2) due to two similar centres of disturbance has been reproduced. The figure shows clearly the displacement of half a wave in the positions of the wave-fronts on either side of the interference lines, as well as the appearance of prongs or forks joining up these fronts. By drawing such figures for the case in which the amplitudes of the interfering waves differ greatly, we may illustrate the smaller and less abrupt changes of phase which then occur and the unsymmetrical curving of the wave-fronts which results therefrom, as may be seen in outlying parts of the field in figure 1.

It is unnecessary to discuss in detail the case of two intersecting trains of unequal wavelength and unequal amplitude, as the features then to be expected would naturally be similar to those already discussed above. In the case of two parallel trains of unequal wavelength and of unequal amplitude, the groups of waves would no longer be separated by lines of zero disturbance with a discontinuous reversal of phase. We would have, instead, regions of maximum

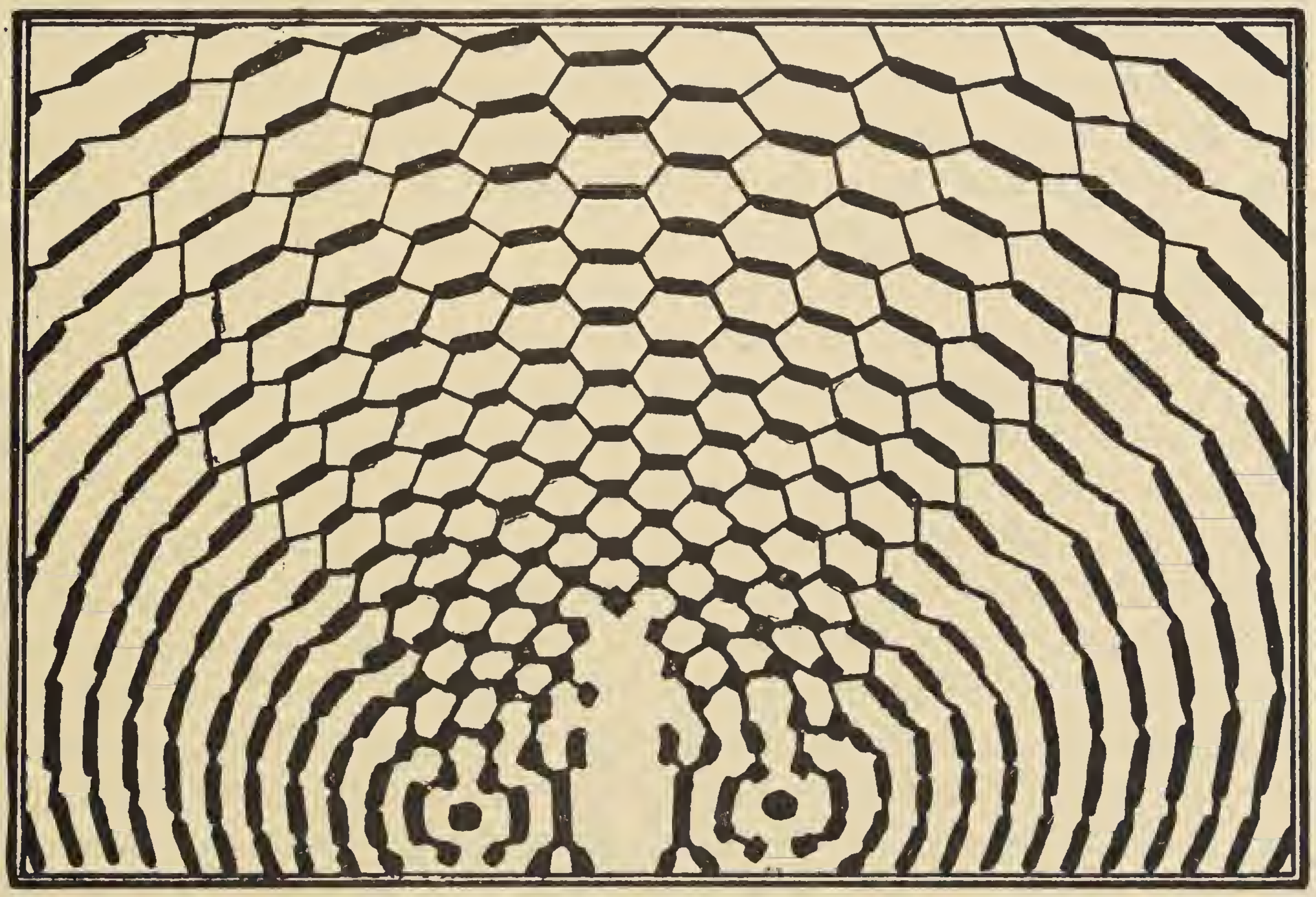


**Figure 1.** Drawing from ripple photograph.

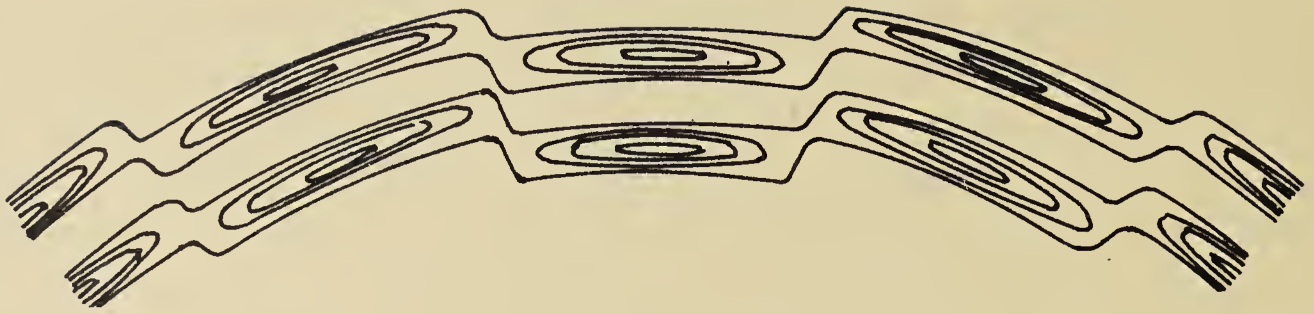


**Figure 2.** Contours of equal amplitude.

and minimum disturbance in which the wavelength, amplitude and phase all exhibit changes as we pass through different parts of the field. We shall not however consider these points in detail.

## 4. Interference of wave-groups

If a source simultaneously emits two wave-trains of equal amplitude but of slightly different wavelength, the resulting disturbance would consist of a succession of wave-groups separated by regions of zero amplitude. If a second and similar source also emits such groups of waves and if the two trains traverse the field intersecting each other, it is clear that their superposition would give, besides the individual waves and groups emerging from the two sources separately, a further superposition effect, namely, two interference patterns corresponding to the two wavelengths emitted by each of the sources separately. These interference patterns would be on different scales corresponding to the two different wavelengths, and as these would again be superposed on each other, and as the phases of the wave-fronts change by a half period across each interference line, the result of such superposition would be a simple interference pattern corresponding to the mean wavelength, and further overlying this we would have a "group interference pattern" in which the interference lines would run parallel to those in the "wave interference pattern" but would be much more widely spaced. This "group interference pattern", which is here regarded as arising from the superposition of the two "wave interference patterns" may equally well be regarded as the result of the mutual interference of the two groups of waves emitted by the two sources separately. Such interference would be possible as the phase of each group as a whole reverses across each line of minimum disturbance in it. The two groups would thus tend to cancel each other's effects where their phases are opposite and to reinforce each other where they are in agreement. The length of the group being much greater than the length of the individual waves, the spacing of the interference lines in the group pattern would be correspondingly greater than the spacing of the interference lines in the wave pattern.

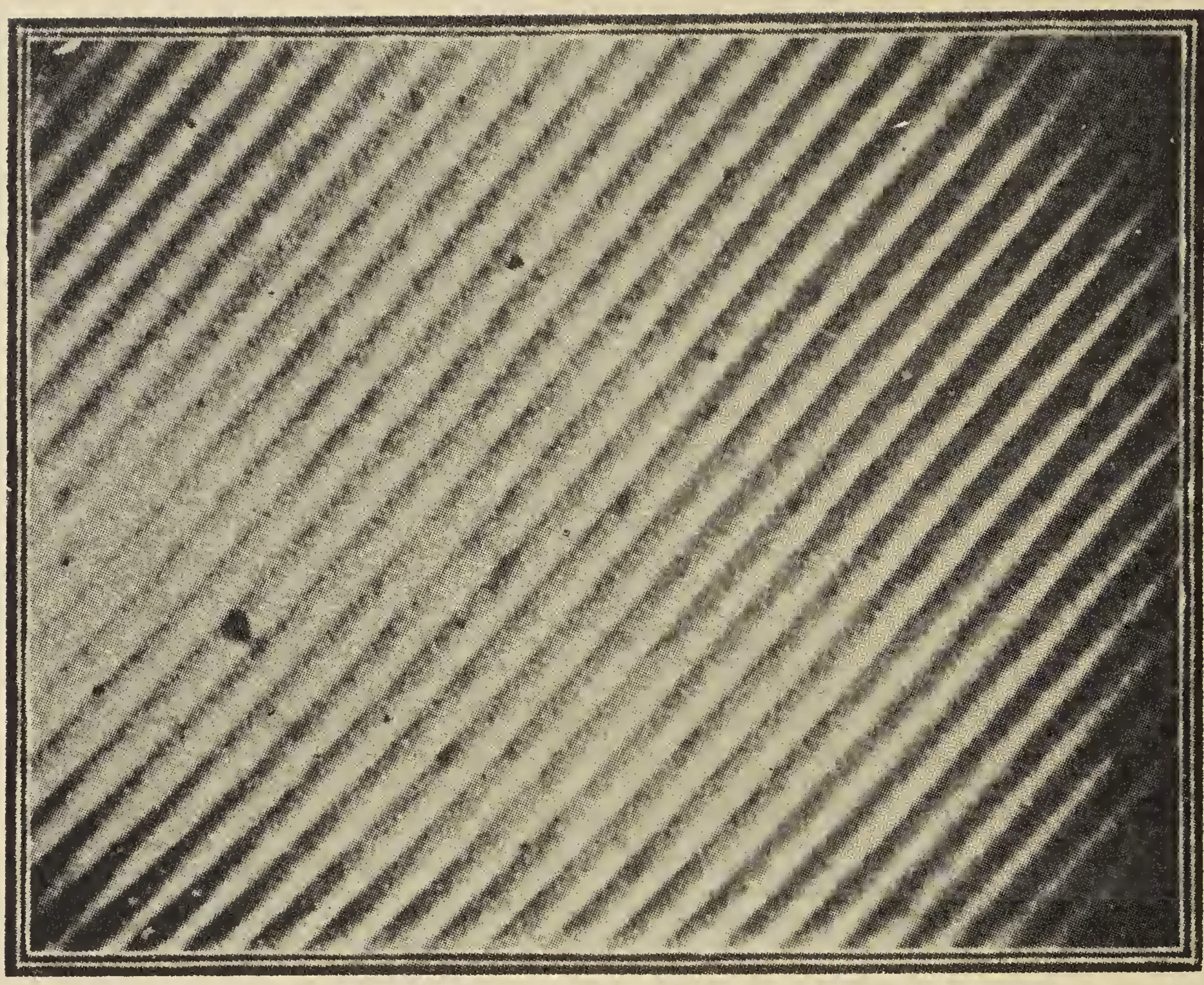
The discussion thus shows that when there are two similar sources emitting groups of waves, we have two kinds of interference pattern, one on a fine scale for

the individual waves, and another on a much larger scale for the groups of waves. The two patterns would co-exist, but would be entirely separate and distinguishable. Further, we would also have the usual discontinuities of the wave-fronts in the wave interference patterns, and similar discontinuities of the group fronts in the group interference pattern but on a much larger scale. It should be remarked that the "group fronts" and the "wave-fronts" would everywhere run parallel. If, however, the discontinuities are replaced by continuous curves, forks or branches, the wave-fronts and group fronts would not be parallel to each other in the vicinity of the interference minima. The deviations from parallelism should be specially noticeable in the vicinity of the group interference minima, as these are on a relatively larger scale.

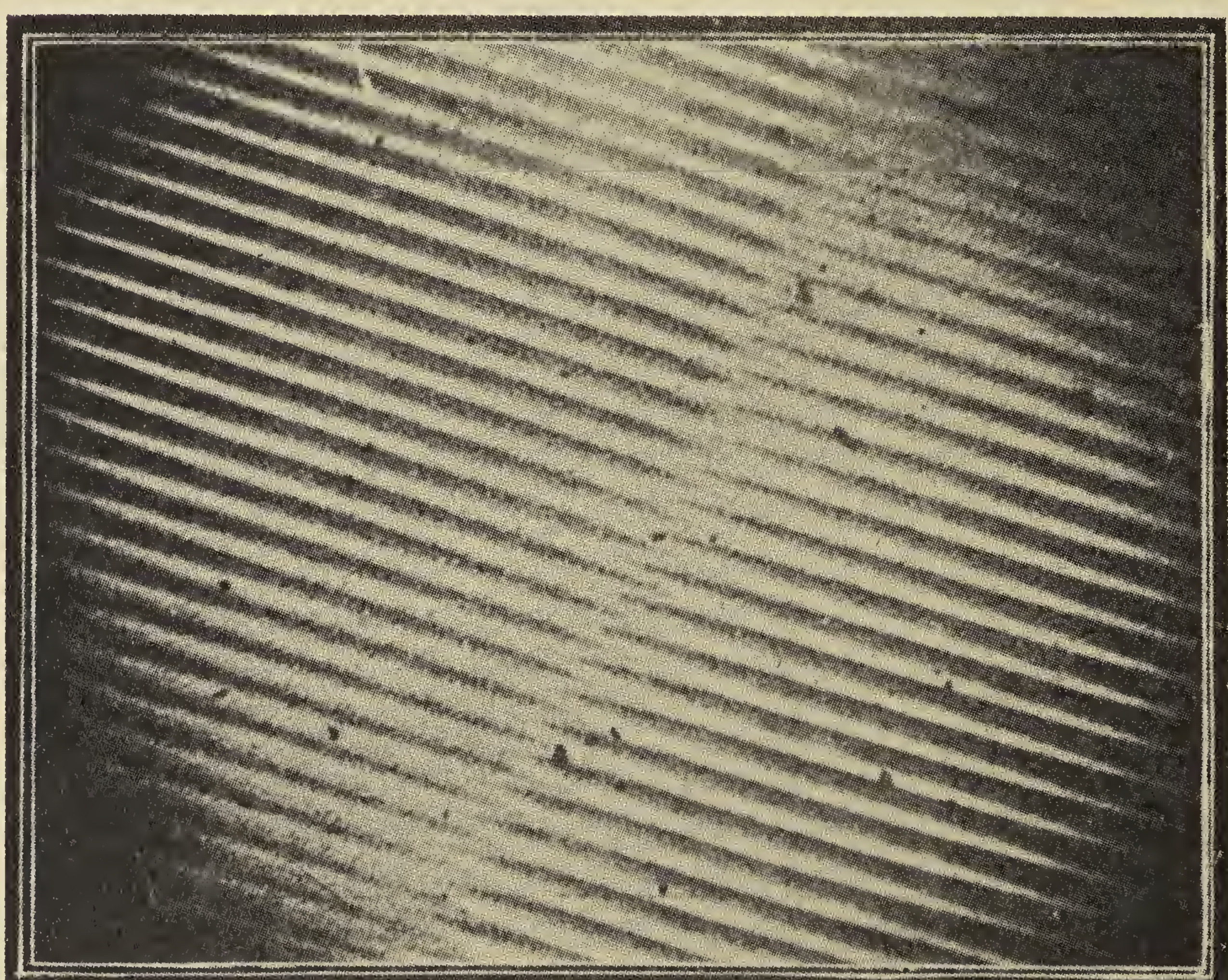
## 5. Wave-interferences in Liesegang patterns

In the examples of rhythmic precipitation which illustrate the treatment of the subject in treatises on colloid chemistry and even in special monographs, we usually observe only a few unequally spaced bands of which the width is small compared with their mutual separation. The suggestion of a resemblance between such precipitates and the infinite train of harmonic waves postulated by the mathematician would seem indeed rather far-fetched. The case is however, different when we consider the rhythmic precipitates of silver chloride or silver phosphate in gelatin which are described more fully in the following paper. In these cases, we observe under the microscope thousands of rings following each other in regular succession, and the width of the rings is everywhere comparable with their spacing. Such a periodic precipitate indeed strikingly suggests an analogy with an extended harmonic wave-train. The analogy becomes more specific if we compare the regions where the precipitate appears with the parts of the wave where the amplitude is positive and the empty spaces without deposit to the negative part.

Figure 3 in plate I illustrates a small area in a rhythmic precipitate of silver chloride in gelatin, reproduced from a photomicrograph with a magnification of 160 diameters. In this area, we have a remarkably uniform and uninterrupted periodicity of the deposit. This however is the exception rather than the rule. Usually, even when the preparation is made with very careful technique, the rings exhibit non-uniformities of a very significant character, illustrations of some of which are reproduced as figure 4 in plate I and figures 5 and 6 in plate II. In figure 4, the interruption takes the form of a succession of forks running somewhat obliquely across the lines of the deposit near the centre of the field. A short distance away from this region of the forking, the lines of deposits become very sharp and straight and also much more intense than in the region of the forks. It will also be seen that the lines of deposit on the left side when produced correspond with the dark regions of no deposit on the right and *vice versa*. In the extreme left-hand top and the right-hand bottom corners, the forkings broaden

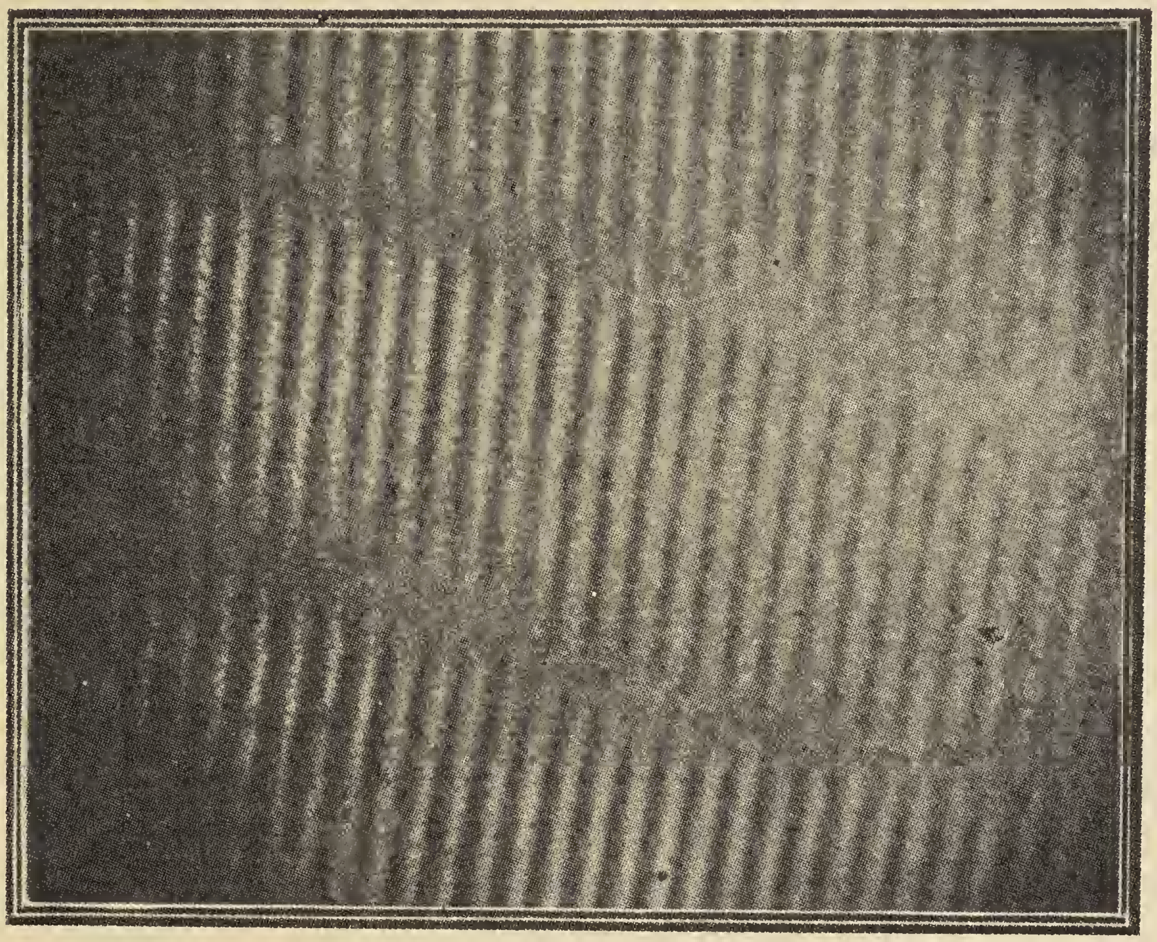


(3)

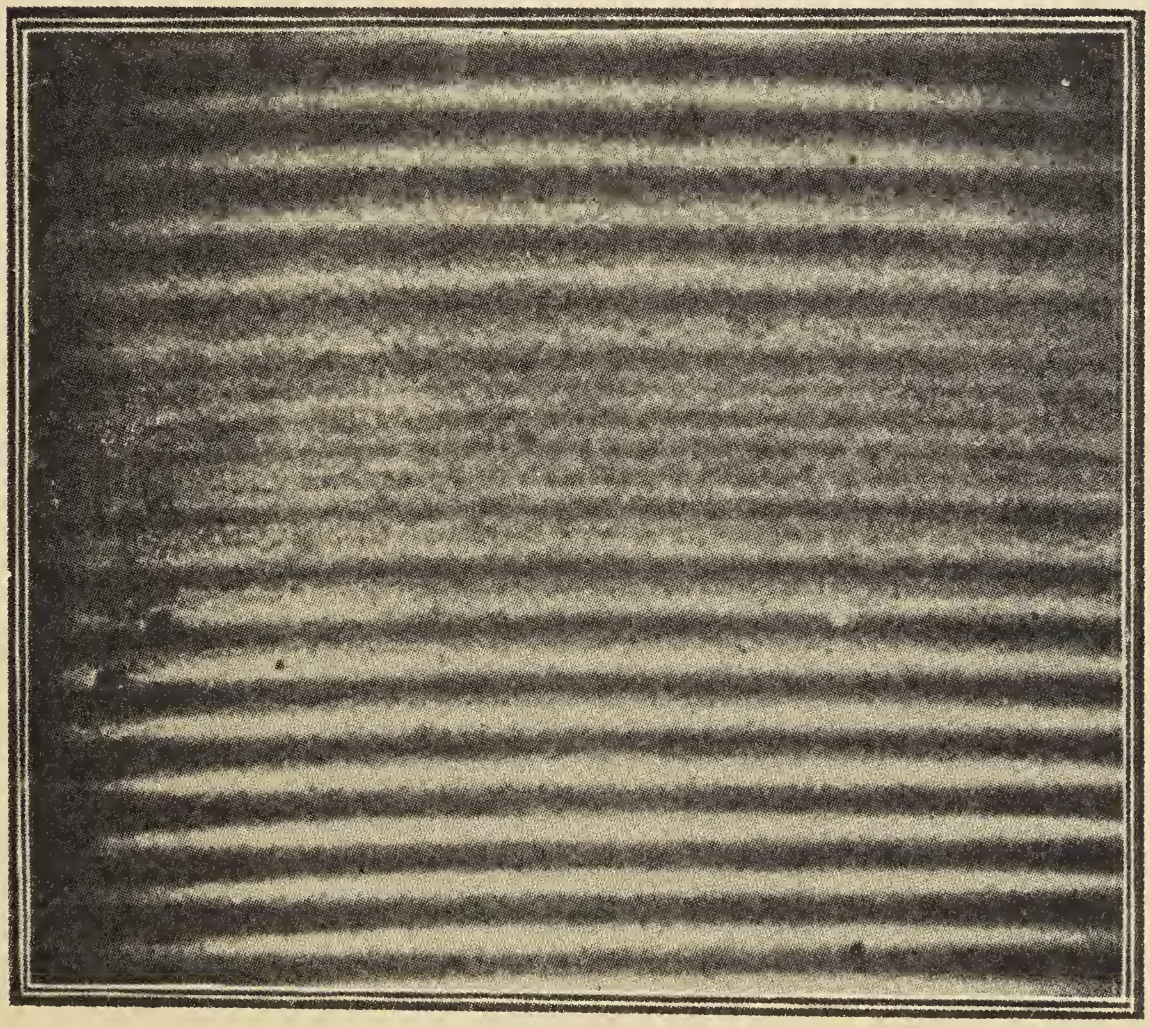


(4)

Plate I



(5)



(6)

Plate II

giving rise to a distinct doubling of the lines of deposit. These features when considered together leave little doubt that we are here observing the effects of the "coherent interference" of two "periodic waves" of deposit of slightly different wavelength and direction and of nearly equal amplitude superposed on each other as discussed in sections 2 and 3 of this paper.

Further confirmation of this view is furnished by figure 5. One of the effects indicated by the theory of interference is that when the area over which two intersecting wave-trains are superposed is sufficiently extended, we should observe a succession of equidistant interference lines running parallel to each other. Observation of the silver chloride precipitates shows that parallel and equidistant or nearly parallel and equidistant lines along which there is a sharp break or dislocation of the lines of precipitate occur so frequently as to exclude the possibility of their being accidental. Figure 5 is an example of two such "interference lines" running parallel to each other and cutting across some thirty or more "waves" of the deposit, the wave-front exhibiting a sharp bend or a forking at each point of intersection.

Examination of silver chloride deposits shows that still another effect is common in them, namely, that of a succession of interference lines running parallel to the wave-fronts, due evidently to parallel "waves of precipitation" of slightly different wavelength having been superposed. Figure 6 is a photomicrograph of a portion of the field exhibiting such an effect. Over the greater part of the field, the two series of waves are "coherent" and form single sharp lines of heavy deposit, while in those regions where the waves are out of step, the deposits are relatively light. In the regions of light deposit, the individual waves and in the manner in which they fall out of phase can actually be observed in the distribution of the precipitate. Though this particular feature does not support the "interference" theory, it is not inconsistent with it, in view of the fact that the distribution of the precipitate is evidently not of the simple harmonic type. Superposition of two waves which are not of harmonic type and which resemble periodic "pulses" would result in changes of form being observable in the vicinity of the intensity minima analogous to those actually seen in figure 6.

## 6. Groups and group-interferences in rhythmic precipitates

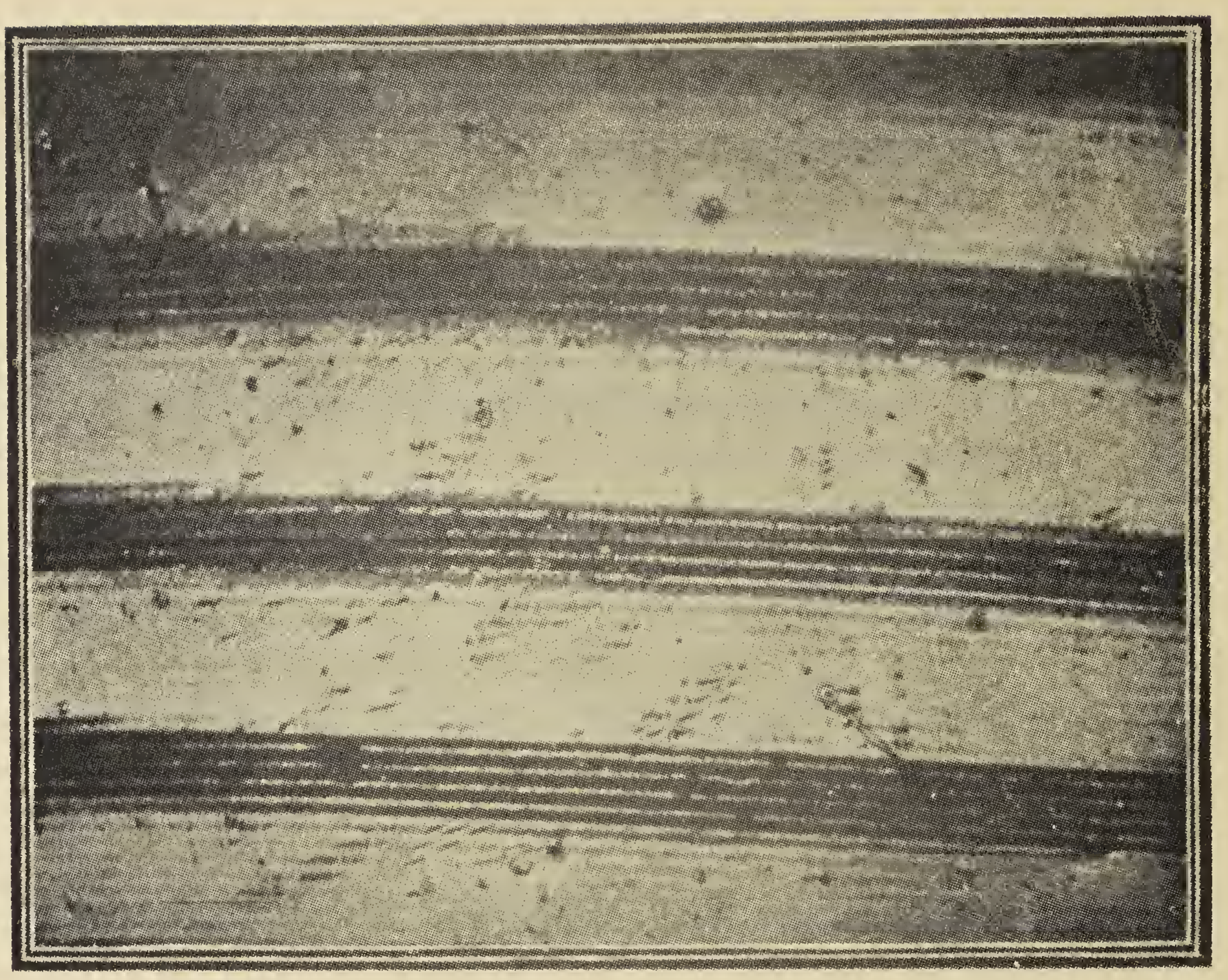
The peculiar fine structure of periodic precipitates of silver chromate and the circumstances in which it is observable have been fully discussed in the following paper by one of us. The present trend of opinion appears to be that the so-called secondary structure of very finely spaced rings is due to presence of impurities in the gelatin leading to the formation of closely-spaced silver chloride or phosphate rings. Reasons have however been given in the paper* in support of the

*K Subba Ramaiah, *Proc. Indian Acad. Sci.* A9 467–478.

view that such impurity is probably not essential for the formation of secondary structure, the items of evidence being: (1) a thoroughly washed gelatin which shows only a slight turbidity but no rings with silver nitrate, produces both secondary and primary rings when pure potassium chromate is added to it; (2) the spacing of the secondary silver chromate rings is not so close as that of the silver chloride rings; and (3) the primary and secondary rings form a continuous periodic system observable everywhere in the field. An explanation has also been offered for the fact in certain circumstances the primary rings may be obtained without the secondaries being present.

A careful examination of the silver chromate rings in gelatin in various circumstances of formation reveals a variety of interesting effects, which fit beautifully into the theory of wave-interferences and group-interferences developed earlier in this paper. It should be remarked that this interpretation is wholly independent of any decision on the question whether or not the presence of a minute quantity of impurity in the gelatin is essential for the production of a periodic precipitate and on the question whether or not, such impurity plays an important part in the spacing and distribution of the precipitate. We shall base ourselves solely on the observed facts regarding the configuration of the rings in various circumstances. Figures 7 and 8 in plate III, figures 9 and 10 in plate IV and figures 11 and 12 in plate V are photomicrographs of small selected areas in silver chromate precipitates intended to illustrate the relationship between the so-called primary and secondary ring systems and the appearance of interference phenomena in them. Figures 13 and 14 in plate VI, figure 15 in plate VII and figure 16 in plate VIII are enlargements of the Liesegang patterns themselves; these are of the type from which selected areas have been enlarged and reproduced as figures 7–12. Figures 13–16 are in fact intended to explain and illustrate the physical origin of the effects shown in figures 7–12. A careful study of the plates shows that the configuration of the rhythmic precipitates is strikingly influenced by the initial geometrical distribution of the precipitating agent. In figure 13 we have a small circular drop of silver nitrate solution starting the precipitation; in figure 14 we have an elliptical drop, while in figure 15 we have two circular drops running into an oval figure; in figure 16 we have two separate drops separated by an interference dead space. Even in the case of the apparently circular drop illustrated in figure 13, we notice radial dislocations in the rings running outwards from the margin of the drop. The very striking perturbations of the rings seen in figure 14 with the elliptical drop indicate clearly that the form of its boundary is responsible for these effects. Further, in figures 15 and 16, the perturbations in the form of the rings are most prominent in the central regions where the effects of the two drops are superposed, and less prominent in the marginal areas where they are practically independent.

A study of the Liesegang patterns illustrated in figures 13 to 16 brings out the following interesting facts. Firstly, where the primary rings run most regularly, that is, without inflections or changes of curvature, the secondary rings run most nearly parallel to the primaries; on the other hand, where the primaries bend



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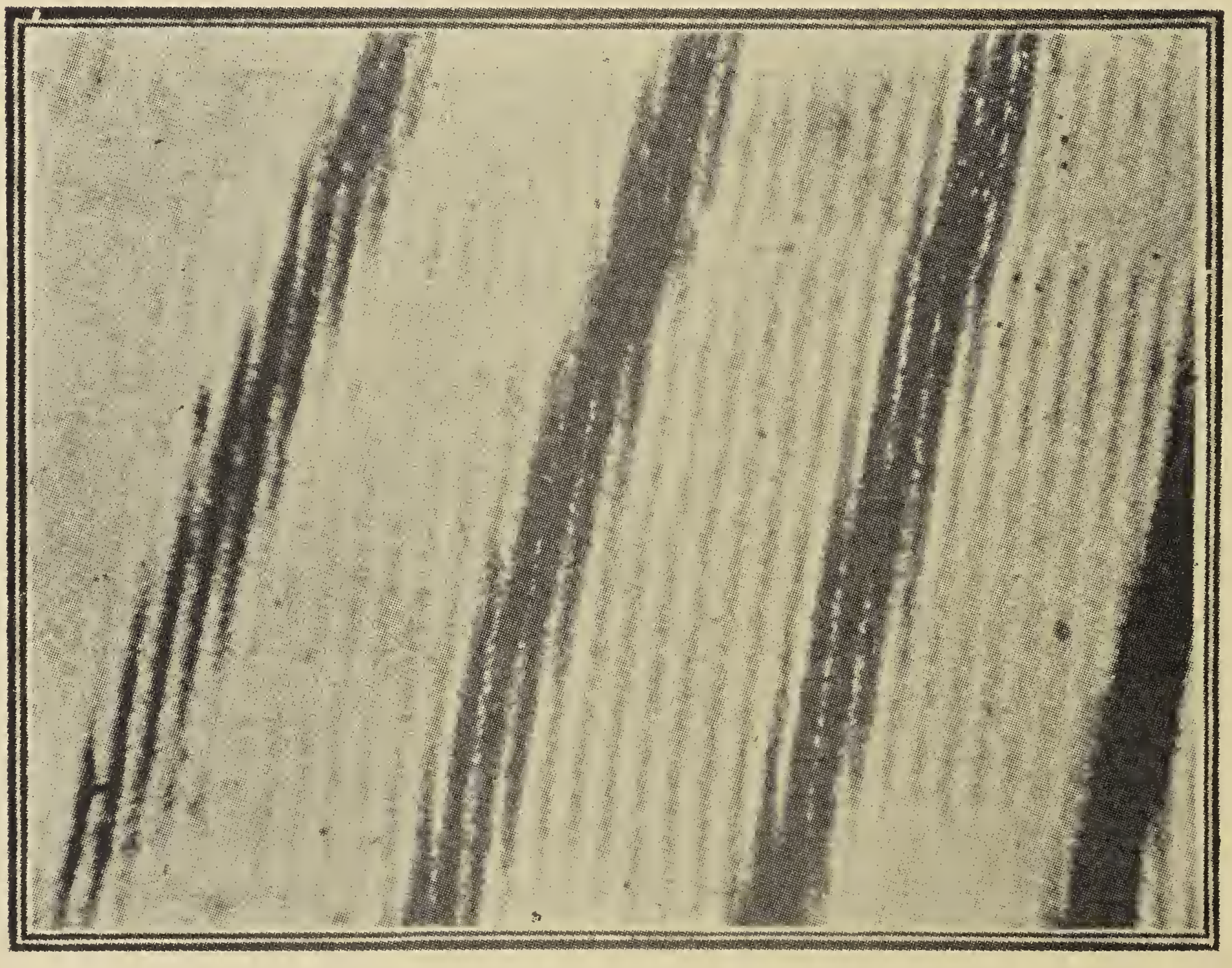

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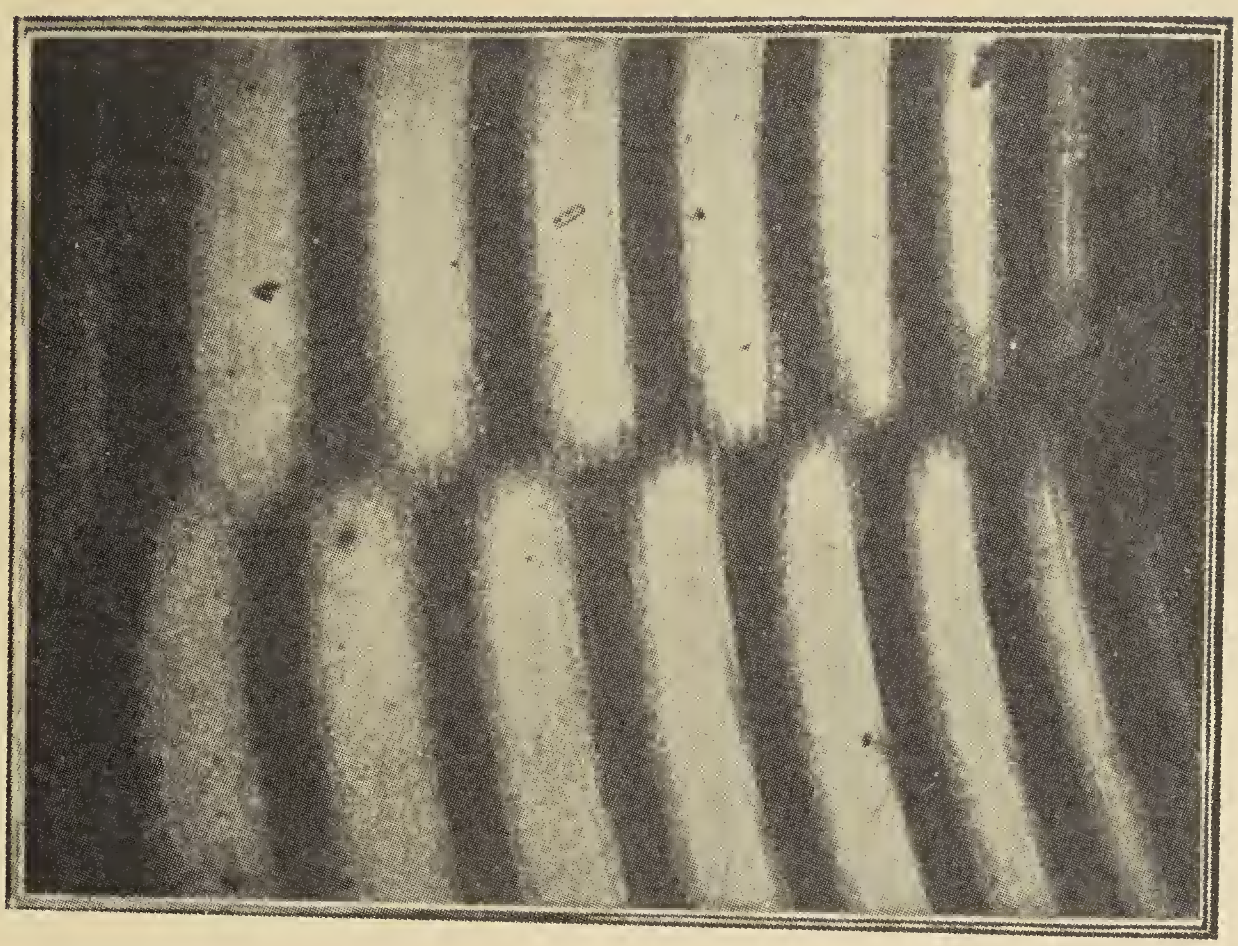
Plate III



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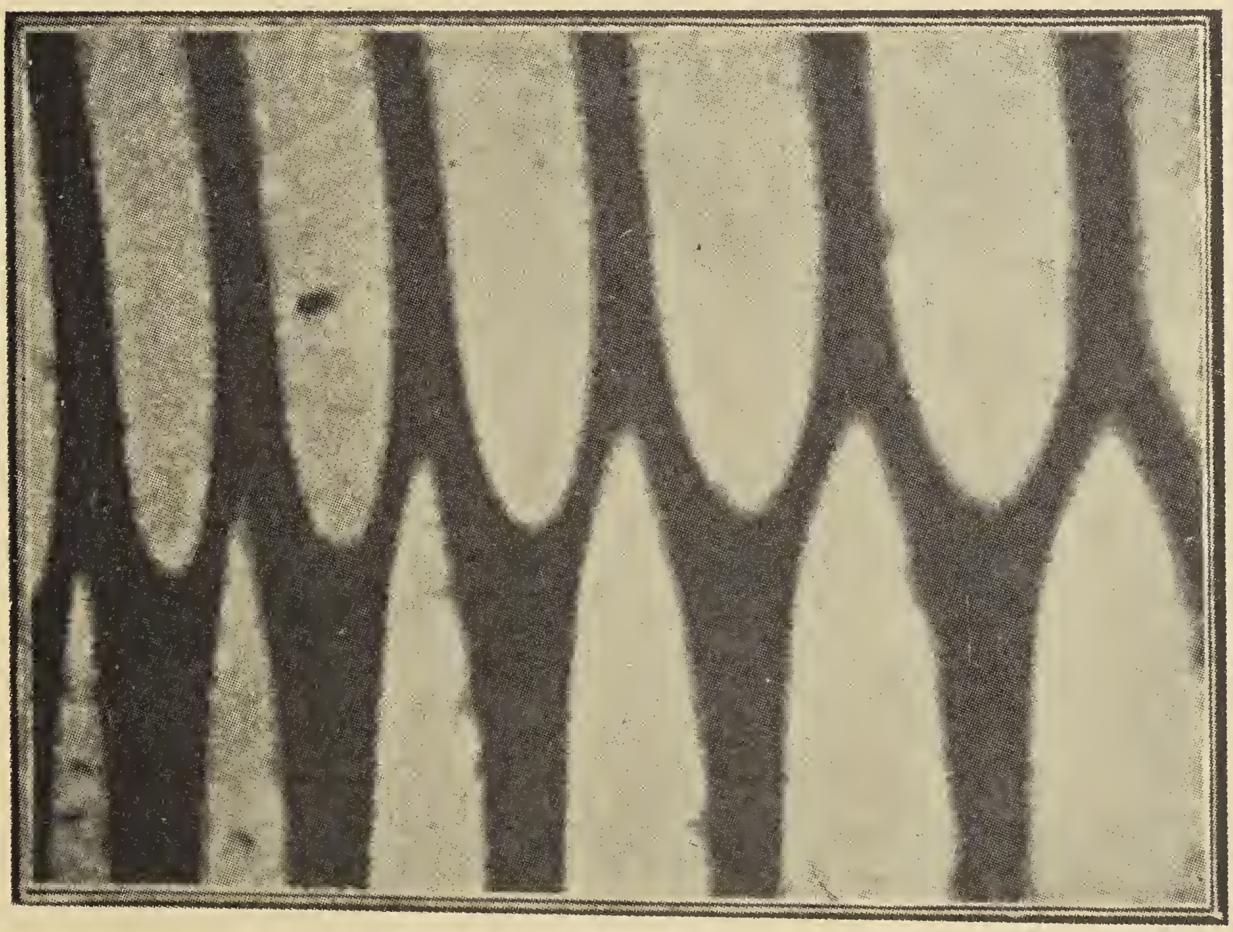

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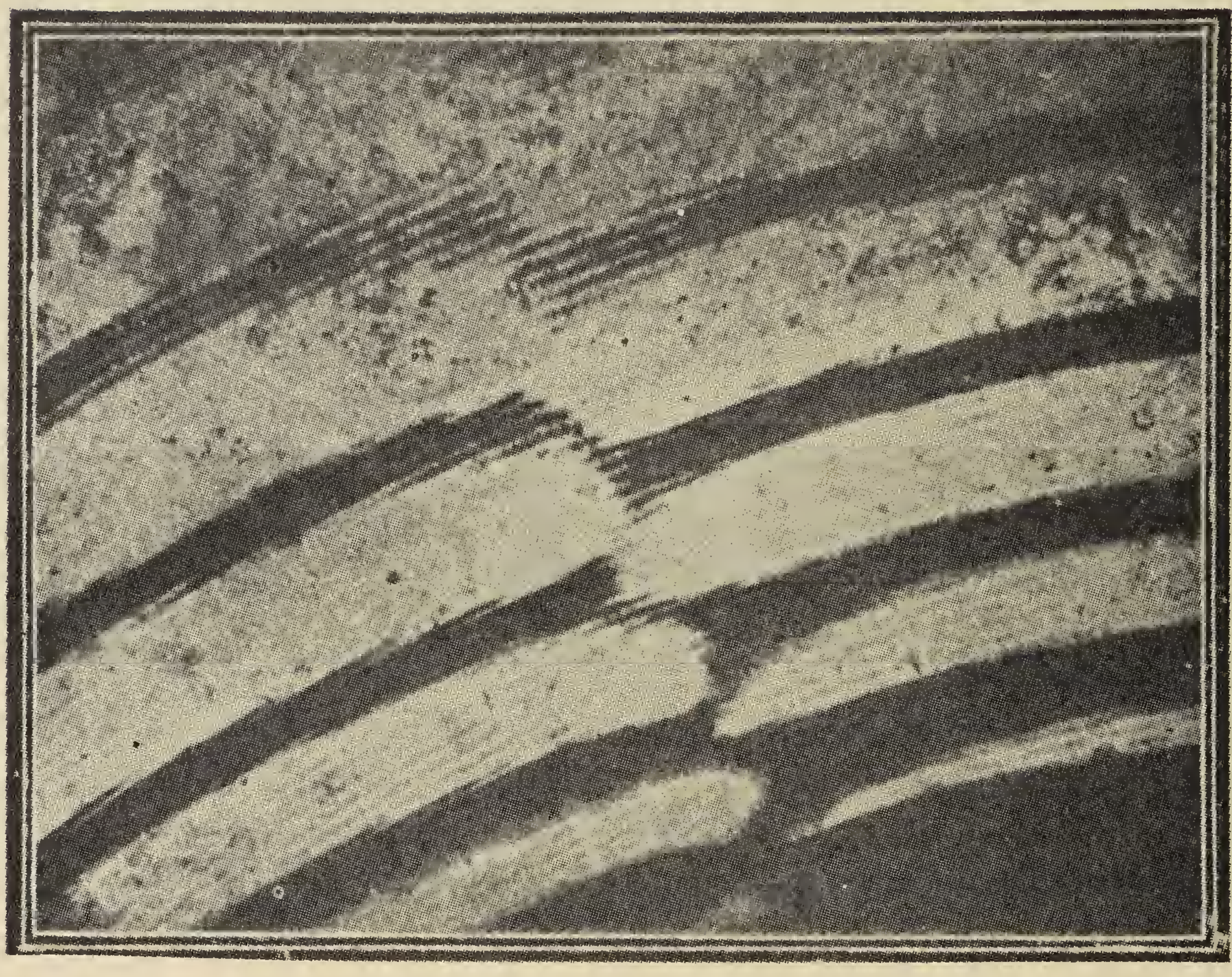
Plate IV



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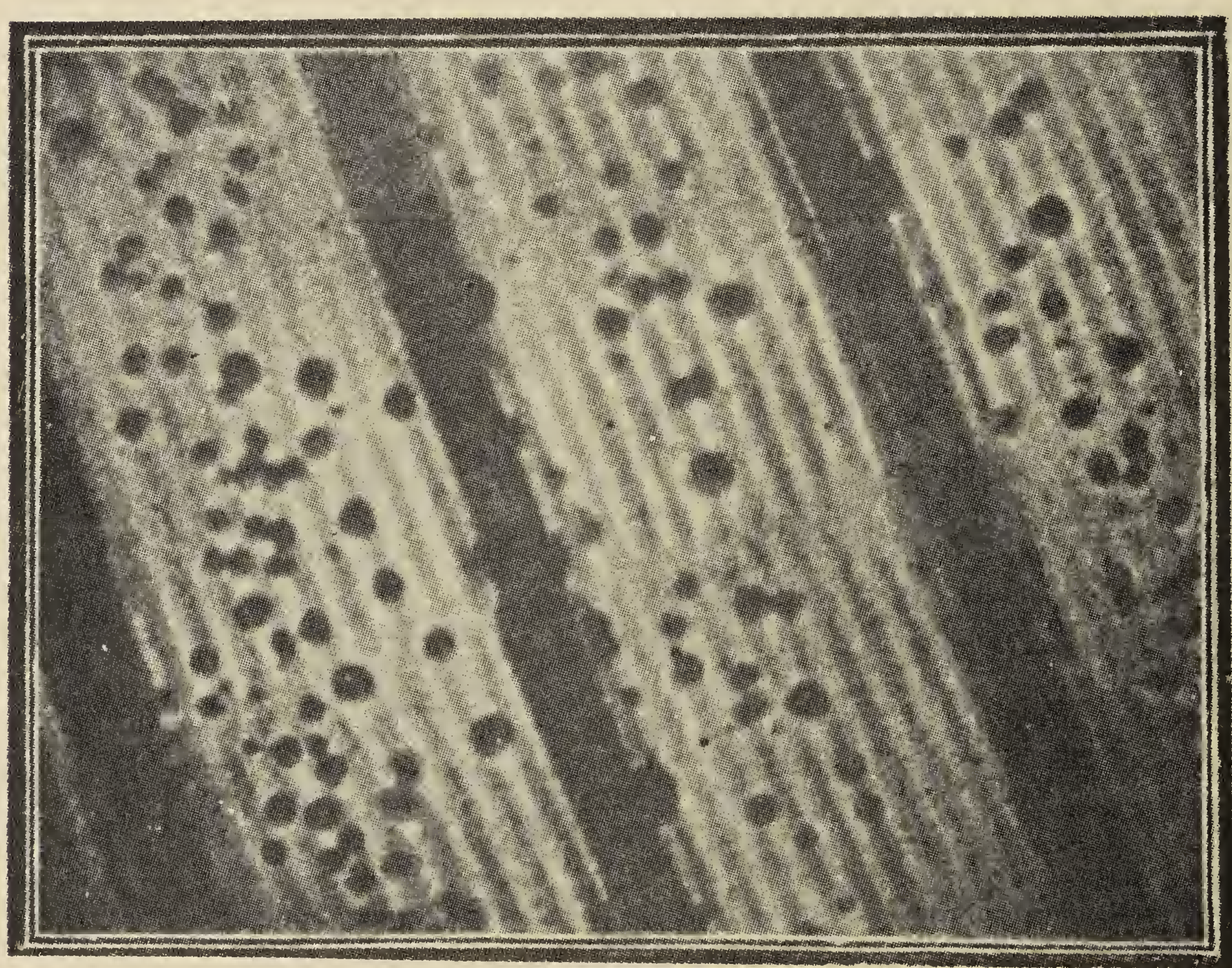

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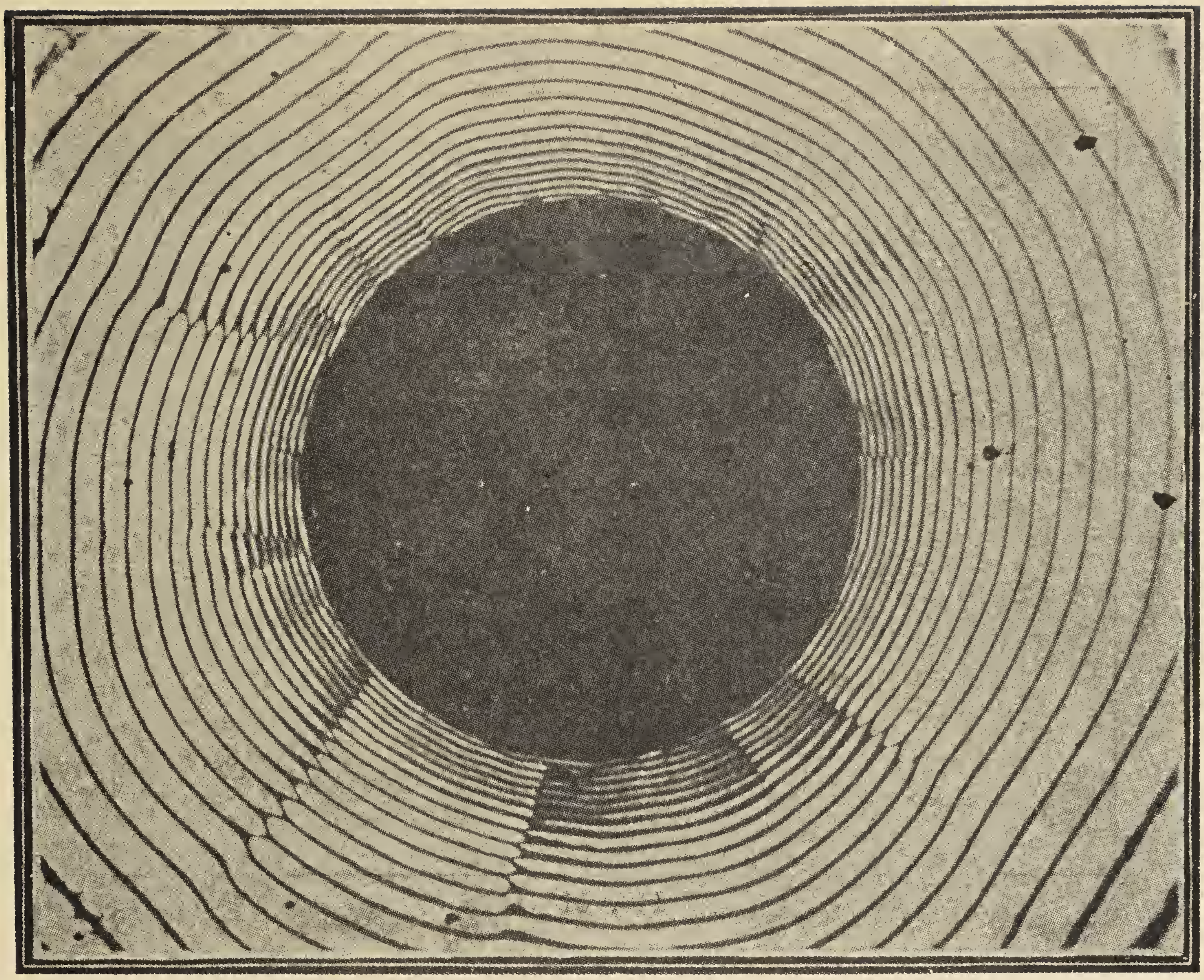
Plate V



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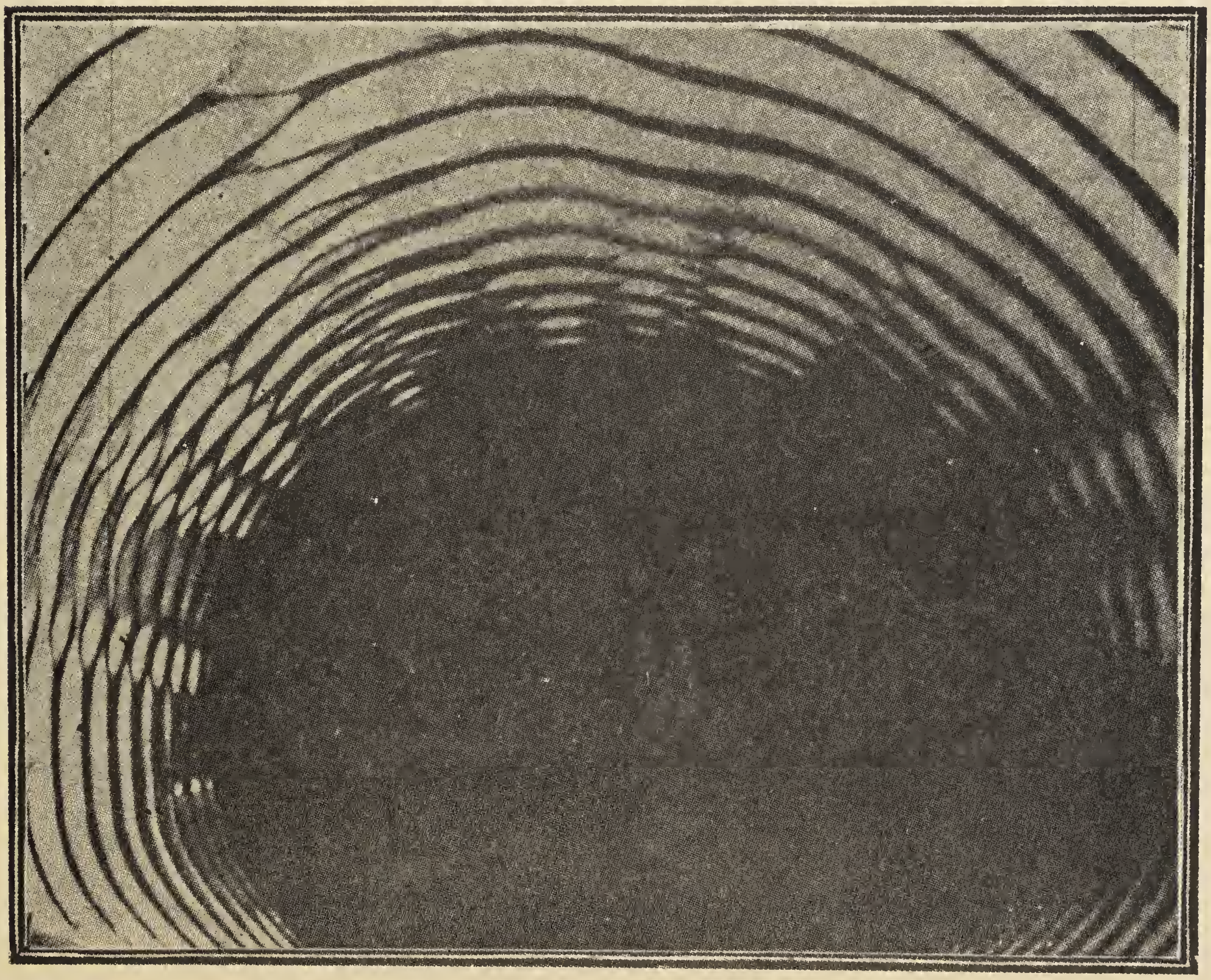

(14)

Plate VI

(15)

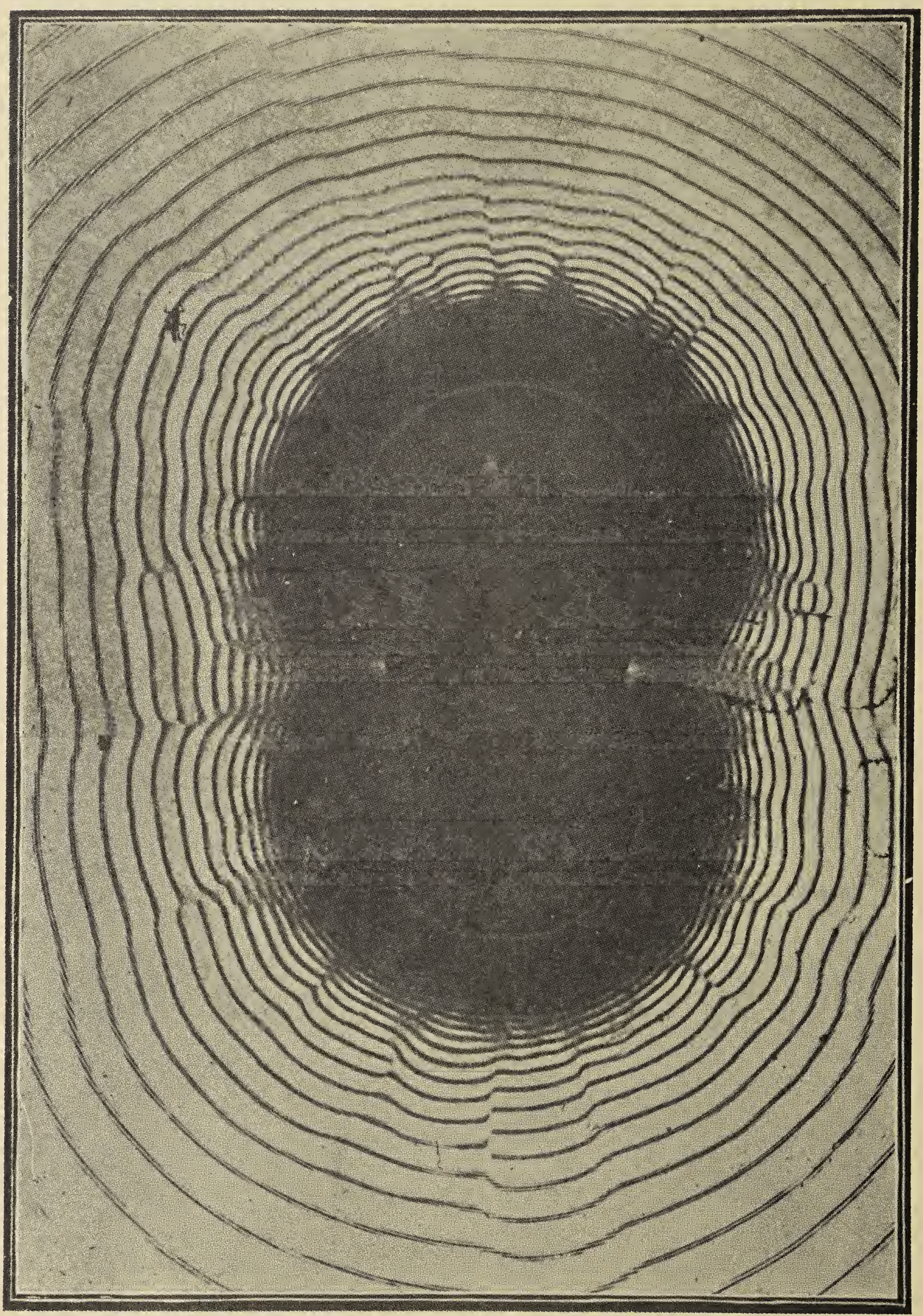

Plate VII

(16)

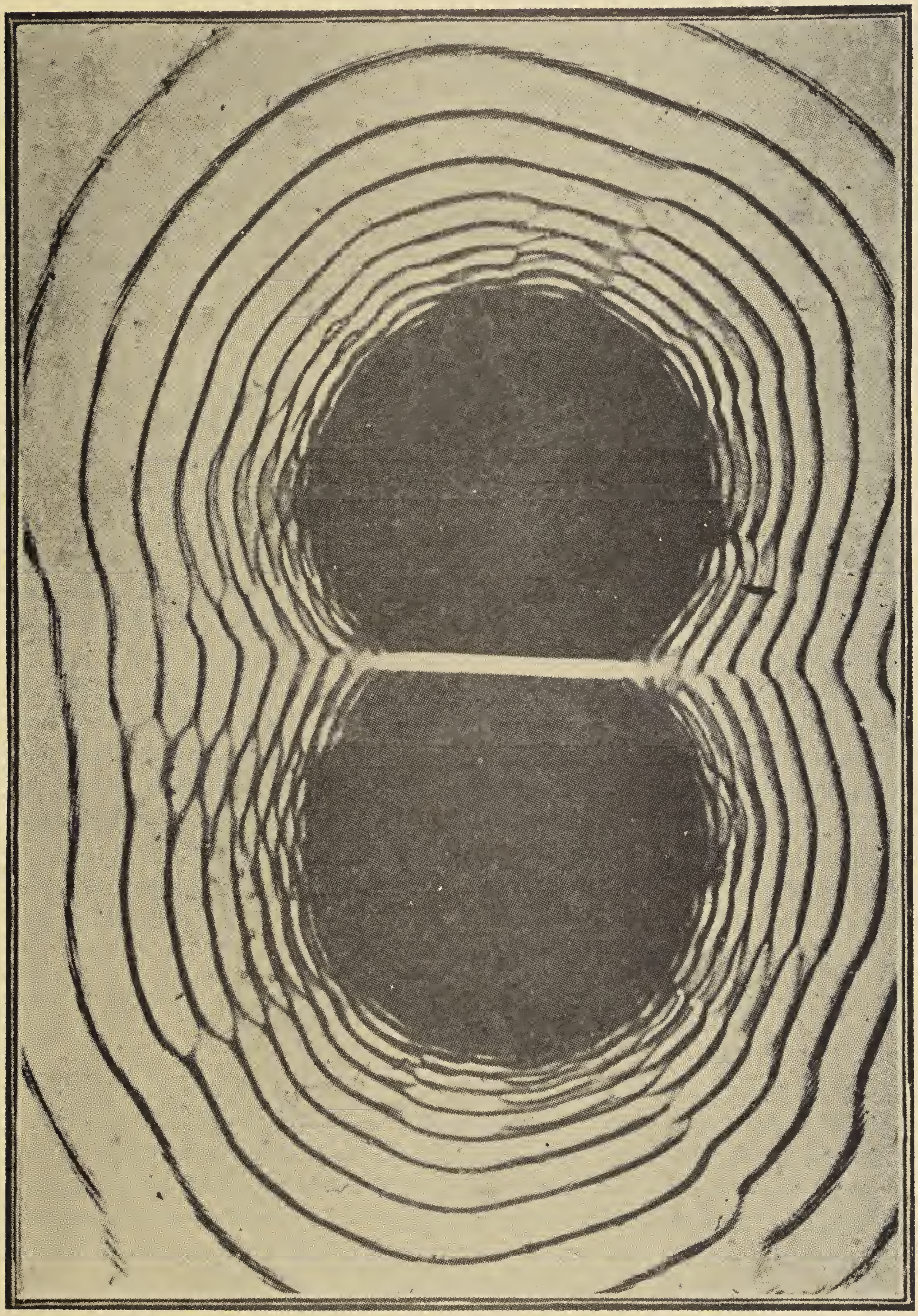

Plate VIII

round sharply, the secondaries tend to preserve their direction and therefore become inclined to the primaries. Secondly, when the primaries cut across a clear region or when they broaden out, the secondaries traversing them become very intense and are clearly seen, even though elsewhere in the clear regions they may be too feeble to be visible. Further, an examination of the patterns under a microscope shows that when the secondaries are at all visible, and irrespective of whether they are parallel or inclined to the primaries (see for instance, figures 7 and 8 in plate III, the two systems run continuously into each other as part of a simple periodic distribution of precipitate. These three features together make it clear that the primary and secondary ring systems are organically connected with each other and form part and parcel of a single phenomenon. It becomes therefore justifiable to regard the so-called primary rings as the groups, and the secondary rings as the waves of which the group is composed.

Figures 9 and 10 in plate IV are photomicrographs of a "group interference line". In figure 9 the secondaries are visible and sharply terminated by the interference line along which there is a discontinuous shift of half a group-length in the phase of the groups on either side of the minimum. On the other hand in figure 10, the secondaries are not visible and we have instead of a discontinuity, prongs or forks joining up the group maxima on either side of the interference line. It will be noticed that in figure 9 the disposition of the group maxima is quite symmetrical, while in figure 10 it is distinctly not so. Evidently in the latter case the interfering groups were not quite of equal amplitude.

Figures 11 and 12 are intended to be a contrast to each other. Figure 11 exhibits a "group-interference line" on either side of which the groups are discontinuous while the waves run continuously through. In figure 12 on the other hand, we see "a wave-interference line" crossing the whole field, including the intense primaries or wave groups and everywhere exhibiting a discontinuity in the phase of the secondaries or wave-fronts on either side of it. The groups as a whole, however, are entirely unaffected. Figures 11 and 12 thus illustrate the fact that group-inteference and wave-interference are two distinct phenomena, one on a large scale and the other on a fine scale.

## 7. Origin of the interference effects

It may be remarked that interference effects have also been observed with silver phosphate rings in gelatin, and with lead iodide rings in agar gels. In the former case, the effects are similar to those observed with silver chloride rings and in the latter they resemble those observed with the primary rings in silver chromate. These instances are sufficient to show that the phenomena we are considering are of a general nature. The question now arises how the actual interferences arise. From the facts already stated, we may infer that the effects owe their origin in the main to the irregular form of the boundary from which the diffusion starts, as the result of which we have two or more waves traversing the field simultaneously in different directions. It must be remembered that irregularities in the thickness of

the gelatin film or in the concentration of the reacting materials present in it may also be responsible for the perturbations and result in interference or diffraction effects becoming observable in the field. It would obviously be desirable to make experiments in which pre-determined effects with rhythmic precipitates similar to the standard optical experiments in interference and diffraction are produced by suitable technique. Some attempts in this direction were being made when the work was brought to a standstill by the departure of one of us from Bangalore. It is to be hoped that this experimental work as well as the theoretical interpretation of periodic precipitation on a wave-basis will again receive attention from us at an early date.

## Summary

It is pointed out that the suggestion of an analogy between periodic waves and rhythmic precipitation would be without physical significance unless it can be shown that the characteristic superposition effects observable with waves are also demonstrable with Liesegang patterns. Fourteen photomicrographs and enlargements of Liesegang precipitates of silver chloride and of silver chromate are reproduced with the paper which show that such superposition effects are actually to be observed, both in regard to the intensity of the deposits and in respect of their geometrical configuration. These examples illustrate the varying effects of the superposition of two wave-trains when their wavelength, direction and amplitude are individually or collectively different. It is shown theoretically that when groups of waves interfere, we have a group-interference pattern on a large scale in addition to a wave-interference pattern on a fine scale, the two being separate and distinguishable effects. It is observed that the so-called primary and secondary rings obtained with silver chromate precipitates in gelatin are related to each other as a group is to the individual waves of which it is composed. Independent wave-interferences and group-interferences are exhibited by such precipitates.

## References

1. Leduc *Physico-chemique de la Vie*, Paris, 1910.
2. Wo. Ostwald *Kolloid-Z.*, 1925, **36**, 380.
3. Michaleff, Nikiforoff and Schemjakin *Kolloid-Z.*, 1934, **66**, 197.
4. Christiansen and Woulff *Z. Phys. Chem.*, 1934, **36B**, 187.
5. Nikiforoff and Kharamonenko *Acta Physicochem. URRS*, 1938, **8**, 95.
6. Datta "Sir Asutosh Mookerjee Commemoration Volume," *Science*, Calcutta University, 1921.
7. Photographs very clearly showing the dislocations in periodic precipitates appear in Suzanne Veil's monograph on *Les Periodicites De Structure*. Hermann and Cie, Paris, 1934, plates I and II, and not so clearly in Hedges' *Liesegang Rings*, Chapman and Hall, London, 1932, figure 1. A beautiful example of a series of parallel dislocations is reproduced with a note by Dr Liesegang in the *Z. Wiss. Mikrosk.*, 1936, Band **53**, p. 438, the photograph being by Manfred Kohn of Freiburg i. B. A reprint of the note was very kindly sent by Dr Liesegang to Sir C V Raman in May 1939.

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# Interference patterns with Liesegang rings

Anyone who is experimentally familiar with the production of Liesegang rings in gelatine films and other allied phenomena might well feel tempted to believe that such periodical precipitates are to be regarded as wave-patterns. Indeed, several workers in the field appear to have felt that the analogy between the Liesegang phenomenon and a wave-effect is not merely superficial, and have sought for more positive evidence in support of it. Leduc and others, for example, claimed that Huygens' well known optical principle gives an explanation of the form of the rings observed when a precipitating agent diffuses through a narrow aperture in an obstacle cutting across the film. More recently, some Russian workers [Nikiforov and Kharmonenko, *Acta Physicochemica URSS*, **8**, 95 (1938)] have gone further and suggested that the periodic precipitation itself is to be explained in terms of the de Broglie waves associated with the movement of the precipitating agent, and claim to have been able to measure the 'refractive index' of such waves in passing across a boundary separating regions of different concentration of the gelatine.

The distinguishing character of a true wave is the existence of phase relationships, and connected therewith, the possibility of interference effects. In the course of some studies made by us, we have observed some phenomena with Liesegang precipitates which are unmistakably in the nature of interference effects. To make the significance of our results clear, it is necessary to make here a remark regarding the structure of an interference field. When two wave trains crossing at an angle are superposed, we have, of course, regions of maximum and minimum disturbance. If the minimum disturbance is actually zero along a given line, the wave fronts on either side of it show a difference of phase of half a wave. This is an exceedingly characteristic interference effect and can easily be recognized in ripple photographs.

When on a gelatine film containing a very small concentration of sodium chloride a drop of silver nitrate is placed, the Liesegang pattern consisting of thousands of closely spaced rings of silver chloride precipitate may be observed. On an examination of the precipitate, it is often seen that the patterns are not of uniform intensity everywhere, but show lines of minimum and maximum disturbance, and the effects observed are closely analogous in some cases to beats, and in other cases to interferences of the individual waves. In the latter case, the difference of phase of half a wavelength on either side of a line of zero disturbance is invariably to be observed.

Even more striking are the interferences which we have observed in suitable circumstances with silver chromate rings in gelatine. In this case, the pattern really consists of a great number of fine rings, the intensity of which varies in such manner that they form a succession of widely spaced groups. Not only the individual waves, but also the groups, show interference phenomena with the characteristic discontinuity of phase of the group on either side of a line of zero disturbance. The accompanying photograph* shows this in a striking way.

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Indian Institute of Science, Bangalore
July 13

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*See figure 9 in p. 253.

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# Haidinger's rings in curved plates

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The paper describes and illustrates the Haidinger interference patterns observed in curved plates of uniform thickness but of arbitrary form, and discusses their theory. Using an illuminated screen as light source and a limited viewing aperture, it is possible to observe such patterns with curved plates, their configuration depending upon the form of the plate as well as the position of the aperture with reference to the centers of curvature of the surface of the plate. It is pointed out that the interference pattern for a curved plate is geometrically similar in form to the image of the standard Haidinger pattern of circular rings as seen viewed by reflection at the appropriate distance from the curved surface of the shell.

## 1. Introduction

The "interference curves of equal inclination" in transparent plates first observed by Haidinger are of great importance in physical optics. For their practical application in the field of spectroscopy, the plane-parallelism of the plate exhibiting these interferences is so essential that great stress is naturally laid on this requirement, and the tendency has therefore been to regard such plane-parallelism as a *sine-qua-non* for the observation of Haidinger's phenomenon. Actually, however, this is far from being the case, and by suitable arrangements the interference curves of equal inclination may be observed in a variety of other circumstances. In particular, these figures are readily to be seen in the transmission or reflection of light at the surface of curved transparent shells of uniform thickness but of arbitrary shape. They then assume a variety of forms depending on the circumstances of observation. From the standpoint of general optics, this enlarged field for the study of Haidinger's phenomenon is not without interest, and it is proposed in this communication to place on record some remarks and observations dealing with it.

## 2. Methods of observation

To observe or photograph the Haidinger rings by reflection, a convenient arrangement is to use an opaque diffusing screen which contains a small aperture

at its center. This is placed behind the plate under observation and is illuminated on its front by the light of a mercury lamp, while the eye or the camera is directed from the rear through the aperture and the plate towards a dark background. The arrangement permits of the observing aperture being brought very close to the plate, which is of great advantage if the latter is uniform over only restricted areas on its surface. For instance, interferences exhibiting hundreds of concentric rings may easily be observed with this arrangement in suitable areas on any ordinary glass plate such as a cleaned photographic negative. The advantage of restricting the aperture of observation and of bringing it close to the surface is equally evident in the observation of the transmitted system of rings. For the latter purpose, all that is necessary is to view an illuminated screen through the plate held close to the eye. As the plate is moved about, the complete system of rings suddenly springs into view when areas of sufficiently uniform thickness come in front of the eye, while with other parts of the plate nothing is to be seen on the viewing screen.

It is worthy of remark that the Haidinger interferences in a transparent plate may also be exhibited, either in reflection or in transmission, using a point source of light. For this purpose, we merely reverse the roles played by the aperture and the diffusing screen in the arrangements described above. A small aperture in an opaque screen closely backed by a mercury lamp is the origin of a divergent pencil of monochromatic light in the path of which the transparent plate is held. The reflected system of rings is then formed on the rear surface of the screen containing the illuminating aperture. The transmitted system of rings may similarly be observed when the diverging beam after passage through the plate falls on a viewing screen. Though these arrangements are in certain respects less satisfactory than those in which an extended light source is employed, they are nevertheless of interest as showing that such a source is not indispensible for the observation of Haidinger's rings. In what follows, however, it is to be understood that we are referring to the method of observation first described above, unless otherwise explicitly stated.

## 3. Some geometrical considerations

Using an extended light source and a limited aperture of observation, the interference lines of equal inclination may readily be demonstrated with curved plates of any shape. If such plate be of uniform thickness, it is not necessary that the observer's eye should be placed close to its surface. Indeed, a special point of interest in the case is that the configuration of the interference curves varies with the position of the observer's eye. With the eye sufficiently close to the plate, the rings have the same form as for a plane-parallel plate. As the eye recedes from the plate, the curvature of the surface modifies the inclination of the normals to the reflected or transmitted rays which reach the eye; when the eye is sufficiently

remote from the plate, its curvature determines such inclinations almost exclusively. The interference curves of equal inclination therefore progressively alter in form and finally reach a limiting configuration which is determined by the shape of the curved shell and which would, in general, be quite different from the familiar arrangement of concentric circular rings.

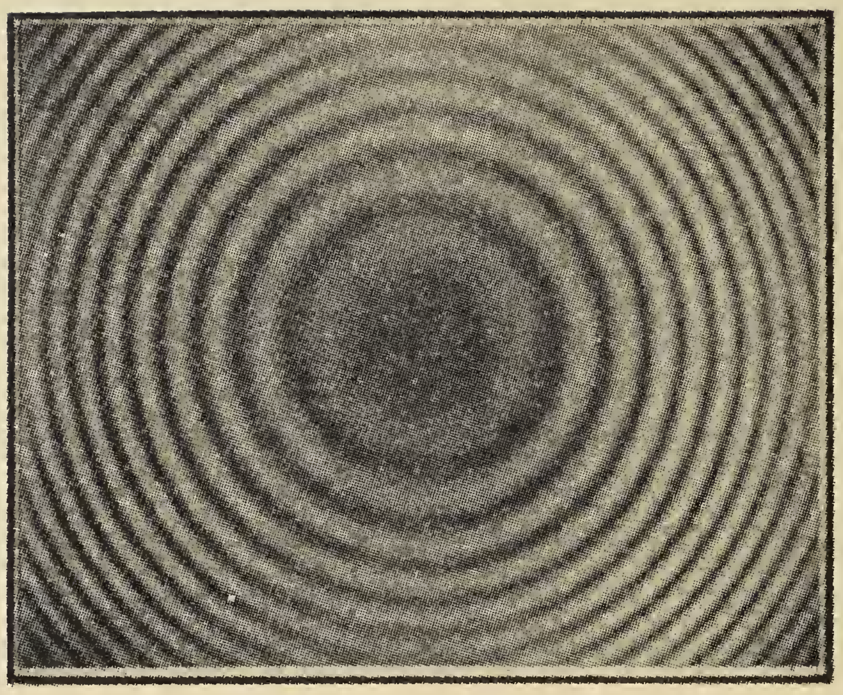
A closer consideration of the geometric problem which here arises indicates that the configuration of the interferences would depend greatly on whether the principal curvatures of the plate are turned towards the observer or away from him, as also upon the situation of the eye with respect to the centers of such curvature. To appreciate the reason for this, we may consider the case of a curved shell in the form of a spherical bowl with its concavity towards the observer. When the eye is exactly at the center of the bowl, all the rays reaching it will be normal to the surface, while if it be situated either nearer or further from the shell, the inclinations of the rays to the normals would increase from the vertex outwards. For this position of the eye, therefore, the circles on the sphere representing varying inclinations of the normal to the light rays would be very widely spaced and they would also shift rapidly as the eye is moved. For the same reason, in a case in which only one of the principal curvatures of the shell faces the observer, both the form and the spacing of the curves of equal inclination would alter rapidly with the position of the eye when the latter is near the center of curvature. On the other hand, if both the principal curvatures are turned away from the observer, no such singularity would be noticed, but the configuration of the interference curves would alter continuously as the eye recedes from the surface of the shell. In general, the form of the interference curves would depart the more widely from circular symmetry the greater the difference is between the principal curvatures at the vertex of the shell opposite to the observer.

## 4. Observations with curved plates

The theoretical considerations set out above are readily capable of experimental test. That a plane-parallel plate is not necessary for the observation of Haidinger's rings may be readily demonstrated with a spherical bulb of glass blown rather thin but uniform and examined by the method already described. The material most suitable, however, for a detailed study of the interference phenomena of curved plates is mica, which was indeed the substance with which Haidinger discovered his rings.[1] Mica is readily bent and held in a cylindrical form, and the various points discussed above may readily be examined with it. When such a cylinder is held with its convexity towards the observer and its axis horizontal

[1] See also Rayleigh, *Philos. Mag.* **12**, 489 (1906) and Chinmayanandam, *Proc. R. Soc. London* **A95**, 176 (1918).

(1)



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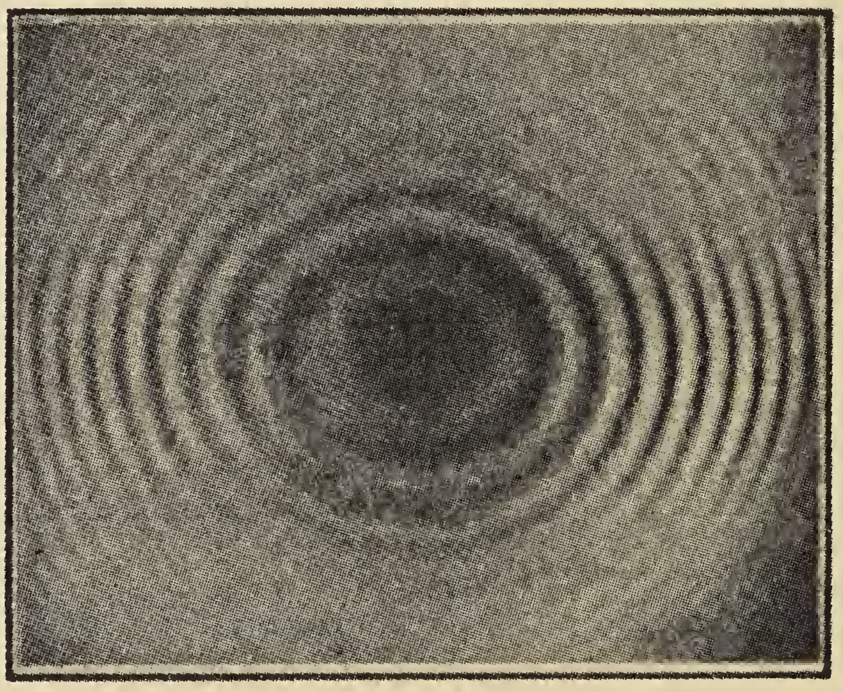

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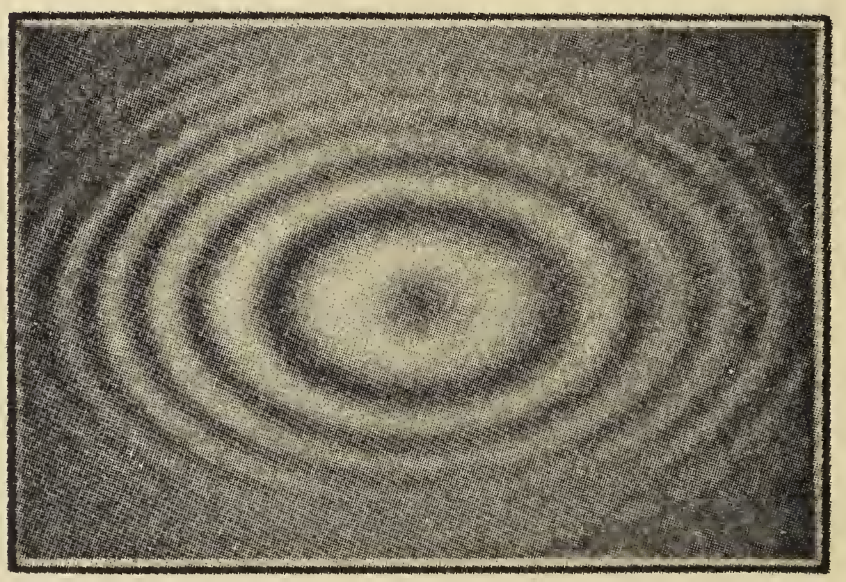

**Figures 1–3.** Haidinger's rings: (**1**) in a plane sheet of mica; (**2**) in a convex cylindrical sheet of mica held near the camera lens; (**3**) same as (**2**), but with sheet farther from camera lens.

and is gradually moved away from the eye, the Haidinger curves progressively change in form from a set of concentric circular rings, to concentric elliptic rings with the major axes horizontal and finally to a set of straight lines parallel to the generating lines of the cylinder.

The accompanying reproductions illustrate the foregoing remarks. Figure 1 is a picture of the Haidinger's rings in a plane sheet of mica. Figure 2 is a picture of the rings in a convex cylindrical sheet held fairly near the lens of the camera. Figure 3 is a picture taken with the sheet moved further away from the lens, and it will be seen by comparison with figure 2 that the ellipticity of the rings is thereby increased. All the three photographs are of the reflected system of rings taken by the method described earlier in this communication.

When the cylinder of mica has its concavity towards the observer, the axis being horizontal, the sequence of changes with increasing distance of the eye from the mica is quite different. We have at first concentric circular rings which change to concentric elliptical rings with the *major axes vertical*, and after passage of the eye through the center of curvature, to circular rings once again, then to elliptical rings with the *major axes horizontal* and finally to straight lines parallel to the generating lines of the cylinder. In the passage through the center of curvature, the configuration of the rings is very varied and depends on the exact form of the mica. When this has the shape of a hyperbolic cylinder, the pattern is observed to consist of two sets of rings joined by oval curves similar to the well known figures exhibited by a biaxial crystal in the polarization microscope.

## 5. Relation to geometry of image formation

It has already been mentioned that the Haidinger pattern of a plate may be observed with a point source of light and a viewing screen to receive the light transmitted or reflected by the plate. A remarkable fact observed with curved plates in this connection is that *the transmitted and reflected systems of rings behave in a wholly different manner.* The transmitted ring-system seen in this way alters in form with the shape of the plate and its distance from the light-source in the same manner as in the usual method of observation. On the other hand, the reflected interferences remain of invariable form as a system of concentric circular rings, whatever may be the shape of the plate or its position. In both cases, however, the linear dimensions of the pattern seen on the screen increase with its distance from the source of light. The explanation of these facts becomes clear when we trace the formation of the interference curves, by following the rays delineating them from the source to the screen. The curves of equal inclination as traced on the surface of the plate would be of identical form in the two cases. In the case of the reflected system, however, the approach of the curve of a particular order of interference towards the vertex of the shell produced by the curvature of the latter is exactly set off by the recession from the vertex produced by the same

curvature in the act of reflection of the ray before it reaches the screen. The effect of the curvature of the shell on the reflected system is thus to alter the distribution of intensity of light on the screen without producing any change in the form or position of the interferences. On the other hand, in the transmitted system, the interference curves seen on the screen are the direct projections of the curves as traced on the surface of the plate and hence exhibit forms depending on the shape of the plate and its distance from the source.

The foregoing considerations indicate that the Haidinger pattern of a curved plate may be regarded as the geometric image of a pattern of the standard type consisting of concentric circular rings as seen by reflection at the curved surface of the plate. The facts as already described then become easier to grasp in the light of the well known theory of image formation in geometrical optics. That this idea can form the basis for an exact theory of the Haidinger patterns of curved plates can be shown in the following way. A small white circular disk with a central aperture in it is illuminated by monochromatic light and held close to the eye. It is then viewed through the same aperture by reflection at the surface of a curved plate of mica. It is then observed that the configuration of the circular edge of the disk runs exactly parallel to the course of the interference rings seen projected on its surface. Indeed, the changes in the form of the reflected image of the disk in the most varied circumstances describe also the changes in the configuration of the interference rings, thus establishing the thesis stated.

## 6. Position of best focus

An interesting question which is closely related to the topic discussed above relates to the position in space where the Haidinger pattern of a curved plate is seen most clearly in focus. As the visibility of the pattern essentially depends on the limitation of the aperture of observation, and as this aperture is necessarily finite, a certain lack of definition is inevitable. This may however be minimized by suitable accommodation of the observer's eye or focusing of the camera employed to photograph the pattern. The focus would evidently be best when the angle between the normals and the corresponding rays reaching a given point in the image vary as little as possible. When the curvature is small or when the aperture is close to the shell, we should therefore focus for infinity in the same way as for a plane-parallel plate. For greater curvatures of the shell or with increasing distance of the aperture from it, the variation of the angle referred to over the surface of the shell becomes more rapid. We may reduce this variation by altering the focus so as to approach the surface of the shell, but this on the other hand would introduce a variation of the inclination of the rays from the same area of the shell which come to a focus at an image point. The best focus is determined by these opposing considerations which indicate that as the form of the interference curves alters progressively from the standard configuration of concentric circular

rings to one determined completely by the form and curvature of the shell itself, the focus of the pattern also shifts steadily from infinity towards the surface of the shell. This general statement must, however, be qualified in certain respects. If the principal curvatures of the shell differ greatly in magnitude or in sign or in both, it is clear that the focusing would show a certain astigmatism, features in the pattern running in one direction being more clearly seen than features running in a perpendicular direction. It is also to be remarked that the focusing of the pattern as a whole would be difficult when the observing aperture is placed at or near a center of curvature of the surface.

From what has already been stated, it is evident that the interference curves of equal inclination for a strongly curved shell at oblique incidences would be best seen when the eye is focused on the surface of the shell itself. A limitation in the area of the light source employed to the extent of practically restricting it to a line-source running parallel to the interferences should then be of marked advantage. For, such a restriction would result in the angle of incidence of the light being better defined and therefore in a better definition of the fringes, since these depend for their visibility on the variation of the angle of incidence over the surface of the shell. In these respects there is a certain similarity with the behavior of the interference curves in plates of varying thickness, though the behavior in other respects is wholly different.

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# Haidinger's rings in soap bubbles

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## 1. Introduction

The colours of soap films are the classic illustration of the interferences of light in thin plates for which the theory was first given by Thomas Young. To exhibit these colours most advantageously, it is usual to stretch a film of soap solution on a wire frame and allow it to drain by placing it in a vertical plane. Horizontal bands of colour then develop on the film which are best seen in the light reflected at its surface. These bands of colour result from the varying thickness of the film and become more conspicuous as it grows thinner. The colours seen depend both on the thickness of the film and the angle at which the light falls on it before being reflected and reaching the eye of the observer. This angle of incidence does not differ appreciably for different parts of the surface of a plane film, provided that the eye of the observer is sufficiently distant. The circumstances are, however, quite different in regard to the colours exhibited by a spherical soap bubble. In this case, it is obvious that the surface of the bubble presents itself to the eye at all angles varying from a normal to a tangential aspect. The colours of a spherical soap bubble necessarily therefore exhibit the effect of the varying obliquity of incidence of the light reflected at its surface. Indeed, in the ideal case of a spherical bubble of completely uniform thickness, the variations of colour exhibited by the surface would be determined exclusively by such variations of obliquity. The colour pattern in this ideal case would consist of a set of circular rings localised on the surface of the bubble and arranged concentrically around that diameter of the sphere which when produced meets the eye of the observer. It is the purpose of this paper to show that this effect can actually be observed and to emphasise that it is essentially similar to the well known rings of Haidinger exhibited by a plane-parallel plate in monochromatic light. Indeed, as will be shown in this paper, there is a simple geometric relation between the configuration of the interference curves of equal inclination as exhibited respectively by a spherical film and a plane-parallel one of the same thickness. The position of the observer's eye enters into this geometric relation, and when the eye is sufficiently close to the surface of the plate, the two cases become indistinguishable.

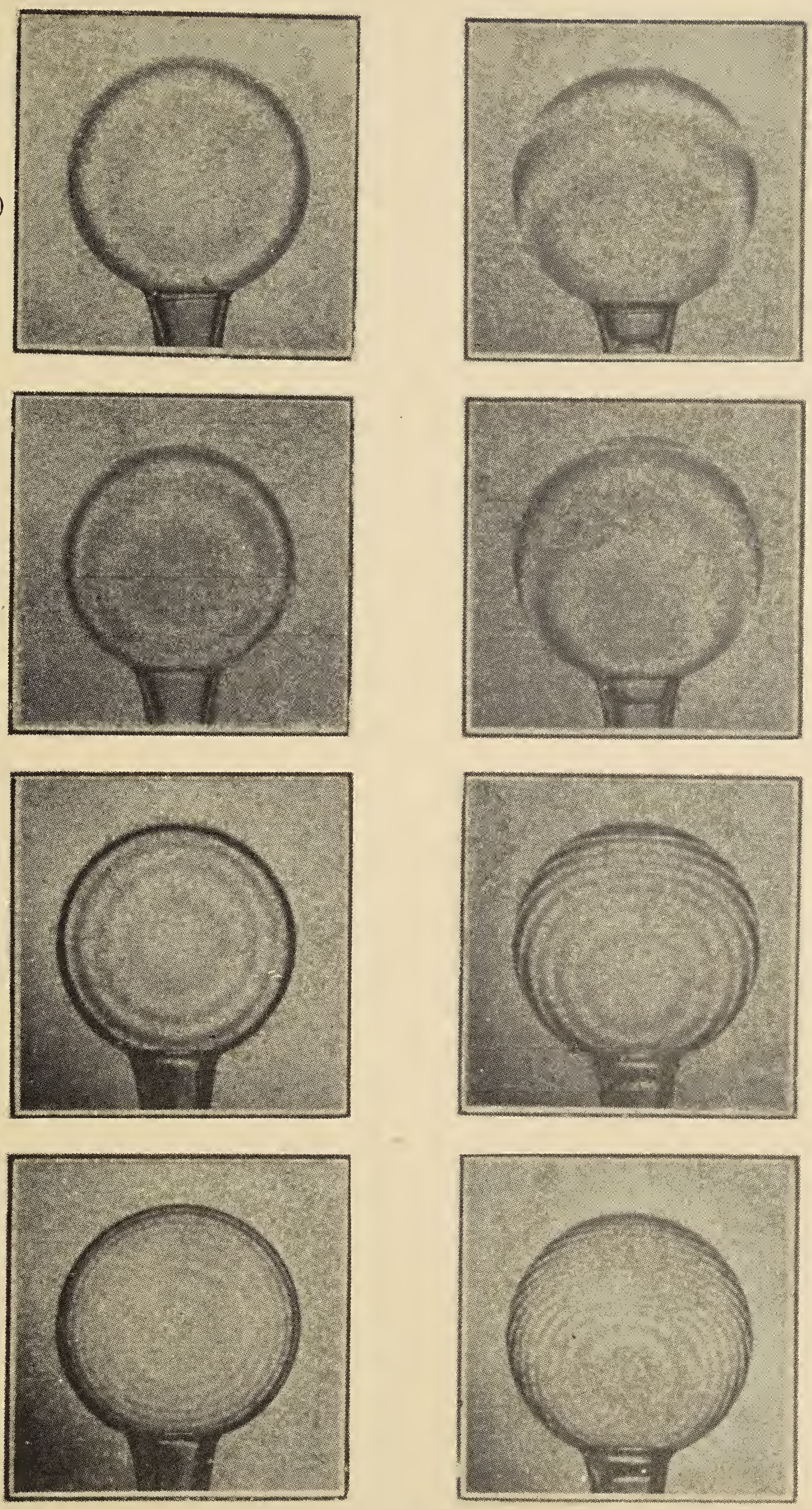



**Plate I. Figures a–h.** Interference figures in soap bubbles by transmitted light.

(a) (e)

(b) (f)

(c) (g)

(d) (h)

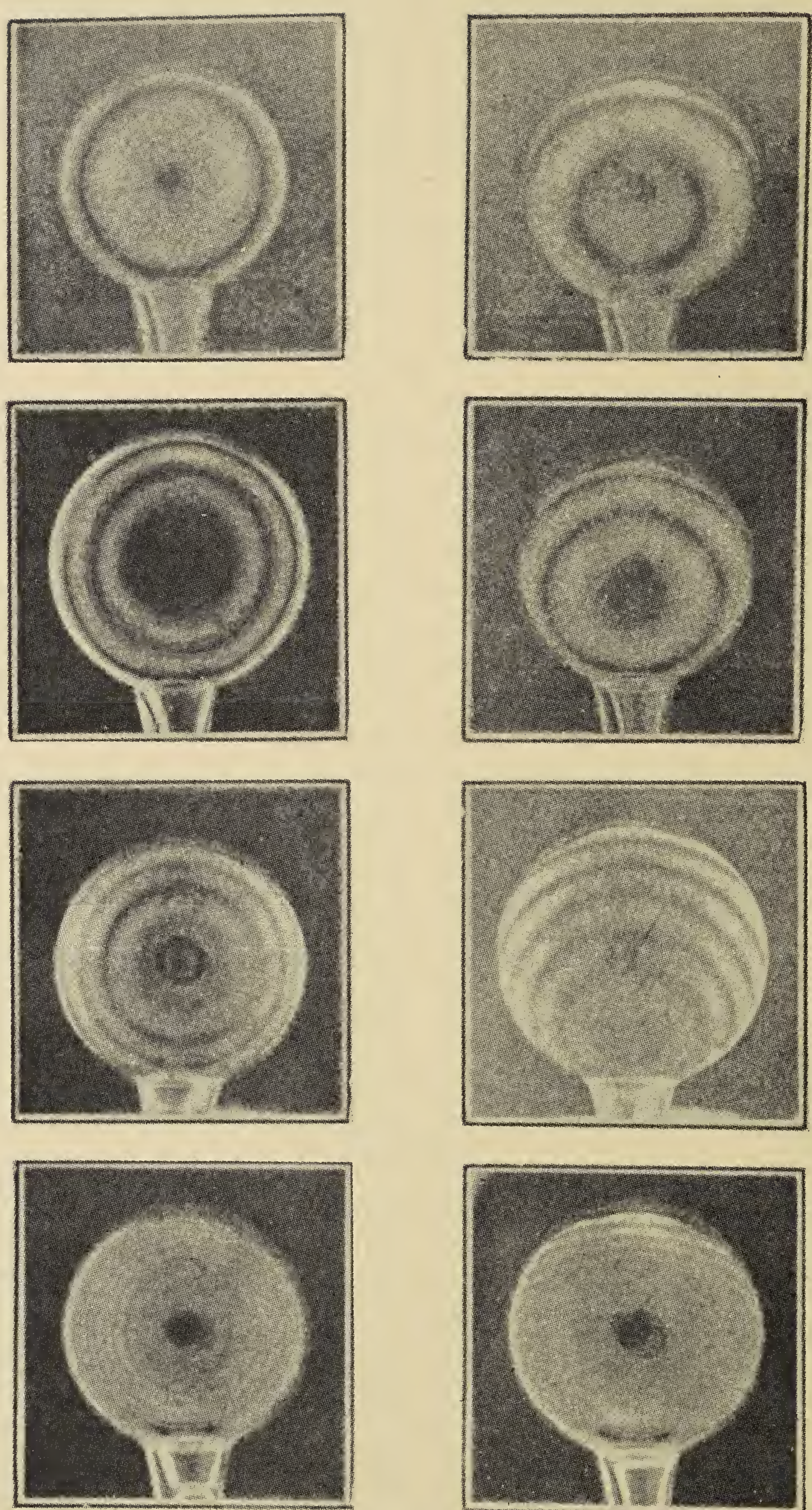


**Plate II. Figures a–h.** Interference figures in soap bubbles by reflected light.

## 2. Methods of observation

The interference figures of soap bubbles in transmitted light, though not so striking as those exhibited by reflection, are in a sense the more easily observed. All that is necessary is to view the bubble against an illuminated background. Using monochromatic light (a mercury lamp with a green ray filter), the interference curves may be observed and photographed with very brief exposures, and the changes which occur with time may therefore be readily followed and recorded. To observe the complete interference figures of a soap bubble as seen by reflection, it is necessary to place the bubble inside an illuminated enclosure. This may conveniently be done by surrounding the bubble by a cover of translucent material, e.g., paper, and lighting up the latter from outside. It is necessary, of course, to provide the enclosure with two apertures, one to serve as a dark background for the bubble, and the other as an opening through which it may be observed or photographed. Plates I and II reproduce photographs obtained in this way. Plate I illustrates soap bubbles as seen in transmitted light and plate II as observed by reflection. Figures (a) to (d) in both plates illustrate the Haidinger rings exhibited by soap bubbles of uniform thickness (different in each case); while figures (e) to (h) in the two plates illustrate the interference curves of non-uniform bubbles. The form of the interferences in the latter case is evidently determined jointly by variations of thickness and of obliquity.

## 3. Behaviour of spherical soap-films

Optical observation as described above enables us readily to follow both the configuration of thickness in a spherical bubble when it is first blown and also the changes that occur in it with lapse of time. It is quite possible, using suitable technique, to blow bubbles of almost ideal perfection in regard to uniformity of thickness. This is largely a question of avoiding any superfluity of liquid in the first instance and of adjusting the rate of blowing to the quantity of liquid used. The use of a capillary tube with its ends ground flat appears to be the most convenient method of securing both these aims, and bubbles blown with its aid exhibit, in the first instance, a perfect system of Haidinger rings. Soon after the bubble is blown, however, the natural flow of the liquid downwards within the film causes the lower levels to gain in thickness at the expense of the upper. This non-uniformity of thickness reveals itself immediately in interference figures of which the form is no longer independent of the direction in which the bubble is observed. When the bubble is seen horizontally, the ring-system ceases to be centred round the line joining the eye and the bubble and gradually moves downwards making the pattern unsymmetrical. Successive rings as they go down become incomplete arcs meeting the edge of the bubble. Finally, the innermost

ring touches the bottom, and the interference curves above gradually straighten out and stretch more or less horizontally across the bubble, reaching and cutting its periphery. A distinct curvature, however, remains as an indication of the effect of varying obliquity on their form. The complete sequence of changes observed and its progress with lapse of time naturally depend upon the initial thickness and size of the bubble and the viscosity of the soap solution used. The changes are naturally the more rapid, the greater the thickness of the bubble in the first instance and the less the viscosity of the solution used.

Any initial excess of the liquid used in blowing a bubble reveals itself in a striking non-uniformity of its thickness. The contour lines of constant thickness, are of course, horizontal circles, and these congregate densely in the lower levels of the bubble; their position is indicated by the interference lines of equal thickness which appear crowded together in this region and indeed seem to bear no relation to the more widely distributed curves seen in the upper parts of the bubble. Viewed from a point vertically above or below it, such a non-uniform bubble presents a symmetrical aspect, the interference curves being horizontal circles. These, however, are crowded together in the lower parts of the bubble and very wide apart in the upper levels.

It may be pointed out that as the sequence of changes described above for a spherical soap bubble may be readily photographed and measured, it offers the possibility of a quantitative test of the theory that viscous flow of liquid within the film is the cause of its thinning. According to Willard Gibbs (*Scientific Papers*, p. 307, *et seq.*), the changes occurring within a soap-film are due not so much to the action of gravity as to the suction exercised by the ring of liquid formed along the line where the film meets its solid supports. Indeed it appears that in the case of a plane film this suction is the agent principally responsible for its thinning down. It would, therefore, seem to be important to investigate whether this is the case also for a spherical bubble when the perimeter of its support is reduced to the absolute minimum necessary.

## 4. The crossed air-jets method

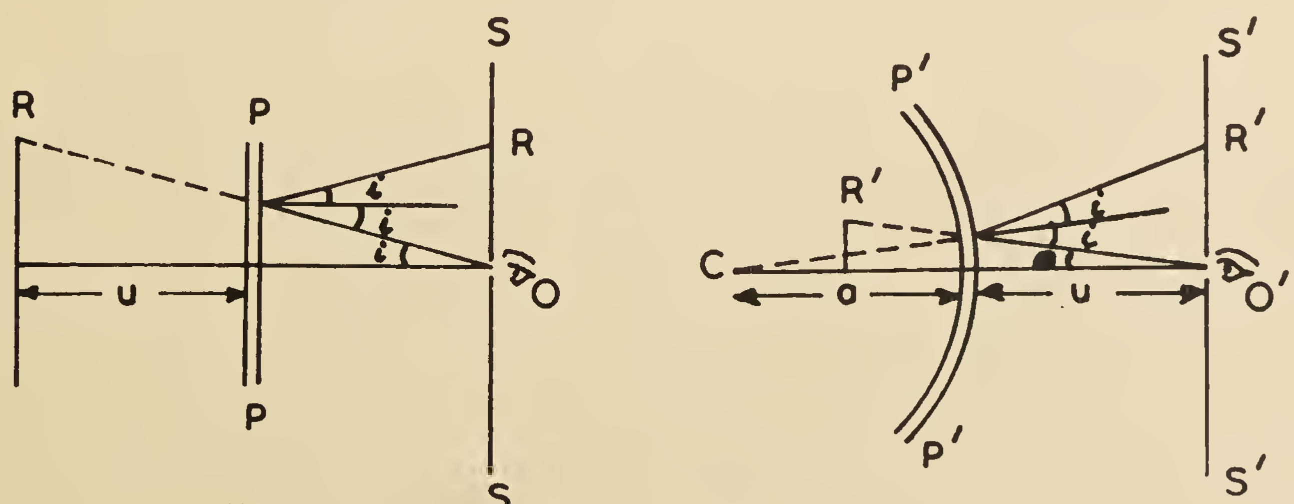
A simple and efficacious technique has been devised by the authors which enables a soap bubble to be reduced to uniform thickness even when it is initially non-uniform and by which the uniformity of thickness thus obtained can be maintained. A soap bubble is blown and allowed to sit on the circular end of a vertical glass tube. Two *very gentle* currents of air are blown upwards from two glass tubes placed below the level of the support of the bubble and displaced from it in two directions exactly a right angle apart. The effect of the air currents impinging at an angle on the surface of the bubble is to set up an upward drift of the liquid within the film in two directions 90° apart. The resulting circulation soon results in the thickness of the film becoming the same everywhere. The

continued flow of the two air currents with just sufficient force to counteract the tendency of downward movement ensures the thickness of the bubble remaining uniform in spite of its steady thinning by evaporation. By using a mechanical blower with a control stop-cock, the currents of air can be made sufficiently weak to accomplish this purpose without causing the bubble to oscillate on its support and without setting up any visible vortex motion on its surface. The success of the device is most clearly exhibited by its optical results. The interference pattern takes the form of perfect circles and becomes identical from whatever aspect the bubble may be viewed. The adoption of this device has been found very useful in obtaining photographs of the Haidinger rings, especially those seen in reflection, for which somewhat longer exposures are necessary than in the case of the transmitted system of rings. The photographs reproduced in the two plates illustrating the paper were actually obtained in this way.

## 5. Optical characters of the interferences

As is to be expected, the transmitted system of rings exhibit less striking contrasts between the maxima and minima of illumination than the interferences observed by reflection. Towards the margin of the bubble, however, the minima of illumination in the rings observed in transmission are seen to become much more conspicuous (plate I, and on the extreme edge of the bubble they become broad and very dark, the maxima, at the same time, becoming bright and narrow lines cutting through them. The reverse is seen to be the case in the reflected system of rings, the minima becoming sharp dark lines cutting across the broad and bright maxima. These effects, of course, are due to the effect of multiple reflections within the film becoming sensible at oblique incidences.

Reference may appropriately be made here to the photographs of soap bubbles between crossed nicols observed in monochromatic light by transmission




(1) (2)

**Figures 1 and 2**

obtained by one of us and published some years ago in the *Indian J. Phys.* (1929, **4** plate X, facing page 390). Striking differences will be noticed between the effects there reproduced and the photographs of the Haidinger rings in transmitted light which illustrate the present paper. Apart from the appearance of a black cross, it is noticed the rings as seen by transmission between crossed nicols, resemble rather the rings as seen in reflection without them and illustrated in the present paper. The explanation for this will be clear from the theoretical discussion of the results by Prof. K S Krishnan in the paper appearing in the issue of the *Indian J. Phys.* quoted above; indeed Krishnan definitely drew attention to the feature just mentioned.

## 6. Angular diameters of the rings

We may now refer to the point already indicated in the introduction, namely that the interference figures exhibited by a spherical film of uniform thickness are essentially similar to the well known rings of Haidinger exhibited by a plane-parallel plate. This will be clear on a consideration of the geometric relationship between the configuration of the rings in the two cases.

Figures 1 and 2 represent respectively the arrangements (similar in both cases) for observing the interference figures in a plane-parallel plate PP and a curved plate P′P′ of the same thickness. An opaque diffusing screen SS or S′S′ which is illuminated on the side facing PP or P′P′ serves as the source of light. The eye of the observer is placed behind an aperture O or O′ in the screen and views the illuminated surface of the latter as seen reflected at the faces of the plate PP or P′P′. A ray of light from a point R or R′ on the screen reaches the eye after reflection at an angle of incidence $i$ on the surface of the plate PP or P′P′ which is the same in both cases. It will however be seen after such reflection at different angles to the axial ray in the two cases, namely, $i$ in one case, and $\theta$ in the other. If $a$ is the radius of curvature of the plate and $u$ is the distance between it and the observer's eye, it can be deduced from the diagram that

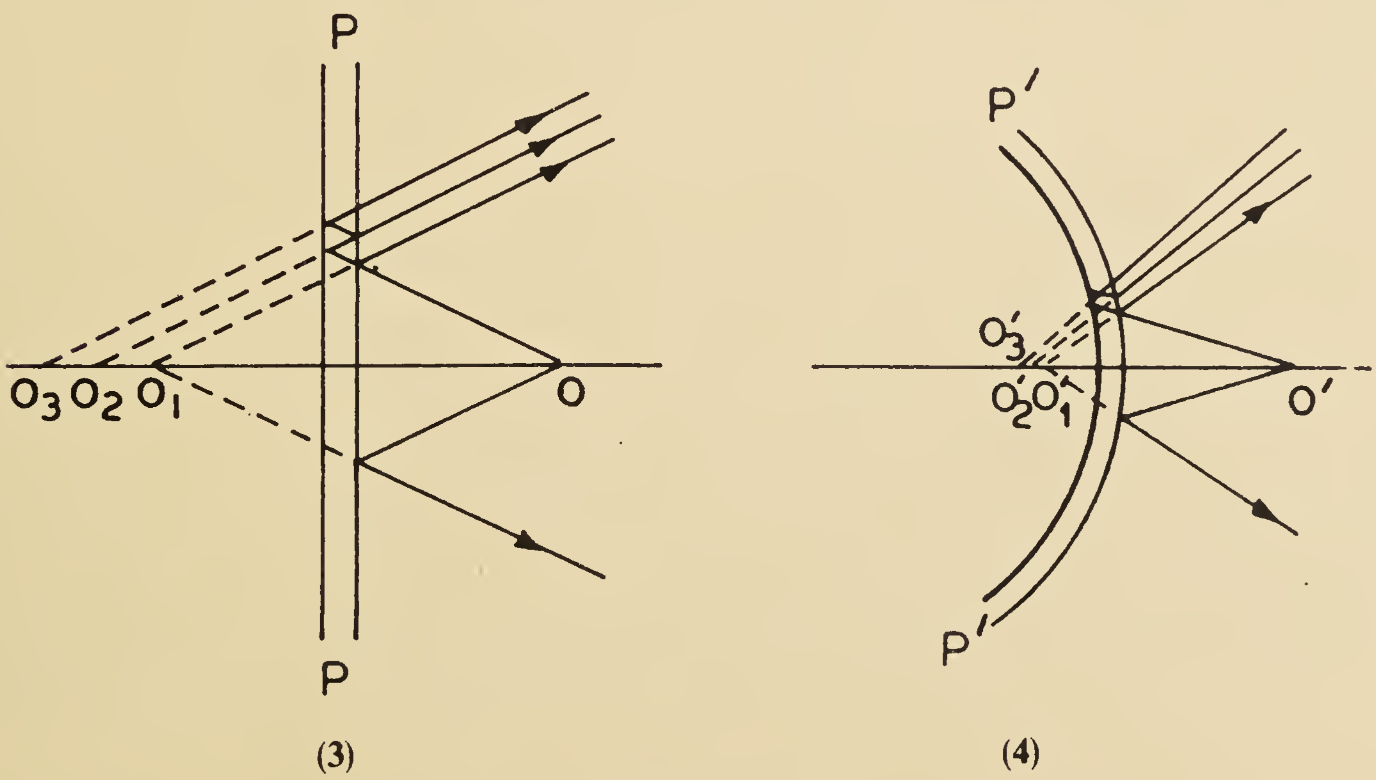
$$\theta/i = a/(a+u).$$

In other words, the system of interference rings produced by the curved plate is geometrically similar to that for the plane-parallel plate, but appears reduced in size; the ratio of the angular diameters of the rings in the two cases is the same as the angular diameters of an object at the position of the observer's eye as seen by reflection in the flat and curved plate respectively bear to each other. If the plate is concave instead of being convex towards the eye, we have merely to consider $u$ and $a$ in the formula as of opposite sign, but the relation as stated above remains unaltered. It will be noticed that when $u$ is sufficiently small in relation to $a$, in other words, when the eye is sufficiently near to them, a curved plate and a plane one exhibit interference figures which are identical. On the other hand when the

distance between the eye and the plate is sufficiently great, the angular diameters of the rings for a curved plate are inversely proportional to such distance, in other words, the rings appear as of fixed linear dimensions relatively to the plate. In such a case, also, it makes no difference whether the plate is convex or concave towards the eye of the observer. The Haidinger rings on the front and rear surfaces of a spherical soap bubble appear, therefore, coincident to an observer at a sufficient distance from it. But this would not be the case when the eye is placed near the bubble.

## 7. Localisation of the rings

As is well known, the Haidinger rings due to a plane-parallel plate may be seen with an extended source of light through a telescope of any aperture focussed for infinity, whereas, as we have seen, the interference figures of a curved plate vary in their size and location with the position of the observer's eye, and accordingly demand a limited aperture of observation, though an extended source of light may be used. It appears, therefore, worth-while to emphasise the essential similarity between the two cases by considering an arrangement in which the interference rings instead of being virtual are received on a screen. This may be done by merely reversing the roles of the aperture and the screen in figures 1 and 2 above. The aperture O is illuminated and the light diverging from it falls on the plate PP or P′P′ and after reflection at its surfaces is received on the screen SS or S′S′ which becomes the surface on which the interference pattern is actually formed and observed. It will be obvious from figures 1 and 2 that the interference figures of the plane and curved plates seen on the screen would then be completely




**Figures 3 and 4**

identical. Further, as the position of the screen SS or S′S′ is quite arbitrary, it is clear that the Haidinger rings are not localised at any particular position in the field but are observable everywhere to the right of the plate PP or P′P′. The reason for this will be seen better from figures 3 and 4 in which O or O′ is a point source of light on one side of the plate PP or P′P′ and $O_1$, $O_2$, $O_3$, etc., or $O'_1$, $O'_2$, $O'_3$ are its virtual images formed by successive reflections at the two surfaces. As these sources are coherent amongst themselves, they must give rise to a system of interference rings symmetric about the line $O\ O_1\ O_2 \cdots$ or $O'\ O'_1\ O'_2 \cdots$ which extends from the surface of the plate to an infinite distance on the right side. The special feature which distinguishes the case of the plane-parallel plate from that of the curved one is the circumstance arising from purely geometrical considerations that in the former case, any movement of the source O either along the line $O\ O_1$ or perpendicular to it leaves the positions of the interference maxima and minima *at infinity* undisturbed.

## Summary

By suitable arrangements which are described, it is possible to obtain soap bubbles which are perfectly uniform in thickness and to maintain them in that state. Such bubbles exhibit by transmission or reflection, interference figures consisting of concentric rings which are essentially of the same physical nature as the Haidinger rings due to a plane-parallel plate, besides being geometrically similar to them in configuration. Sixteen photographs are reproduced showing the interference figures of soap bubbles of uniform thickness as also of the changes which occur in them when the films are allowed to drain. The question of the localisation of the interference figures of a curved plate is discussed.

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# Conical refraction in naphthalene crystals*

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## 1. Introduction

The phenomena of internal and external conical refraction in biaxial crystals predicted by Sir William Hamilton and observed by Humphrey Lloyd are amongst the most beautiful and striking effects arising in crystal optics. Following Lloyd's original experiments, these phenomena are usually exhibited with aragonite, a polished plate of this crystal suitably mounted between apertures and a viewing lens being employed for the purpose. The angles of internal and external conical refraction in aragonite are however small, ($\chi = 1°\,52'$ and $\psi = 1°\,42'$ respectively), and the use of other crystals, e.g., tartaric acid with $\chi = 3°\,54'$ and $\psi = 3°\,58'$, and of sulphur for which $\chi = 7°\,11'$ and $\psi = 7°\,33'$ has therefore been sometimes suggested. It may be pointed out, however, that organic crystals of the aromatic class are specially suitable for the purpose. Naphthalene, in particular, exhibits birefringence in an exceptional degree, having as its principal indices 1·525, 1·722 and 1·945 respectively for $\lambda = 5461$ Å and its conical angles ($\chi = 13°\,44'$ and $\psi = 13°\,51'$) are enormously larger than in aragonite. Large single crystals of naphthalene can easily be prepared (Hilmi-Benel, 1940; Nedungadi, 1941), and the substance is thus well suited for exhibiting the optical characters of biaxial crystals and especially conical refraction in a striking way.

It may be remarked that apart from the purely geometrical aspects of optical theory illustrated by the Hamilton-Lloyd experiments, certain physical aspects of the propagation of light in biaxial crystals arising in conical refraction are of great interest. One of these is the enormous concentration of energy which occurs along the axis of single-ray velocity within the crystal and along the axis of the cone of external refraction outside it (Raman 1921; Raman and Tamma 1922). The converse phenomenon associated with internal conical refraction has long been known and is referred to in the literature as the Poggendorff dark circle. This was explained by Voigt (1905) as due to the attenuation of the energy of the incident pencil which occurs in single-wave propagation within the crystal. Both of these phenomena are well shown by naphthalene and in such manner as to bring out clearly their theoretical significance.

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*A preliminary note on this subject appeared in *Nature* (*London*) of the 1st March 1941.

Conical refraction is often studied by viewing an illuminated pin-hole in focus through the crystal plate with a microscope or magnifying lens. It is generally supposed that what is then seen is internal conical refraction. That this is not quite correct was long ago pointed out (Raman, *loc. cit.*), but the matter was not then adequately discussed. Since the illuminated pin-hole is usually held close to the crystal and is backed by an extended source of light, the beam of light entering it is not restricted to any particular direction, and the effect observed is not therefore ascribable to internal conical refraction. Neither would it be altogether correct to ascribe it to external conical refraction; for, though with the pin-hole close to the crystal, a cone of light is incident on its first surface, no aperture limits the exit of the light from the second surface as in the Lloyd experiment. The focussed image of an illuminated pin-hole as seen in the microscope through the plate of crystal is formed by the entire bundle of rays issuing from the pin-hole and passing through the crystal and is thus a phenomenon distinct from either internal or external conical refraction, though related to both. As will be shown in this paper, the form of this image is determined by the curvature properties of the wave-surface in the crystal in the vicinity of the conical points. It is specially worthy of remark that when the microscope is focussed on the second surface of the crystal and not on the illuminated pin-hole, we see in the field of view an illuminated picture of the two sheets of the wave-surface, the conical point where they meet appearing as an intensely luminous centre, and the circle of contact with the tangent plane appearing as a dark ring (figure 7 in plate II).

## 2. Preparation of the specimen

A clear block of naphthalene can be grown by slow crystallization from a melt. Pure naphthalene redistilled several times is collected in a pyrex glass tube of about half-an-inch diameter, with its lower end drawn out tapering to a sharp point. The tube is suspended in a vertical furnace kept at a temperature from 10° to 15° C above the melting point of naphthalene and gradually lowered out of it automatically by clock-work mechanism. Crystallisation starts at the tapering end of the tube and develops upwards. By proper control of the temperature of the furnace and of the rate of lowering of the tube, it is possible to get clear flawless blocks of the single crystal of any desired length. It is removed from the container by momentarily heating the walls of the glass tube to a high temperature; the portion of the crystal in contact with it then melts and the crystal slips out.

As the crystal blocks prepared in this way do not possess any natural faces, advantage is taken of the fact that the axes of the optical and magnetic ellipsoids of the crystal roughly coincide to determine their orientation. The three magnetic axes of the crystal block may be determined by marking its preferred orientations in a strong magnetic field with different modes of suspension. The crystal block may be then cut with faces making any desired angle with these axes. To exhibit conical refraction, the naphthalene block should have its faces approximately normal to one of the primary optic axes, these being inclined at 42° to the acute

bisectrix of the angle between them. For mounting the cut crystal, a flat surface is first ground and then quickly pressed on to a microscope cover slip kept at a temperature of about 40° above the melting point of naphthalene. The crystal face melts and wets the glass plate and immediately cools, thus resolidifying the melted layer as part of the single crystal in addition to making good optical contact with the glass. The second face of the plate is then treated in the same way. An alternative method of mounting is to grind the surfaces of the block smooth on a ground glass plate and then to polish them by rubbing quickly on a soft cloth stretched over a glass plate and moistened with a drop or two of xylene. Thin microscope cover clips may then be stuck on the faces with Canada balsam. The mounted crystal may be conveniently fixed on a disc of aluminium having a central opening.

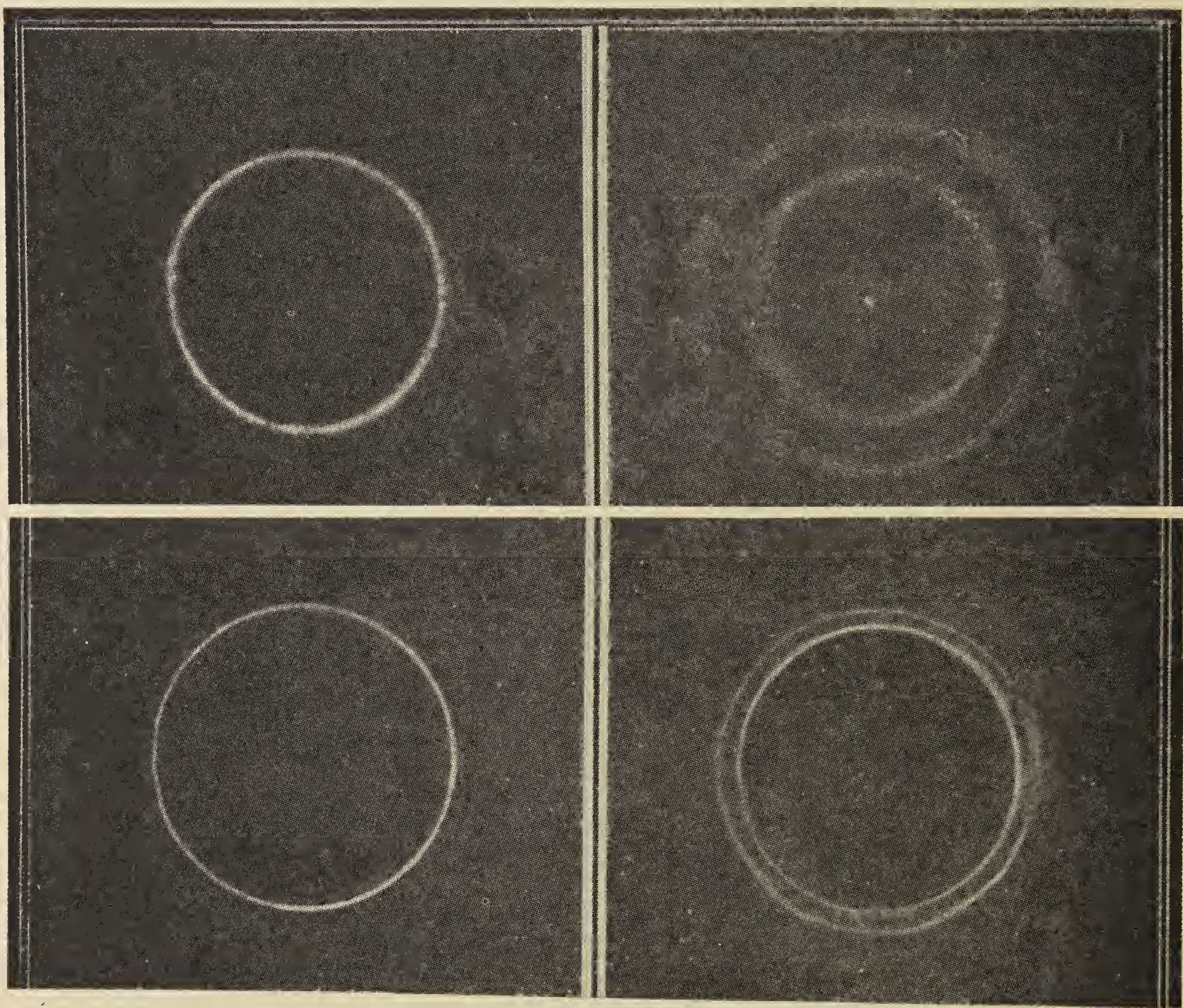
## 3. Method of observation and results

The angles of conical refraction in naphthalene are so large that with a fairly thick piece, the phenomena can be seen directly with the simplest possible arrangements. For a critical study of the effects, however, and especially for securing satisfactory photographs, it is convenient to use a microscope with a revolving and centering stage and a Federov universal stage attachment on which the crystal plate is placed so that it can be tilted and set with its optic axis accurately parallel to the axis of the microscope. A low-power objective and a high-power ocular should be employed so that, for the same effective magnification, the largest working distance between the upper surface of the crystal and the objective of the microscope can be secured. For observing conical refraction under the microscope, a relatively thin plate of naphthalene (say two to three millimetres thick) is quite suitable. It is easy with these arrangements to photograph the cone of external conical refraction and the cylinder of internal conical refraction outside the crystal in the manner of the Hamilton-Lloyd experiments. It is also possible to examine the relationships between these effects and the nature of the optical images obtained when a point-source of light is viewed through the crystal either in or out of focus.

Figures 1–4 in plate I and figures 5–8 in plate II reproduce a series of photographs obtained with the microscope camera attachment to illustrate the phenomena of conical refraction in naphthalene, the monochromatic green light $\lambda 5461$ Å of the mercury arc being employed to avoid all disturbances due to chromatic aberration or dispersion. Figure 1 shows the hollow cone of external conical refraction as seen above the crystal; to photograph this, both the upper and lower surfaces of the crystal are covered up except for small apertures situated at the ends of the axis of single-ray velocity, the lower aperture being illuminated by a convergent pencil of light. Figure 2 shows the cylinder of internal conical refraction seen outside the crystal when a parallel beam of light is incident in the direction of the optic axis on the lower face of the crystal; to observe this, the lower face is covered by a screen with a small aperture and the second face is left

uncovered. Figures 3 and 4 in plate I and figures 5, 6, 7 and 8 in plate II reproduce a consecutive series of photographs of a point source of light held close to the first surface and viewed through the crystal; figure 3 is the image seen in focus, while the other photographs in the series are ultra-focal images obtained when the microscope objective is gradually drawn away from the crystal. As already mentioned in the introduction, figure 7 is the ultra-focal image of the point source of light obtained when the microscope is focussed on the second surface of the crystal. Figure 8 is the ultra-focal image obtained when the microscope is still further drawn up. In obtaining this series of six pictures, the source of light was an extremely fine hole (1 $\mu$ in diameter) in an aluminium foil covering the lower surface on the crystal and illuminated by a convergent beam of light, while the second face of the crystal was left uncovered. The extreme sharpness of the circular ring seen in figure 3 is particularly significant. *It is*

(1) (2)



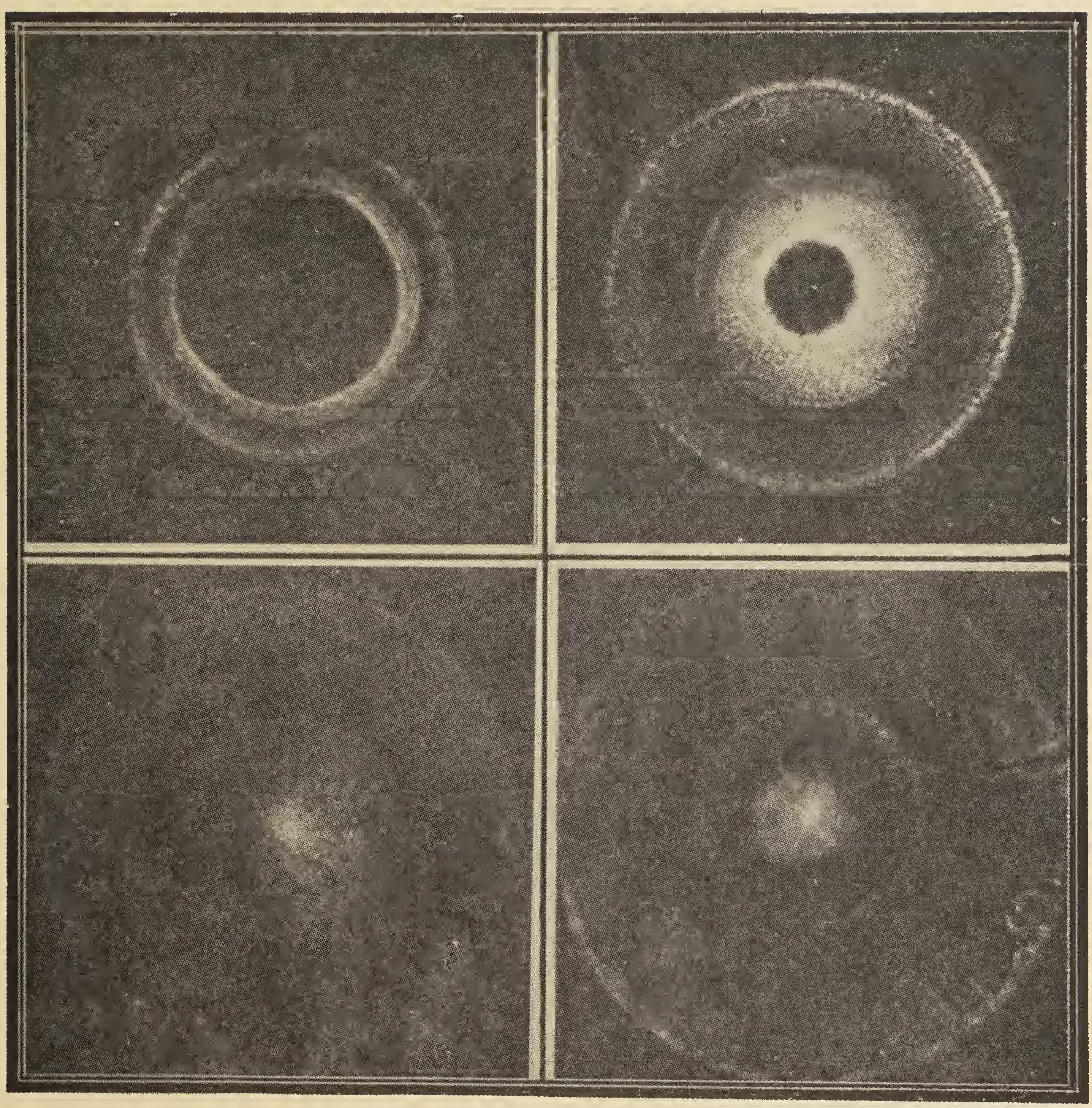
(3) (4)

**Figures 1–4.** Illustrating conical refraction in a naphthalene crystal.

**Plate I**

(5) (6)

(7) (8)



**Figures 5–8.** Illustrating conical refraction in a naphthalene crystal.

**Plate II**

*noteworthy also that the so-called Poggendorff dark circle does not appear in the focal image of the point source and develops only in the ultra-focal images.* The extremely bright point seen at the centre in figure 7 (as also in figure 8 and very feebly in figure 2) is a noteworthy feature. This bright point in the ultra-focal image coincides with the end of the axis of single-ray velocity meeting the second surface of the crystal. This is shown by the fact that the second aperture for observing the cone of external conical refraction above the crystal (figure 1) has to be placed exactly at the same point so as to admit the light passing through the crystal. It is evident from the series of pictures *that the axis of single-ray velocity and the conical point on the wave-surface are loci of intense concentration of energy*

*within the crystal, while the circle of contact where the wave-surface touches the tangent plane is a locus of vanishingly small energy.*

## 4. Image formation with a biaxial crystal

It is well known (Stokes 1877; Walker 1904) that the image of a point source of light seen through a crystalline plate exhibits astigmatism, being drawn out into a line perpendicular to the plane of principal curvature. For a biaxial crystal there are, in general, no fewer than four distinct positions of best focus determined by the orientation of the plate and by the principal radii of curvature of each of the two sheets of the wave-surface. In our present problem, we are concerned with the curvature of the wave-surface in the vicinity of the conical point and especially along the circle of contact with the tangent plane. At the conical point, one of the principal radii of curvature for each of the two sheets of the wave-surface vanishes, while the other two radii are

$$\rho_1 = b \text{ and } \rho_2 = (a^2 + c^2 - b^2)^{3/2}/ac.$$

At points along the circle of contact, one of the principal radii of curvature of each of the two sheets becomes infinite, while the other radius of curvature is

$$\rho = b\cdot(a^2 - r^2)(c^2 - r^2)/(a^2 - b^2)(c^2 - b^2),$$

$r$ being the length of the line joining the origin with any specified point on the circle of contact. At the two points where this circle cuts the circular and elliptic sections of the wave-surface respectively, the radii of curvature are

$$\rho_1' = b \text{ and } \rho_2' = b^3/a^2c^2.$$

In the case of naphthalene, $b^2$ and $ac$ are practically identical, as is readily seen from the numerical values of the principal refractive indices. As a consequence of this, also, the angles of internal and external conical refraction are practically identical. Hence, while one of the principal radii of curvature of the wave-surface is infinite along the circle of contact, the other radius of curvature is practically constant and equal to $b$ at all points on the circle and changes only slowly as we move away from the circle along the wave-surface either towards or away from the conical point. Accordingly, the astigmatism of the rays emerging from the plate results in an exceptionally simple form of the image, namely a sharply focussed circular ring having the same diameter as the circle in which the wave-surface makes contact with the second surface of the crystal. As the microscope objective is drawn away from the crystal, the ultra-focal image necessarily alters continually. The rays reaching the upper surface of the crystal within the circle of contact bend inwards, while those outside the circle bend outwards, a gap appearing between them owing to the vanishing intensity at points along the circle. The rays that bend inwards appear to gain rapidly in intensity as they approach the centre of the field; the latter appears as a luminous point from which the rays appear to diverge, when the focal plane of the microscope coincides with the upper surface of the crystal.

The radiations from the point source entering the crystal may be regarded as an assembly of plane waves with coherent phase-relationships crossing each other at that point. Entering the crystal, their directions of travel are altered, and the resultant distribution of the energy stream within the crystal is determined by their superposition. Along the axis of single-ray velocity, the lines of energy flow of numerous sets of plane waves coincide and the density of the energy flow is therefore a maximum on this line. On the other hand, along the so-called cone of inner conical refraction, the energy-flow of a single set of plane-waves is divided up and the energy flow is therefore a minimum. Since the disturbance emerging from the crystal is determined by the superposition of the plane waves refracted out from it, the energy flow outside would be closely related to the special character of the energy flow within the crystal. Actually, the bright spot at the centre of the field may be traced for a great distance outside the crystal. The bright spot is, in effect, a spectral image of the original point source, its position varying with the wavelength of the light used (Raman and Tamma, *loc. cit.*).

## 5. Summary

The angles of internal and external conical refraction for naphthalene are exceptionally large (both about 13° 45′), and the substance is therefore exceptionally well suited for exhibiting these phenomena as well as for a critical study of the same. A series of eight photographs is reproduced with the paper and is discussed in detail. The following noteworthy effects are exhibited by the crystal. The so-called Poggendorff dark circle appears only in internal conical refraction and is an ultra-focal phenomenon, disappearing when the image of a point source of light seen in exact focus through the crystal plate, the image being then a single circular ring which is extremely sharp. In external conical refraction we have an effect converse to the Poggendorff phenomenon, viz., a concentration of energy at the conical point of the wave-surface and therefore also along the axis of single-ray velocity. When the microscope is focussed on the second surface of the crystal and not on the source of light, the field of view exhibits a picture of the wave-surface in two sheets, their intersection appearing as an intensely luminous point and the tangent plane to the surface as a dark ring.

## References

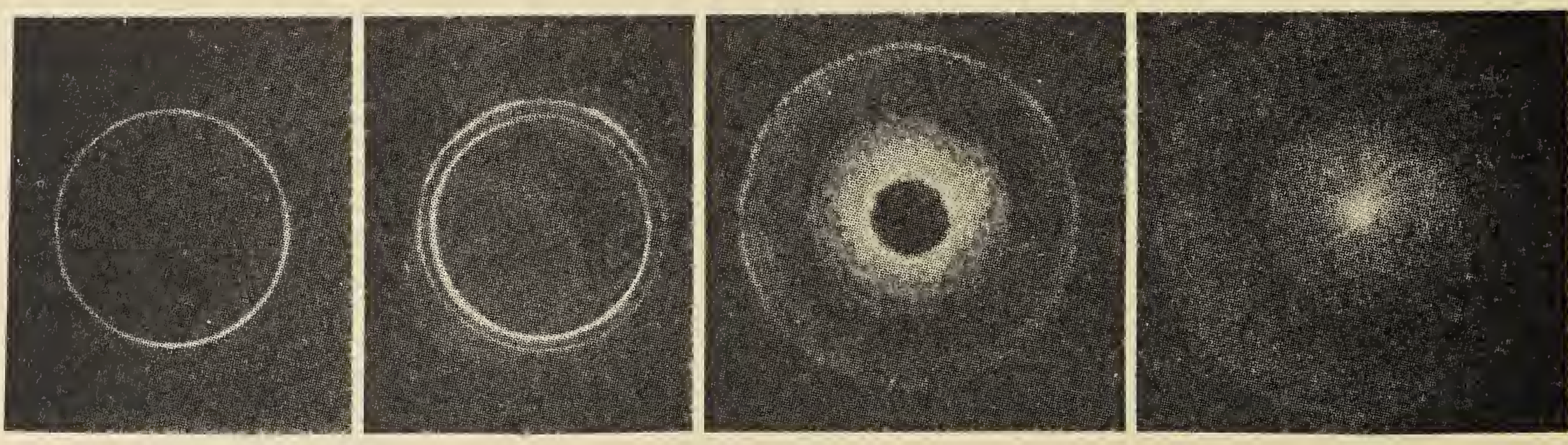
Hilmi-Benel *Theses*, Bordeaux, 1940.
T M K Nedungadi *Proc. Indian Acad. Sci.*, 1941, **A13**, 161.
C V Raman *Nature* (*London*), 1921, **107**, 747.
C V Raman and V S Tamma *Philos. Mag.*, 1922, **43**, 510.
W Voigt *Phys. Z.*, 1905, **6**, 673–818.
G G Stokes *Scientific Papers*, Camb. Univ. Press, Vol. **5**, p. 6.
J Walker *Analytical Theory of Light*, Camb. Univ. Press, Appx. III.

*Nature (London)* **147** 268 (1941)

# Conical refraction in naphthalene crystals

The birefringence of many organic crystals of the aromatic class is large, and when the intermediate index differs widely from the upper and lower indexes, the angles of internal and external conical refraction assume very high values. These angles in naphthalene, for example, are both about 13° 45′, which may be compared with 1° 54′ and 1° 44′ respectively in the classical case of aragonite. By fusion followed by very slow solidification, it is fairly easy to obtain transparent blocks of naphthalene and other aromatic compounds. When suitably cut and mounted between glass cover-slips, naphthalene crystals exhibit the phenomena of conical refraction in a very striking way, and enable their features to be critically examined.

The photographs reproduced with this note are the images of a fine illuminated pinhole held against the face of a naphthalene crystal 2 mm thick and viewed



(1) (2) (3) (4)

Conical refraction in naphthalene crystals.

through it in the appropriate direction. In the first of the series, the pinhole is seen in perfect focus and appears as a very sharp and perfect circle. The three other pictures illustrate the effect of moving the focus from the pinhole, until, finally, in the last picture, the second surface of the crystal is in focus. The central bright spot seen in the fourth picture is actually an image[1] of the fine pinhole used, and appears at the point where the axis of single-ray velocity meets the rear surface. This bright spot continues to be the most conspicuous feature in the field for a great distance behind the crystal.

It will be seen on a comparison of the first and second pictures that, at least in the case of naphthalene, *the so-called Poggendorf dark circle vanishes when the pinhole is seen in perfect focus.*

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19 December 1940

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[1]Raman, C V, *Nature*, **107**, 747 (1921), and *Phyl. Mag.*, **43**, 510 (1922).

*Curr. Sci.* **11** 44–46 (1942)

# The phenomena of conical refraction

SIR C V RAMAN

The newly developed techniques for growing large transparent crystals by slow solidification from melts are of great value for those interested in optical investigations with such crystals. In two recent communications,[1,2] attention was drawn to the very striking demonstrations of conical refraction possible with crystals of aromatic organic compounds. A transparent block of naphthalene, a centimetre square and half a centimetre thick, prepared by Mr T M K Nedungadi and mounted between parallel glass plates has enabled me to pursue the subject further and make some observations which appear well worthy of being placed on record.

As mentioned in the earlier communications, the angles of internal and external conical refraction are both large in naphthalene, so much so that conical refraction can be shown in the same way as ordinary birefringence, viz., by viewing a line of print through the crystal block. Figures 1(a) and (b) illustrate the effects observed in this way. It will be seen that a dot in print appears as a circle, and a circle as two concentric rings, when seen through the crystal at the correct orientation.

Several years ago, I noticed and described a very remarkable optical effect[3] associated with conical refraction which is observed when a small luminous object faces a parallel plate of aragonite suitably orientated and held at some little distance from it. A bright erect image of the luminous object superposed on a field of general illumination may then be seen anywhere on a line behind the crystal which is a prolongation of its join with the object. The same effect is shown in a much more striking way by a naphthalene block. Not only is the image seen much more intense than with aragonite, but it can also be traced to much greater distances, indeed up to about a metre, and is visible even when the luminous object is similarly far removed from the crystal. The effect is illustrated by figure 2(a) which is a human profile scratched with a needle on a glass plate covered by black varnish and placed in front of the naphthalene plate, while figure 2(b) reproduces the image of the same received on a photographic plate placed behind the crystal. It will be noticed that the features of the profile are recognizable in the

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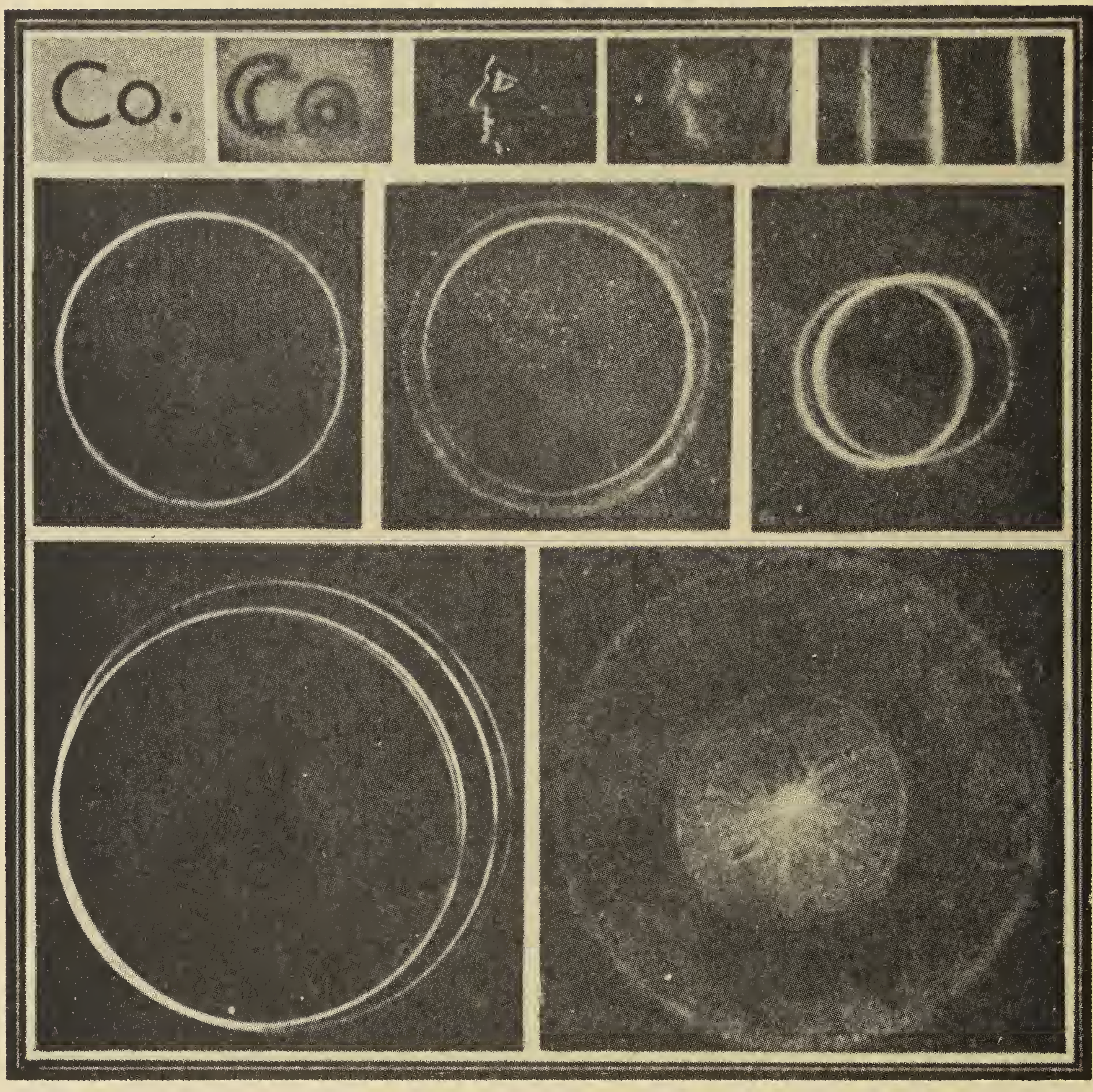
[1] *Nature* (*London*), 1941, **147**, 268.
[2] *Proc. Indian Acad. Sci.*, 1941, **A14**, 221.
[3] *Nature* (*London*), 1921, **107**, 747.




**Figures 1–8.** Illustrating conical refraction in biaxial crystals.

image, though slightly distorted owing to the optical imperfections of the crystal. The images formed in this way by the crystal plate show a strong chromatic dispersion, and it is therefore necessary to use monochromatic light (the green rays of the mercury lamp) in photographing the effect.

An explanation of this remarkable property of biaxial crystals was given by me in 1922.[4] This is completely confirmed by the present experiments which show that the phenomenon is of fundamental significance in relation to the physical theory of conical refraction. The image-formation in the rear of the crystal arises from the intense concentration of luminosity which occurs at the singular or

[4] *Philos. Mag.*, 1922, **43**, 510.

conical point on the wave-surface within the crystal. This is shown by the fact that the image is most intense at the rear surface of the crystal where the waves emerge from the crystal at the terminus of the axis of single ray velocity (figure 8). Using a point source of light and covering up the illumination in the field behind the crystal except for a small aperture at the position of the image, the light is found to diverge from the bright spot in the form of a hollow cone. When both the source and the aperture are situated on the faces of the crystal, this is identical with the Hamilton-Lloyd experiment demonstrating external conical refraction.

In the usual discussions of conical refraction, the geometric aspects of the problem receive attention, while the closely related physical aspects are practically ignored, though the latter are as interesting and important as the former. Corresponding to the two geometric properties of the ray-surface discovered by Hamilton, we have two physical properties, namely the intense concentration of intensity at the singular or conical point and the vanishing of the intensity along the circle of contact of the tangent plane with the ray-surface. Both the bright spot and the dark circle surrounding it may be traced to a considerable distance behind the crystal using a point-source of light and a low-power magnifier.

The phenomena exhibited by a naphthalene crystal in certain respects present an exceptional simplicity. The image of a fine pin-hole illuminated by monochromatic light and viewed in perfect focus through the crystal appears as a single sharply defined circle (figure 4), the so-called Poggendorff dark circle being then non-existent. Figure 5 shows the ultra-focal image where the two circles are just separated, and figure 8 the case when the microscope is focussed on the second surface of the crystal, the bright spot at the centre being then conspicuous. Figure 6 is the appearance of a fine illuminated pin-hole seen through a fairly thick plate of aragonite at as near a perfect focus as possible. It will be seen that we have now not a single circle, but two *intersecting* curves, one of which notably departs in shape from circularity. This feature is a general one shown by all biaxial crystals of which the angles of internal and external conical refraction differ sensibly, the coincidence of the two curves in naphthalene being a special feature due to the identity of the two angles.

Figures 3 and 7 illustrate the dispersion of conical refraction as observed with a naphthalene crystal in two different ways. In the former, a straight slit illuminated by the total light of a mercury lamp is employed as the source, and the crystal itself forms the spectral images of the source as already explained. In figure 7, on the other hand, a fine pin-hole is employed as the source (as in figure 4 but with a thicker plate) and is viewed in exact focus. Four distinct circles are then seen corresponding to the four brightest rays of the mercury lamp.

The photographs illustrating this article were obtained for me by Mr V S Rajagopalan.

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# The theory of the Christiansen experiment

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## 1. Introduction

In the well-known experiment due to Christiansen (1884), an optically isotropic solid, e.g., glass, is powdered and put inside a flat-sided cell which is then filled with liquid and the refractive index of the latter is adjusted suitably by varying its composition or altering its temperature. Beautiful chromatic effects are observed when the refractive index of the liquid is thus brought into coincidence with that of the powder for some chosen wavelength in the spectrum. The cell becomes transparent for a restricted region of the spectrum in the vicinity of that wavelength, while the rest of the incident light passing through the cell is diffused out in various directions and appears as a halo surrounding the light source. The range of wavelengths regularly transmitted by the cell diminishes as its thickness is increased and is also influenced by the other conditions of the experiment. Coarser powders give a sharper transmission band, while, *per contra*, the range of wavelengths transmitted may be made as large as we please by making the particles sufficiently fine. The difference in the dispersive powers of the solid and the liquid is also of importance. When it is large, the transmission band is sharp, while *per contra*, if it be small enough, nearly the whole spectrum can get through.

From the facts stated, it is clear that the effects observed in the Christiansen experiment are not capable of being understood on a purely geometric basis but require to be considered on the basis of wave-optics. This was appreciated by Rayleigh (1899), and he suggested that by considering the fluctuations in the number of particles encountered by a ray of light traversing the cell and the resulting variations in optical path, it might be possible to make a theoretical estimate of the width of the transmission band in the spectrum. The subject was, however, not further pursued by him. At the suggestion of the present writer, N K Sethi (1920) undertook a study with a view to obtain some experimental data and develop a theory capable of explaining the facts. A considerable measure of success was achieved by him in both of these directions. His work was followed up by C M Sogani (1926), who made a detailed examination of the optical phenomena exhibited by the so-called chromatic emulsions. Later,

B Mukhopadhyaya (1932) undertook to investigate the case of very finely powdered non-isotropic crystals. More recently, also, G N Ramachandran (1943) has sought to elucidate the subject further by considering the case in which the particles are of spherical shape and applying to it the principles of diffraction theory.

The present paper considers the subject afresh from a point of view which is different from that originally suggested by Rayleigh and is also simpler. The theory as now developed gives us a clear account of the phenomena and yields results in satisfactory accord with the facts of observation. Its publication has appeared desirable in view of the fact that of recent years, the importance of the Christiansen effect has been more widely appreciated. Many papers have been published and many references to it have appeared in text-books, concerning themselves chiefly with its practical application in optical filters capable of isolating narrow regions in the spectrum with the minimum loss of light. Strangely enough, however, one does not find in the literature of the subject any recognition of the fact that the performance of such a filter is determined by the principles of wave-optics.

## 2. Some general considerations

The powder-liquid mixture contained in a Christiansen cell is an optically heterogeneous medium, and its functioning depends on the fact that while this heterogeneity vanishes for the particular wavelength for which the two refractive indices ($\mu_1$ of powder and $\mu_2$ of liquid) are identical, it persists for adjoining wavelengths and disturbs the regular wave-propagation in their cases. In actual practice, the thickness of a Christiansen filter is of the order of a centimetre or even several centimetres, and hence, we shall not be justified in assuming a simple rectilinear propagation of the light rays for such wavelengths through the entire distance. To find the effect of the cell on the passage of the incident light-beam, we have to conceive of its total thickness as divided up into a sufficiently large number of individual layers, each of which in its turn produces its own independent effect, namely that of diverting part of the energy of the incident wave-train away from its original path in the form of diffracted waves. The wave-train finally emerging from the cell is that which has had its intensity cut down in this manner by the cumulative effect of the successive layers through which it has passed. To complete the picture, one has also to consider the diffracted radiations having their origin at these layers and emerging from the cell, since they are responsible for the halo observed in the experiment. The characters of the halo would evidently be determined by the intensities of these radiations as well as by their distribution in angle for various wavelengths.

In considering the problem from the point of view indicated above, it is evident that various quantities need to be known, viz., the size, shape and orientation of

the particles of the powder, as well as the manner in which they are disposed within the cell with respect to their neighbours. In the circumstances of the actual experiment, however, none of these quantities can be considered as invariable. It is precisely this situation which justifies us in adopting the approach indicated above and dividing up the thickness of the cell into a large number of layers which could be considered as acting more or less independently of each other. Layers of the same thickness would not necessarily be similar in their behaviour, but such differences would be averaged out when the total effect of the whole cell is under consideration. Hence, it is permissible to base our discussion on the behaviour of a single layer which is representative of the material contained in the cell. The question arises as to what should be the choice for the thickness $\Delta$ of an elementary layer, if the theory developed on these assumptions should correctly describe the facts. It is obvious that the fluctuations in optical path arising within a given layer would be *relatively* the largest when its thickness is smallest. On the other hand, it would be clearly not permissible to carry the subdivision beyond the point at which the dimensions of an individual particle would exceed $\Delta$. The choice which we shall accordingly make for $\Delta$ is that it is equal to the maximum distance which a light ray could travel within a single particle of average dimensions contained in the cell.

## 3. Formulation of the theory

By way of introducing the most general case, we shall first consider a particular example in which the elementary layers are assumed to be so constituted as to produce the maximum disturbance of the wave-propagation. This would obviously be the case when the entire thickness $\Delta$ of an elementary layer is occupied at various points on its area either by the solid alone or by the liquid alone, one-half of the aggregate area being thus occupied by the solid material and the other half by the liquid. The retardation of the wave-front in its passage through the layer would be $\mu_1\Delta$ in one case and $\mu_2\Delta$ in the other. Hence, if the wave-train before entry normally into the layer is represented by

$$\sin\frac{2\pi}{\lambda}(ct - Z), \tag{1}$$

the wave-train emerging from it is given by the summation

$$\tfrac{1}{2}\sin\frac{2\pi}{\lambda}(ct - Z - \mu_1\Delta) + \tfrac{1}{2}\sin\frac{2\pi}{\lambda}(ct - Z - \mu_2\Delta),$$

and may therefore be written as

$$\cos\frac{\pi(\mu_1 - \mu_2)\Delta}{\lambda}\sin\frac{2\pi}{\lambda}(ct - Z - \tfrac{1}{2}\Delta\cdot\overline{\mu_1 + \mu_2}). \tag{2}$$

The loss of intensity of the wave-train in its passage through the layer is obtained by squaring the amplitudes in (1) and (2) and taking the difference. It is evidently

$$\sin^2 \frac{\pi(\mu_1 - \mu_2)\Delta}{\lambda}, \tag{3}$$

while the retardation in phase produced by the layer is

$$\frac{\pi(\mu_1 + \mu_2)\Delta}{\lambda}. \tag{4}$$

The retardation in phase produced by passage through the entire cell would be merely the sum of the retardations produced by the individual layers. On the other hand, the reduction in intensity would be cumulative. It may be found by writing (3) in the form of a differential equation for the reduction of intensity, viz.,

$$\mathrm{d}I = -I\cdot \sin^2 \pi(\mu_1 - \mu_2)\Delta/\lambda\cdot \mathrm{d}z/\Delta. \tag{5}$$

On integrating (5) we obtain

$$I = I_0\cdot \exp[-\sin^2 \pi(\mu_1 - \mu_2)\Delta/\lambda\cdot z/\Delta], \tag{6}$$

where $z$ is the total thickness of the cell and $\Delta$ may be identified with the effective thickness of the particles of the powder. As an approximation, we may write (6) in the form

$$I = I_0\cdot \exp[-\pi^2(\mu_1 - \mu_2)^2\Delta z/\lambda^2]. \tag{7}$$

We may now readily generalize the foregoing treatment. For this purpose, we divide the cell-thickness as before into elementary layers of thickness $\Delta$. Considering an individual layer of this thickness, we imagine its area divided into a large number $N$ of equal parts. The disturbance incident normally on the layer being

$$\sin \frac{2\pi}{\lambda}(ct - Z),$$

the regularly emergent wave-train is given by the summation of the disturbances emerging from all the $N$ elements of area, viz.,

$$\sum_N \sin \frac{2\pi}{\lambda}(ct - Z - \mu\Delta), \tag{8}$$

where $\mu$ is a quantity defined for each of the $N$ elementary areas of the wave-front by the relations

$$\mu\Delta = \mu_1\Delta_1 + \mu_2\Delta_2 \quad \text{and} \quad \Delta_1 + \Delta_2 = \Delta. \tag{9}$$

$\Delta_1$ and $\Delta_2$ are the paths traversed in solid and liquid respectively in the particular area. On effecting the summation indicated in (8) and evaluating the resulting

intensity, we find the diminution produced by passage through the layer to be

$$\frac{4}{N^2}\cdot\sum_{rs}\sin^2\frac{\pi(\mu_r-\mu_s)\Delta}{\lambda},\tag{10}$$

where $\mu_r$ and $\mu_s$ are the values of $\mu$ respectively at two different elements of area and the summation indicated in (10) is made over all such pairs of elements, $N^2/2$ in number. We may, at this stage, make an approximation and write (10) in the form

$$\frac{4}{N^2}\sum_{rs}\cdot\frac{\pi^2(\mu_r-\mu_s)^2\cdot\Delta^2}{\lambda^2}.\tag{11}$$

If $k_{rs}$ is a number defined by the relation

$$k_{rs}(\mu_1-\mu_2)=(\mu_r-\mu_s),\tag{12}$$

and using the abbreviation

$$k^2=\frac{4}{N^2}\sum_{rs}k_{rs}^2,\tag{13}$$

we may write (11) in the form

$$k^2\pi^2(\mu_1-\mu_2)^2\cdot\Delta^2/\lambda^2.\tag{14}$$

Finally, by integration over the whole thickness of the cell, we obtain as the expression for the transmission coefficient

$$\exp\left[-k^2\pi^2(\mu_1-\mu_2)^2\cdot\Delta z/\lambda^2\right].\tag{15}$$

Our final formula (15) thus differs from (7) obtained earlier for an idealized case merely by the appearance of an additional numerical factor $k^2$ in the exponential. That this factor has a maximum value of unity and would in general be rather less may be shown by considering particular examples. If, for instance, $\mu\Delta$ is $\mu_1\Delta$ over $\sigma N$ elements of area of the wave-front and $\mu_2\Delta$ for the remaining $(1-\sigma)N$ elements, we obtain at once from (12) and (13) that

$$k^2=4\sigma(1-\sigma).\tag{16}$$

$k^2$ has thus the maximum value unity when $\sigma=\frac{1}{2}$, as in the case considered earlier. That $k^2$ would be less than unity when the thickness of the individual particles is less than $\Delta$ over a part of their area, is also fairly obvious on a consideration of the effect on the summations indicated in (12) and (13).

## 4. The case of spherical particles

It is of interest to consider the special case in which the particles of the powder are tiny spheres, since it forms an excellent illustration of the general theory, and also

since it is an experimentally realisable case. We consider a layer of thickness $\Delta$ equal to the diameter of the spherules in which a fraction $\sigma$ of the area of the wave-front is covered by the spherules, while in the remaining fraction $(1-\sigma)$ the light passes entirely through the surrounding liquid. The disturbance emerging from the layer may be written in the form

$$(1-\sigma)\sin\eta + \sigma\int_0^{\pi/2} 2\sin(\eta - \xi\cos\theta)\sin\theta\cos\theta d\theta, \tag{17}$$

where

$$\eta = \frac{2\pi}{\lambda}(ct - Z) \quad \text{and} \quad \xi = \frac{2\pi(\mu_1 - \mu_2)\Delta}{\lambda}. \tag{18}$$

The integration may be effected, enabling (17) to be written as

$$(1-\sigma)\sin\eta + 2\sigma\sin\eta\left[\frac{\sin\xi}{\xi} + \frac{\cos\xi}{\xi^2} - \frac{1}{\xi^2}\right]$$
$$+ 2\sigma\cos\eta\left[\frac{\cos\xi}{\xi} - \frac{\sin\xi}{\xi^2}\right]. \tag{19}$$

It is readily verified that in the limit when $\xi$ is zero, (19) reduces to $\sin\eta$, as it should. We are interested in the cases when $\xi$ is rather small, and (19) may then be written in the approximate form

$$(1 - \sigma\cdot\xi^2/4)\sin\eta - \sigma\cdot 2\xi/3\cdot\cos\eta. \tag{20}$$

Finally, we obtain for the coefficient of transmission, an expression of the same form as the general formula (14) in which the constant $k^2$ is given by

$$k^2 = (2\sigma - 16\sigma^2/9). \tag{21}$$

This has a maximum value 9/16 when $\sigma = 9/16$; in other words when the spherules cover a little over half the area in each layer. It is thus apparent that the spherical shape of the particles would result in a very considerable increase in the spectral width of the transmission band as compared with the case in which the particles are flat platelets.

## 5. The diffraction halo

That the halo seen surrounding the source of light in the Christiansen experiment is a diffraction effect, and not a simple matter of geometrical optics should be evident from the fact that it has its origin in the same set of circumstances which determine the spectral range of the regularly transmitted light, viz., the optical heterogeneity of the medium and its variations with wavelength. As remarked earlier in the paper, the halo is, in essence, a superposition of the diffracted

radiations having their origin in the successive layers within the cell traversed by the incident beam of light.

In considering the problem of the distribution of light in the halo and the spectral character of the radiations composing it, it should be emphasised that the diffracted radiations have their origin at the *layers* parallel to the wave-front traversed by it, and hence that they represent the joint effect of all the particles in a layer. In other words, we have to consider the configuration of the whole wave-front after it has traversed the layer; the positions of the individual particles in the layer would determine the locations of the elevations or depressions (as the case may be) on the wave-front, while the size and shape of the particles and the value of $(\mu_1 - \mu_2)$ for the particular wavelength would determine their magnitude. In effect, each layer would form a two-dimensional laminar diffraction grating, though necessarily an irregular one. The theory of the haloes observed in the Christiansen experiment would thus be based on a description of the optical behaviour of such an irregular grating.

The theory of diffraction gratings has been well studied in the cases in which the rulings are regular and run in one direction only. We are here concerned with phase-change gratings, and the optical behaviour of such gratings may be readily described in terms of a quantity $\zeta$ which corresponds to $\pi(\mu_1 - \mu_2)\Delta/\lambda$ in our problem and represents the amplitude of the (harmonic) corrugations on the wave-front expressed as a phase-angle. When $\zeta$ is zero, the wave-front is plane and all the spectra vanish. When $\zeta$ is finite but small, only the first-order spectra on either side appear. Their intensity increases steadily as $\zeta$ increases and is quite considerable when $\zeta$ reaches the value 1, while the intensity of the higher order spectra is still quite small. The intensity of the first order spectra continues to increase till $\zeta$ is nearly equal to 2. At this stage, the second order has gained considerable strength, though still weaker than the first order, and the two orders between them contain nearly the whole of the incident energy. As $\zeta$ increases further, the spectra of still higher orders successively gain in strength, the orders having the maximum intensity being higher, the larger $\zeta$ is. The energy is distributed between several high orders for large values of $\zeta$, so that individually they are rather weak.

The nature of the results to be expected in our present problem would necessarily be somewhat modified by various considerations. Nevertheless, one can indicate in general terms what they would be, on the basis of the results indicated above for a regular phase-change diffraction grating. Firstly, the diffracted radiations would be absent for the wavelength of maximum transmission for which $(\mu_1 - \mu_2)$ vanishes. They would rapidly increase in intensity as $(\mu_1 - \mu_2)$ becomes numerically larger and attain large values, but provided $(\mu_1 - \mu_2)$ is not too large, they would be concentrated principally in directions adjacent to that of regular transmission, the angular separation from it being determined by the average distance apart of the individual particles. (In a close-packed arrangement, this would also be the size of the individual particles.) As

$(\mu_1 - \mu_2)$ increases numerically beyond a certain limit, the diffracted radiations would spread out over a wider range of angles, and the direction of their maximum intensity would also move away further from that of regular transmission. Their intensity in any particular direction would simultaneously be much weakened.

Superficially, the results inferred above on the basis of diffraction theory resemble those which one might anticipate on the basis of geometrical optics as the result of irregular refractions by the particles of powder immersed in the surrounding liquid. Actually, however, they are different, since according to geometrical optics, the deviations of the rays would be determined by the *shape of the particles*, while in diffraction theory, it is *the size of the particles and their distances apart* in relation to the wavelength of light that principally determine the results.

## 6. Comparison with the facts

That the transmission band of a Christiansen cell does not exhibit a sharp cut-off in the spectrum in either direction was observed by Sethi (*loc. cit.*) in his investigations on the subject. He noticed that the visually observed spectral width could be increased by merely increasing the intensity of the light incident on the cell, and inferred from this that the transmission curve resembled a graph of the Gaussian error function. This was verified quantitatively by decreasing the intensity of the incident light by known large fractions and observing the corresponding decrease in width of the transmission band. Using powdered glass in which the particles had been sorted out into grades of different size, and also using varying thicknesses of the cell, he was enabled to establish that the variable which determined the spread of the exponential curve was proportional to the product of the thickness of the cell and the size of the particles. Finally, combining these observations with theoretical considerations of the kind indicated by Rayleigh (*loc. cit.*), Sethi derived a formula identical with our formula (7) above, except that an empirical constant appeared in the exponential instead of $\pi^2$. Sethi remarks that his data indicated the constant to be about 7. Sogani (*loc. cit.*) carried the matter further and sought to evaluate the constant appearing in the formula theoretically. Considering the case in which the particles had a spherical shape and occupied the largest possible fraction of the total volume, he calculated the constant to be $16\pi^2/9$ or 18, while his experimental observations showed the constant to be about 9. According to Ramachandran (*loc. cit.*), the theoretical value of the constant is 22 for the same case, obviously much in excess of that indicated by the actual facts. It is noteworthy that the formula of Mukhopadhyaya (*loc. cit.*) when evaluated gives the constant as about 5. The lower value is to be traced to his theoretical approach being more like that adopted in the present paper.

The general form of the transmission curve indicated by our theoretical formula (15) is thus supported by the experimental facts. Various authors have remarked upon the unsymmetrical form of the transmission curve which is steeper on the side of shorter wavelengths than on the long wavelength side. This effect, according to our formula, would be due to the joint effect of two factors, viz., due to $(\mu_1 - \mu_2)^2$ appearing in the numerator of the exponential increasing more rapidly towards shorter wavelengths and also due to the factor $\lambda^2$ appearing in the denominator. The increased sharpness of the transmission band when it is pushed towards shorter wavelengths by altering the temperature or varying the composition of the liquid, which has also been noticed in experiment, may similarly be explained as the joint effect of the same two factors in our theoretical formula. The appearance of $\lambda^2$ in the denominator is characteristic of wave-optics, and it is much to be desired that the influence of this factor is demonstrated by fresh quantitative data obtained with suitable material with which the transmission band could be shifted over a wide range of wavelengths, and by disentangling therefrom the effect of the variations of the factor $(\mu_1 - \mu_2)^2$ which appear simultaneously. In respect of the other factors, viz., the thickness of the cell and the size of the particles, the data already on record amply establish their influence. It would be desirable however to carry out further studies enabling the constant appearing in the formula to be evaluated in a number of cases, with a view to demonstrate the influence of the particle shape and of the closeness of packing on its numerical value.

In the practical use of a Christiansen filter, it is necessary, among other things, by suitable methods to separate the light regularly transmitted by the filter from that diffused or scattered by it. The discussion in the preceding section of the characters of the diffraction halo has an important bearing on this question. If the particles are very large, the diffracted light would appear in directions very close to that of the regularly transmitted light, and its separation from the latter would obviously become difficult. If, on the other hand, the particles are very small, the spectral width of the transmission would itself be greatly increased. There is thus an optimum size for the particles if the filter is to function most usefully. The work of Denmark and Cady (1935) is of interest in this connection. It may be remarked that the characters of the halo in directions adjacent to that of the transmitted light, as also in directions remote therefrom were very fully described and illustrated by a series of spectrograms in Sethi's paper of 1920. The features described by him find a satisfactory explanation on our present theory.

## Summary

The effects exhibited by a Christiansen filter can only be explained or understood in terms of wave-optics. A theoretical formula is derived in the paper for the distribution of intensity in the spectrum of the transmitted light, the variables

involved being the wavelength of the light, the average size of the particles of the powder, the thickness of the cell and the difference in the refractive indices of the powder and the liquid for the wavelength under consideration. The characters of the halo observed around the light source are also discussed in terms of diffraction theory. The theory explains the facts of observation in a very simple manner and gives results in satisfactory accord with the available experimental data.

## Bibliography

(In chronological order)

Christiansen C, *Ann. Phys.*, 1884, **23**, 298.
Christiansen C, *Ann. Phys.*, 1885, **24**, 439.
Rayleigh Lord, *Philos. Mag.*, 1885, **20**, 358.
Rayleigh Lord, *Nature* (*London*), 1899, **60**, 64.
Rayleigh Lord, *Scientific Papers*, **2**, 433 and **4**, 392.
R W Wood, *Physical Optics*, All editions.
Bodroux F, *Comptes Rendus*, 1913, **156**, 772.
Sethi N K, *Proc. Indian Assoc. Cultiv. Sci.*, 1920, **6**, 121.
Sethi N K, *Philos. Mag.*, 1921, **42**, 669.
Holmes and Cameron, *Am. Chem. Soc. J.*, 1922, **44**, 71.
Sogani C M, *Philos. Mag.*, 1926, **1**, 321.
Bhalla, Bhatnagar S S, and Yajnik, *Kolloid Z.*, 1927, **43**, 366.
Weigert, F and Staude H, *Z. Phys. Chem.*, 1927, **130**, 607.
Elvegard, E., Staude, H., and Weigert F, *Z. Phys. Chem.*, 1929, **B2**, 149.
Weigert F and Shidei J, *Z. Phys. Chem.*, 1930, **B9**, 329.
Kohn H and Fragstein K Von, *Phys. Z.*, 1932, **33**, 929.
Mukhopadhyaya B, *Indian J. Phys.*, 1932, **8**, 307.
Fragstein K Von, *Ann. Phys.*, 1933, **17**, 22.
Knudsen E, *Kolloid Z.*, 1934, **66**, 257.
Abbot C G, *J. Opt. Soc. Am.*, 1935, **25**, 42.
Denmark H S and Cady W M, *J. Opt. Soc. Am.*, 1935, **25**, 330.
McAlister E D, *Smithson. Misc. Collect.*, 1935, **93**, No. 7.
Barnes R B and Bonner L G, *Phys. Rev.*, 1936, **49**, 732.
Ramachandran G N, *Proc. Indian Acad. Sci.*, 1943, **A18**, 67.
Sinsheimer R L and Loofbourow J R, *Nature* (*London*), 1947, **160**, 674.

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# The Christiansen experiment with spherical particles

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## 1. Introduction

During a visit by the senior author to the works of Messrs. Chance Brothers at Birmingham in May 1948, he noticed in their showroom an exhibit of glass in the form of tiny spherules about a millimetre in diameter. A sample of the material very kindly presented by the firm was brought back to India, and the studies now reported were made with it.

On placing a quantity of the substance in a transparent cell and holding it up against a bright source, it is found that even a moderate thickness of it suffices completely to cut off the light and make the cell opaque. When, however, the empty spaces between the spherules of glass are filled by a mixture of carbon disulphide and acetone of suitably adjusted refractive index, the Christiansen effect is observed. In other words, the cell becomes transparent to a restricted region of the spectrum, while the rest of the light appears as a coloured halo surrounding the source when viewed through the cell. The explanation of these facts is obviously the same as in the usual form of the Christiansen experiment. Nevertheless, the phenomena observed are distinctly more spectacular with the spherules than with a pack of irregular fragments of glass. The spherical shape of the particles influences the propagation of light through the cell in a characteristic fashion and results in some readily observable and rather striking consequences. Then again, the uniformity of size of the particles results in a high degree of regularity in their disposition within the cell, and this becomes conspicuously evident in the experiment.

Though the material as received shows the effects in question clearly enough, it is worth while to eliminate from it the particles deviating largely from the average size. This is effected by the aid of an appliance used in the diamond trade for sorting cut stones according to size. The device consists of a metal receptacle in two parts separated by a plate pierced by a set of circular holes. On placing the

material in the upper chamber and shaking it gently, the particles which are smaller than the rest fall into the lower chamber very quickly. The particles which are larger than the rest, on the other hand, remain in the upper chamber even after prolonged shaking. The particles which are either larger or smaller can thus be separated out from the middle fraction consisting of spherical particles having the same diameter as the circular holes in the sorter, viz., one millimetre. When a cleaning up of the spheres is found to be necessary, it is readily effected by washing with strong acids and subsequently with distilled water and a final drying.

## 2. Some optical consequences of the spherical shape

Acetone and carbon disulphide, the two liquids used, have refractive indices respectively lower and higher than the glass for the whole range of the visible spectrum. Hence, when a cell one centimetre thick containing the spherules is filled up with either one or the other liquid by itself, the medium is incapable of regularly transmitting an incident light beam. Nevertheless, a good deal of light does find its way through the cell in these circumstances. There is, however, a remarkable difference in its appearance in the two cases. When the spherules are surrounded by a liquid of lower index, the cell presents a brilliant sparkling appearance, due evidently to the emergence of light beams of considerable intensity from localised areas on its surface. The addition of a little carbon disulphide, though insufficient to render the cell transparent to any part of the spectrum, enhances the sparkling effect and makes it more attractive by reason of a play of colours similar to the "fire" of a diamond. On the other hand, when the liquid surrounding the spherules has a higher refractive index, these effects are not observed. The emergent light is then faint and diffuse, and the cell presents a dull appearance. The striking difference between the two cases, as well as the intermediate stages in which the cell is transparent to particular regions of the spectrum, can all be simultaneously observed by pouring enough carbon disulphide to fill the lower half of the cell, and then adding acetone to fill the upper half. The acetone, being lighter, floats above the carbon disulphide and inter-diffusion takes place only slowly. The region of mixing appears as a bright band of transmitted colours, violet above and red below, while the upper and lower parts of the cell exhibit the effects arising from a penetration of the light through the cell without regular transmission.

An explanation in general terms may be readily given of the effects described above. A solid sphere surrounded by a liquid of lower index has a converging effect on a beam of light passing through it. *Per contra*, a light beam is made divergent by the sphere, when the surrounding liquid has a higher index. The effect of passage of the light through a series of spheres would accordingly be very different in the two cases. In one case, there would be a high probability that the light would emerge from the cell with its course deviated but nevertheless having

great concentrations of intensity in particular directions or at particular points on the surface of the cell. In the second case, the emergent beam would be strongly divergent and hence weak and diffuse at all points and in all directions. The correctness of this explanation can be checked by viewing a small bright source of light through the cell held in front of the eye. When the refractive index of the liquid is distinctly lower than that for the glass, the field of view surrounding the source exhibits a small number of bright spots drawn out into spectra irregularly distributed over its area. On the other hand, when the refractive index of the liquid is higher than that of glass, we observe an extended and diffuse halo exhibiting no noticeable detail.

The spherical shape of the particles has also other consequences. The deviation in the path of a ray of light by an individual sphere in the circumstances of the experiment would evidently be very small except when the ray is incident almost tangentially on the surface. Reflections of the light at the boundaries between the two media would also be of negligible intensity except in similar circumstances. Thus, the edges of the individual spheres grazed by the incident rays play a special role in determining the observed phenomena. When the cell is viewed directly, they show up as dark circles on a bright background. On the other hand, when the cell is viewed obliquely, the edges appear as bright crescents of light on a dark background. The convex parts of the edges then exhibit the colours for which the liquid has the higher refractive index, while the concave parts exhibit the remaining spectral colours. Hence, when there is an equality of index at or near one end of the visible spectrum, the concave or the convex parts alone (as the case may be) of the edges appear luminous when viewed obliquely, the other parts remaining dark.

## 4. The structure of the halo

In the foregoing, for the sake of simplicity, we have used the language of geometrical optics in describing and interpreting the observed phenomena. Nevertheless, as was stressed in a recent paper in these *Proceedings*, geometric theory is wholly inadequate as a basis for an understanding of the Christiansen effect. For, we are here principally concerned with regions in the spectrum for which the particles and the surrounding liquid have nearly identical refractive indices. The smaller the difference between these indices, the smaller would be the path-difference between the light rays traversing the cell along different routes and hence the more perfectly coherent would they be. Such coherence and the resulting regular transmission is the very essence of the Christiansen effect, and no explanation of the latter is therefore possible except on the basis of wave-optical considerations.

A study of the structure of the halo surrounding the light-source is very instructive in relation to the foregoing remarks. Remarkable changes are found to

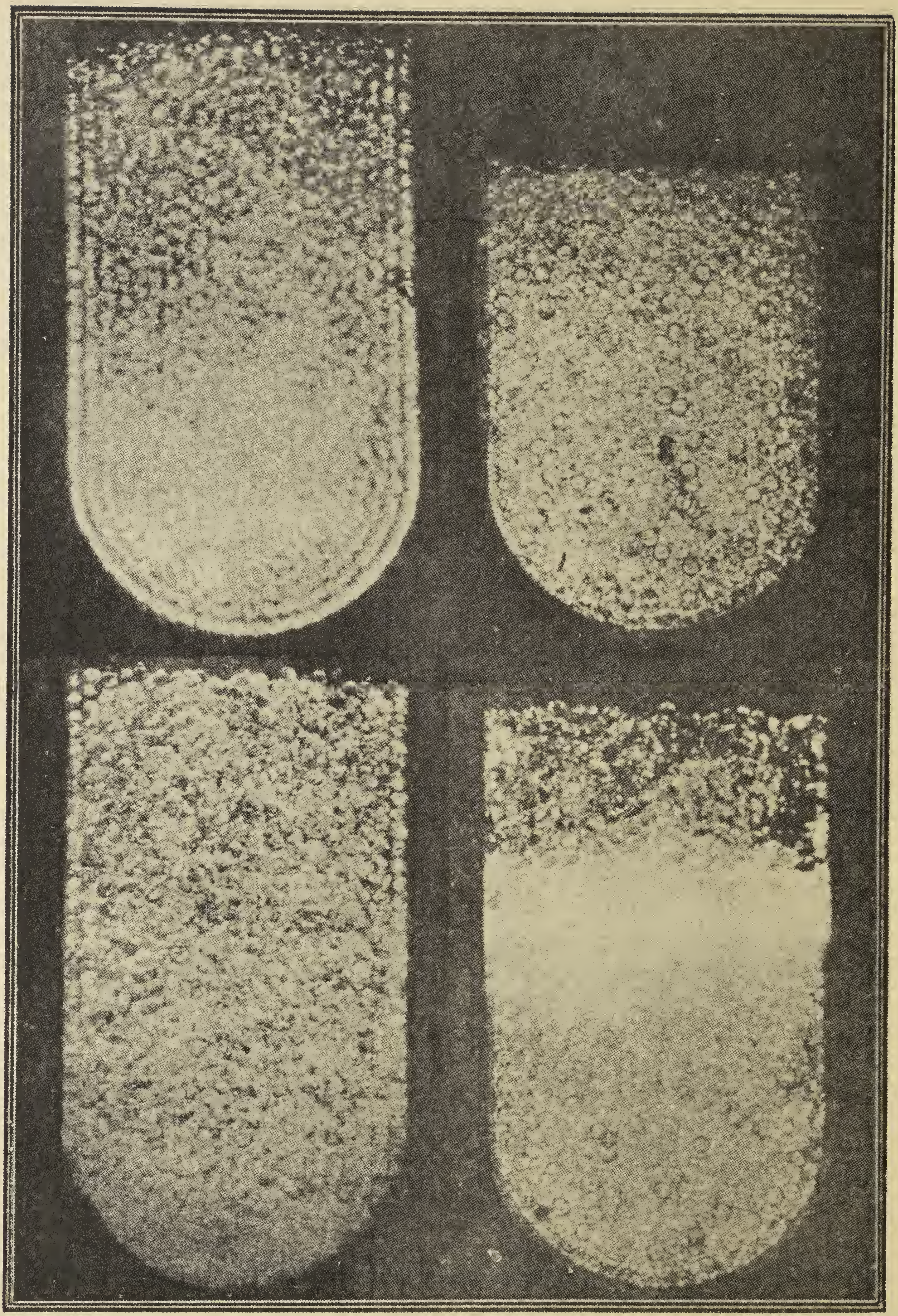
occur in the halo when by progressive additions of carbon disulphide, the refractive index of the liquid is brought into coincidence with that of the spherules successively for different parts of the spectrum. The observations may be made with a small bright source of white light or alternatively with monochromatic radiation from a mercury or sodium lamp. In the former case, the part of the spectrum for which there is equality of refractive index disappears from the coloured streaks of light seen in the field of view and appears instead in the position of the light source. Observations with a monochromatic source reveal corresponding changes in the structure of the halo. The bright spots seen in the field when the liquid has a decidedly lower index are replaced by a complicated pattern of interference streaks and then by a halo with a mottled structure consisting of a great many diffraction images of the light source. This halo contracts in area and finally vanishes when the cell becomes optically homogeneous for the wavelength employed. The regularly transmitted light which builds up in intensity during the latter stages is then at its maximum. With further additions of carbon disulphide, it diminishes in intensity and finally disappears. Simultaneously, the halo reappears and increases in its angular extension until it covers a large area and is so diffuse that no detail can be observed in it.

## 5. The location of the spherules

When the spherules of glass are placed in the cell, they naturally arrange themselves in such manner as to occupy the smallest possible volume. But the walls of the cell necessarily influence and largely determine this arrangement. For, they fix the positions of the spheres actually in contact with them and therefore also, indirectly, the positions of the spheres further out in the interior of the cell. The Christiansen experiment enables us readily to demonstrate this. For, the transparency of the cell permits us to locate the spherules, and observe their positions with respect to each other and with respect to the walls of the cell. Indeed, the influence of the shape of the cell on the disposition of the particles in its interior is strikingly evident in the experiment. The layer of spherules in immediate contact with the walls naturally runs parallel to them. But their influence is observed to extend much further into the interior. At least four layers of particles running parallel to the walls of the cell may be clearly seen. This is irrespective of whether these walls are plane or curved or whether they are horizontal or vertical. When a rectangular cell is employed, five sets of such layers are observed running respectively parallel to the four walls and to the base of the cell and intersecting each other sharply along the edges of the cell.

The observations reported in the present paper are obviously of a qualitative character. It is felt however, that they are of sufficient interest to justify publication. We are engaged in a more detailed study of various aspects of the

(1) (3)



(2) (4)

Plate I

experiment and hope to be able to follow up the present communication by further papers.

## 6. Summary

Glass in the form of tiny spheres is technically available, and by the use of a suitable appliance, the material can be sorted out so as to consist of particles of uniform size. It can then be very effectively used to exhibit the Christiansen effect. The spherical shape of the particles as well as their uniformity of size have some striking consequences which are not observed in the usual form of the experiment in which an irregular pack of powdered glass is employed. Four photographs in plate I illustrate the paper.

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**Plate I.** The photographs in plate I illustrate the appearance of the cell in different circumstances. They represent a threefold enlargement of its actual size. **Figure 1** exhibits the spontaneous arrangement of the spherules in layers parallel to the cell walls. Four such layers can easily be seen. The photograph was taken by transmitted light near the red end of the spectrum. **Figure 2** exhibits the appearance of the cell when filled with acetone and held against a bright source of light. **Figure 3** similarly exhibits its appearance when filled with carbon disulphide. Notice the numerous dark circles which represent the edges of the spheres. **Figure 4** illustrates the effect of filling the cell with carbon disulphide below and acetone above. See the text of the paper for further details of the effects observed in this case.

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# Diffraction of light by transparent spheres and spheroids: The Fresnel patterns

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## 1. Introduction

The phenomena with which we are concerned in the present paper are those arising when a beam of light emerges after traversing a sphere of radius large compared with the wavelength of light. Our attention was drawn to them in the course of some studies on the Christiansen experiment with spherules of glass described in a recent paper in these *Proceedings*. To elucidate more fully the effects described in that paper, it appeared to us desirable to examine the optical behaviour of an individual spherule of glass immersed in a liquid of nearly the same refractive index and traversed by light from a distant point source. Very pretty colour effects were observed with white light, which obviously had their origin in the fact that the convergence of the beam after its passage through the particle was widely different for the different rays of the spectrum. This suggested the use of a monochromatic light source, e.g., a sodium vapour lamp. We then noticed that the light emerging from the spherule exhibits very characteristic diffraction patterns; the nature of these depends notably on the shape of the particle, being widely different for the two most interesting cases in which it is respectively a sphere and a spheroid of revolution. The configuration of the patterns changes progressively as the plane of observation is shifted away from the spherule; the difference in the refractive indices of the sphere and of the liquid in which it is immersed determines how rapid this change is. The diffraction patterns may be observed even when this difference is not small; their changes with the shift of the plane of observation are then rapid. The dispersive powers of the glass and the liquid not being the same, the difference in their refractive indices alters rapidly with the wavelength of the light employed. As a consequence, the patterns change quickly with the wavelength. This is readily observed when the light-source is a mercury arc, even without the aid of colour filters for selecting out the different spectral rays emitted by it.

Our observations were in the first instance made with the same tiny spherules

of glass one millimetre in diameter as those used in the Christiansen experiment. We subsequently found, however, that essentially similar phenomena can be observed also with spheres which are still smaller, viz., one-tenth of a millimetre in diameter and also with spheres of much larger size. A ball of quartz immersed in benzene shows the effects very well; its birefringence, however, results in two sets of patterns being observed instead of one, their separation varying with the direction in which the light traverses the sphere. It would seem that we have here a very simple and convenient method of observing and exhibiting the optical characters of a crystalline solid.

## 2. Geometric theory

As we are concerned with spheres whose dimensions are great in comparison with the wavelength, it is justifiable, at least in the first instance, to regard the problem as one of geometrical optics, and then to supplement the results by considerations based on wave-theory. Using elementary methods, we may trace the course of a bundle of parallel rays incident on a transparent sphere and emerging therefrom. The deviation suffered is zero for the axially incident ray; it increases steadily and then much more rapidly as we approach the marginal ray which is incident tangentially on the surface. The deviations of the rays depend on the refractive index of the sphere relatively to that of the sorrounding medium. In the particular case when the relative index is unity, they vanish and the rays emerge again as a bundle of parallel rays. In the cases which interest us, the relative index is a little greater than unity, and the emerging beam would evidently be convergent, but such convergence would be very different for the axial bundle of rays and for the marginal ones. In actual practice, the sphere is immersed in a flat-walled cell completely filled with liquid. In tracing the course of the rays, the further deviations which occur when they emerge from the cell walls would also have to be considered.

The result of the passage of the light through the sphere and the enclosing liquid is most readily visualised by considering the cylindrical bundle of rays incident on the sphere to be divided up to a great number of concentric hollow cylindrical beams. Each such cylinder of rays would converge to a focus on the axis and subsequently diverge, but the focal points would all be different, being farthest from the sphere for the axial pencils and nearest to it for the marginal ones. In other words, instead of all the rays passing through the sphere converging to a single focus on the axis, we would have a continuous line of foci or concentration of intensity along the axial ray. Further, since each hollow cylinder of rays emerges from the cell as a hollow cone of rays and the convergence of these is different, it follows that the successive cones would intersect each other and form a caustic surface, the cross-section of which by any plane normal to the axis would be a circle. The bundle of rays emerging from the sphere would therefore

exhibit a concentration of intensity along its circular periphery. The area enclosed by this would be largest when the beam emerges from the cell and would contract as we recede therefrom, finally collapsing to a point when we reach the focus of the axially incident rays. Except in the limiting case when the relative index is unity, the diameter of the emergent beam would invariably be less than that of the sphere.

The foregoing remarks are illustrated by figure 1 below, in which the course of a bundle of parallel rays incident on a glass sphere of refractive index 1·54 immersed in water (refractive index 1·33) and emerging therefrom has been computed and shown. Besides drawing the course of the rays, the form of the wave-surface immediately after emergence from the sphere has also been computed and drawn in. It exhibits the cusp-shaped form which in wave-theory corresponds to the formation of a caustic in geometrical optics.

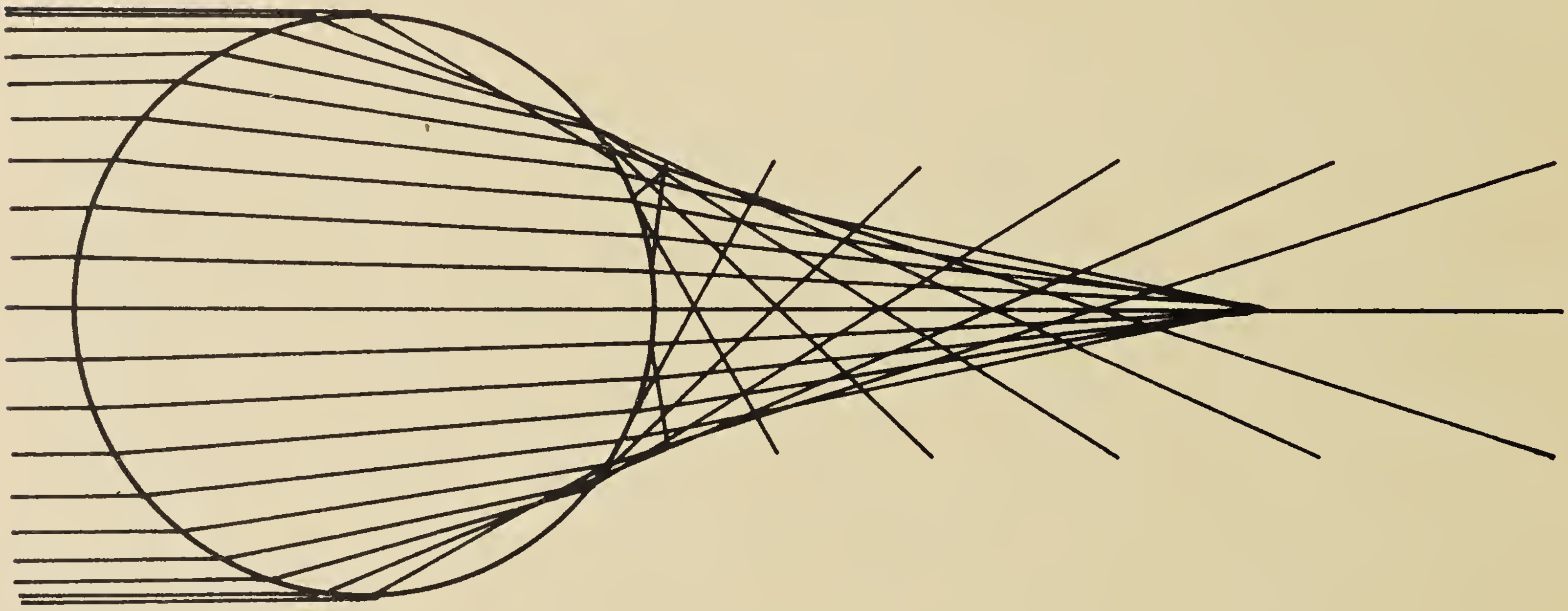

**Figure 1**

## 3. Interference phenomena

As is evident from figure 1, the rays which have traversed the sphere by different paths intersect each other after emergence from it. It follows that interference effects should be observable. We may, in the first place, remark that wave-optical considerations reinforce the results indicated by the geometrical theory that there should be observable concentrations of intensity of *two* kinds in the light-field, firstly along the circular periphery of the beam emerging from the sphere, and secondly along its axis. At the periphery of the beam, rays which have traversed closely adjacent and therefore nearly equal paths within the sphere intersect one other, while on the axis, rays meet which have transversed identically equal paths but emerge from different points forming a ring on the surface of the sphere. It

follows that the axis should everywhere be a locus of maximum intensity, while the periphery should exhibit a maximum of intensity lying close to but not absolutely coincident with it. Corresponding to each of these two loci of maximum intensity, we should have a series of loci of subsidiary minima and maxima of intensity appearing at the points where the intersecting rays differ in optical path respectively by odd and even multiples of half a wavelength. In the case of the interferences running parallel to the circular periphery of the beam, it is evident that the intersecting rays meet at an angle which is small in its vicinity and increases as we move away from the periphery towards the centre of the field. It follows that the interference fringes running parallel to the circular caustic would be very wide at the margin at the field and becomes progressively narrower as we approach the centre of the light beam. The case is altogether different when we consider the interferences of the rays reaching points at and near the axis from points all round the circumference. These rays evidently meet at a fairly large angle which would change very little as we move away from the axis in any given plane of observation. It follows that the interference rings surrounding the axial concentration of intensity would be narrow, but evenly spaced. It is evident also that both the peripheral and axial sets of interference rings should widen out as we move away from the sphere and approach the focal point of the axial rays; for the angles of intersection of the interfering rays then steadily diminish. The superposition of the two sets of interference would obviously become most evident in the same circumstances.

It should not be supposed however, that elementary considerations of the kind indicated above would suffice completely to describe the actual facts of observation. In the first place, the approach is purely qualitative and makes no pretence of giving quantitative indications in respect of the intensity of illumination in the field. Indeed, even if it were to be developed so as to deal also with such questions, we should scarcely expect such a theory to be quite successful. The part of the area of the incident wave-front which gives rise to the experimentally significant phenomena is that which passes through the marginal regions of the sphere. This is a very small part of the whole, especially when the relative index of refraction is only a little greater than unity. Diffraction then necessarily plays a part and determines the observed intensities of illumination in the field. We cannot therefore regard the geometrical approach to the problem as anything more than a useful and easily understood way of interpreting the observed phenomena.

## 4. Methods and results of observation

For observing the diffraction phenomena produced by a small spherule of glass, the most suitable procedure is to immerse it in a thin flat cell which is completely filled up with liquid and then covered by a glass-plate. The cell is then placed on

the stage of a low-power microscope. A convenient light-source is provided by a small aperture backed by a sodium vapour lamp or alternatively a point-o-lite mercury arc. The former is most useful when it is desired to observe the patterns with light of one wavelength, while the latter (unless filters are used) is convenient for observing the patterns with several wavelengths simultaneously. The light from the aperture is sent up through the cell by the plane reflecting mirror below the stage of the microscope, the condenser having been removed. By racking up the microscope, the patterns formed in the successive planes of observation commencing from the upper surface of the cell up to the focal point of the axial rays or even beyond may be conveniently observed. In the case of the larger spheres, no microscope is needed and the observations may be made with a suitable eyepiece or, better still, by receiving the light emerging from the sphere on a sheet of ground glass.

The experimental studies confirm the results of the foregoing theoretical discussion. The caustic and its accompanying interference fringes at the periphery of the field are readily observable features. The bright spot at the centre of the field indicated by the theory is first seen at some distance from the sphere and increases rapidly in intensity as we approach the focal point of the axial rays. That this spot is an optical image of the light source employed is readily verified by making the latter of triangular shape. The bright spot, if observed with a perfectly spherical particle, has then the same shape and increases progressively in size as we recede further and further from the sphere. It is important to remark that the bright spot continues to be visible along the axis of the beam even beyond the geometric focus of the axial rays. This clearly indicates that a purely geometric theory is insufficient to cover the facts.

To observe the uniformly spaced interference rings surrounding the bright central spot, one should use a very small aperture as the source. The necessity for this is easily understood, since the rings are closely spaced. Both sets of rings can

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**Plate I. 1–6.** Diffraction by spheres immersed in a liquid. **1–3.** Diffraction patterns with sodium light of a spherical glass ball one millimetre diameter immersed in xylene, the plane of observation being progressively removed further and further from the cell containing the sphere. Note the bright circular caustic with the accompanying fringes, and the central bright spot which progressively increases in intensity with increasing distance from the sphere. **4.** Diffraction pattern with sodium light of a spherical ball immersed in nitrobenzene, the plane of observation being rather near the focal point of the axial rays. The pattern is highly magnified in the reproduction to exhibit the two superposed sets of interference-rings. **5.** Diffraction pattern of a sphere of glass one millimetre in diameter immersed in a nitrobenzene—monobromonaphthalene mixture, with an unfiltered mercury arc as source. The faint outer caustic is due to the $\lambda$ 4046 radiation. The next bright caustic inside is due to the $\lambda$ 4358 radiation, while those nearer the centre are due to the green and yellow radiations of the mercury arc. **6.** Diffraction pattern in sodium light of a quartz sphere of 5 centimetres diameter immersed in benzene. Notice the doubling of the caustic resulting from birefringence. The bright spot at the centre of the pattern is clearly seen.

(1)

(4)

(2)

(5)

(3)

(6)

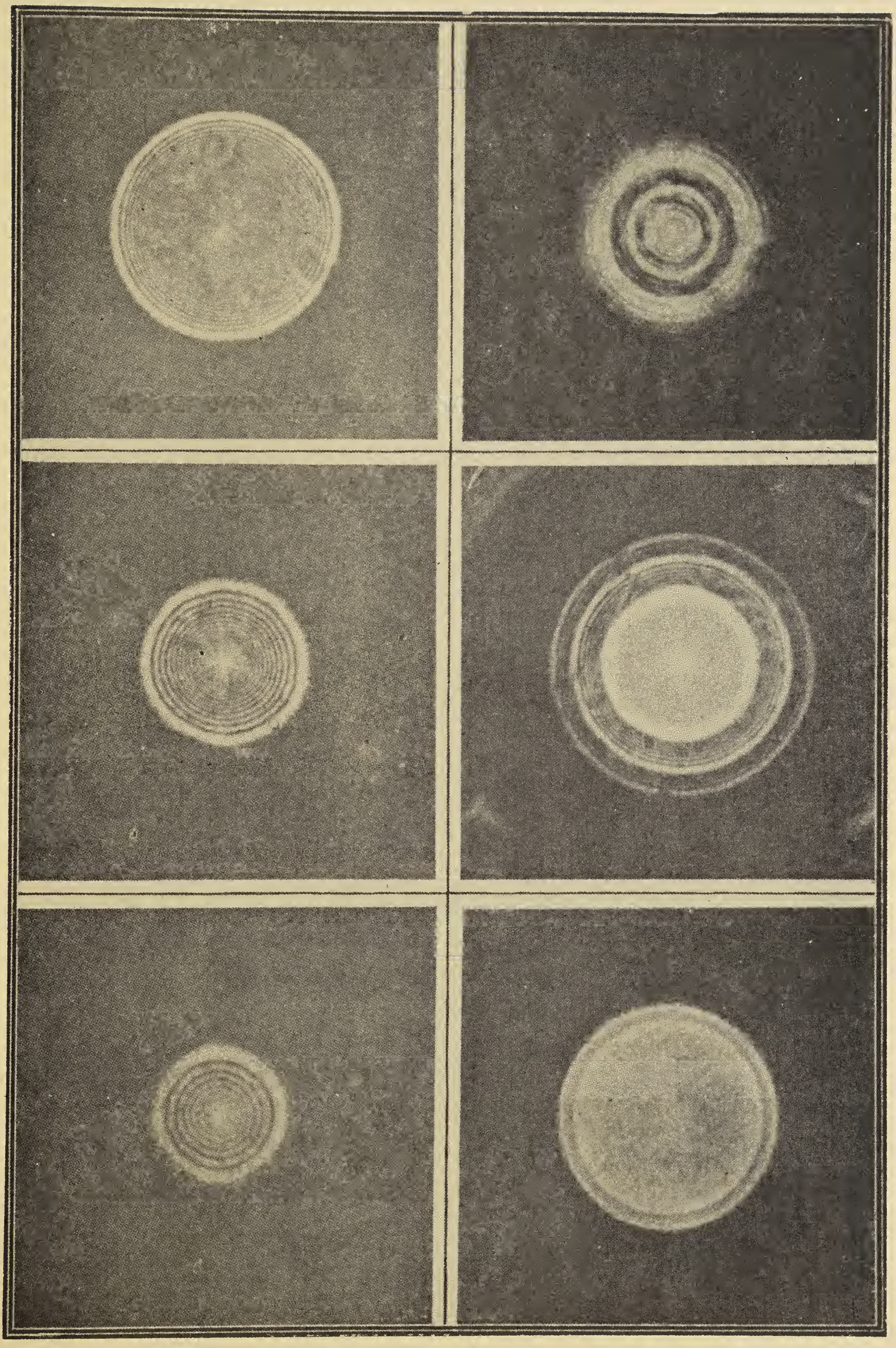

Plate I

(7)
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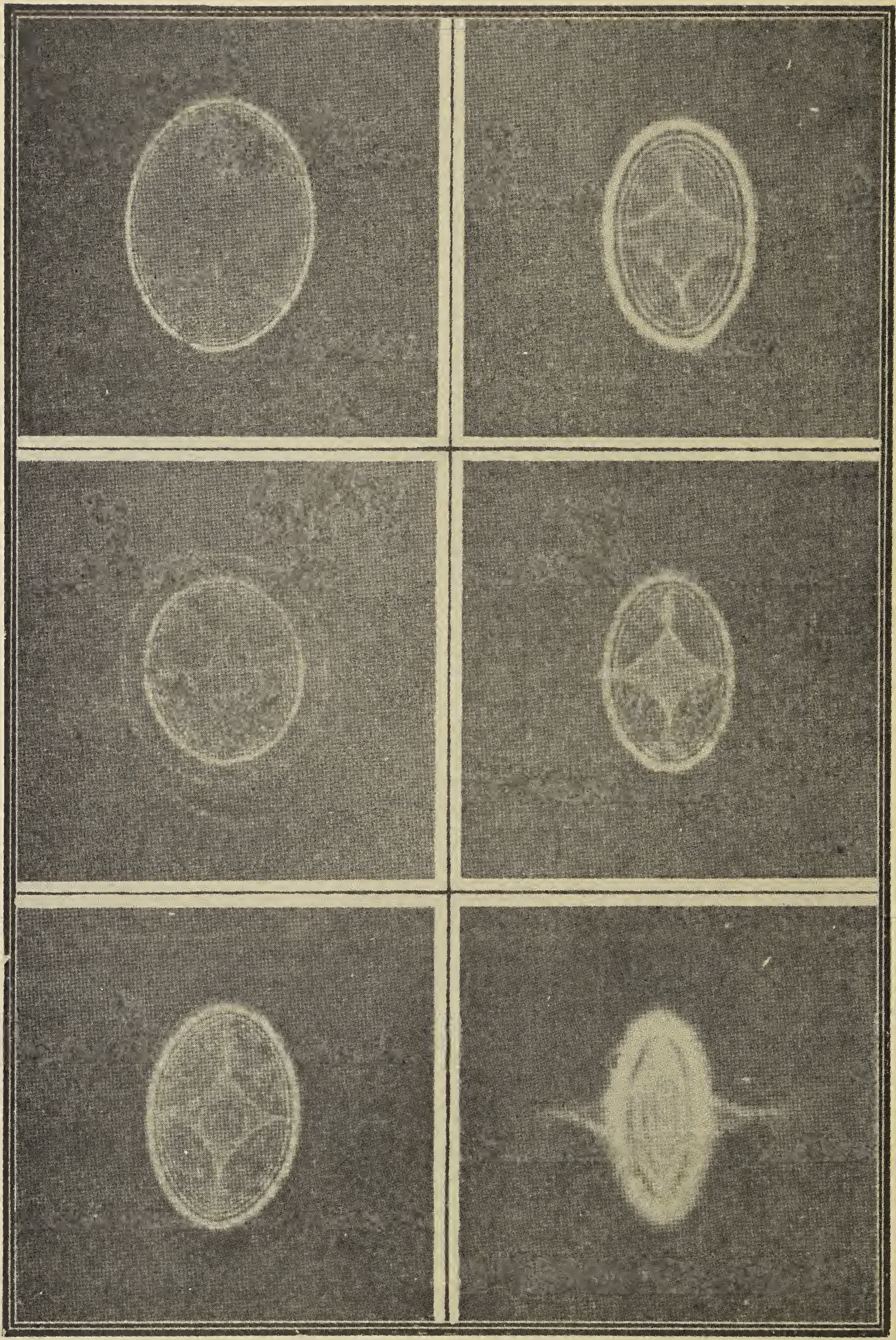
(9)



(10)
(11)
(12)

Plate II

then be readily seen superposed on each other, especially as we approach the focal point of the axial rays.

Using an aperture backed by a mercury arc (without filters) as source, the caustics corresponding to the indigo, blue, green and yellow rays of the arc are seen well separated. With a quartz sphere, two sets of circular caustics are seen instead of one. That these represent the two refractive indices of the crystal corresponding to the direction of the incident beam is readily shown by placing a polaroid in the path of the incident light and rotating it. One or the other of the two caustics is then periodically extinguished.

It is very common to find particles having a spheroidal shape amongst those used for the Christiansen experiment. With such particles, the phenomena observed are strikingly different from those exhibited by spherical particles. The caustic appearing at the periphery of the emerging light beam is not circular but elliptic in shape, as is to be expected. Instead of the bright central spot given by a spherical particle, we find a light-figure having the shape of the geometric evolute of the boundary of the beam emerging from the spheroid. This is clearly a diffraction effect having its origin at the part of the wave-front which has traversed the interior of the particle near its margin, almost grazing the surface.

## Summary

The paper describes and discusses the diffraction effects observed when a beam of light traverses a transparent sphere immersed in a liquid of slightly lower index and emerges therefrom. The two most interesting features are firstly, a concentration of intensity along the periphery of the emerging light beam which is evidently in the nature of a caustic and secondly, a concentration of intensity along the axial ray which is in the nature of a continuous focus. These two features are each accompanied by a set of interference-rings and these appear superposed on each other. Significant alterations appear in these features when the particle has a spheroidal shape. With a birefringent sphere, two sets of caustics are, in general, observed. 12 photographs in plates I and II illustrate the paper.

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**Plate II. 7–12.** Diffraction by spheroids immersed in a liquid. Diffraction patterns of a spheroidal ball. All were obtained with sodium light except figure 8 which was recorded with a mercury arc and exhibits the caustics due to $\lambda$ 4046 and $\lambda$ 4758 separately. The immersion liquid was xylene in all cases except figure 12 which was obtained with a nitrobenzene—monobromonaphthalene mixture and has been reproduced on a highly magnified scale.

*Curr. Sci.* **22** 31–33 (1953)

# The Christiansen experiment*

The present article is concerned with the phenomena observed in the well known optical experiment embodying the principle of the so-called Christiansen filters used for isolating monochromatic radiation from white light. A transparent isotropic solid is powdered and placed inside a flat-sided cell of glass, and the latter is then filled up with a liquid of which the refractive index is adjusted to equality with that of the powder for any desired wavelength in the spectrum. The cell then becomes optically transparent for such wavelength, which the rest of the spectrum is not transmitted but only diffused in its passage through the cell.

The material usually recommended for use in Christiansen filters is powdered glass which needs to be specially prepared. We have found that a convenient substance to employ in experimental studies of the Christiansen effect is hexamethylenetetramine, also known as hexamine or urotropin, which is both inexpensive and readily available as a crystalline powder. Hexamine is optically isotropic and its refractive index is intermediate between those of benzene and carbon disulphide in either of which it is nearly insoluble. Beautiful chromatic effects are observed when hexamine powder is placed in a cell and filled up with a mixture of benzene and carbon disulphide in the proportion of roughly one to four. For visual observations, it is convenient to employ, instead of a flat-sided cell, a stoppered hollow prism of 60° angle to contain the material. The advantage of doing this is that the prism functions both as a containing cell and as a dispersing apparatus. All that is necessary is to view the incandescent filament of an electric lamp from a distance through the prism held close to the eye. The spectral character of the transmitted light then becomes immediately evident, and by moving the eye to different positions on the prism face, the various effects described and illustrated below may be observed.

The spectral character and intensity of the transmitted light in the Christiansen experiment is influenced by several factors, of which the depth of the column through which the light filters is of particular importance. The set of six spectrograms reproduced in figure 1 exhibits the sharpening of the spectral band of transmission with increasing thickness of the filter; the uppermost corresponds to a thickness of 18 millimetres and the lowest to 1·5 millimetres. The exposure given had to be greatly increased with increasing thickness of the material so as to record the transmitted light with approximately the same intensity.

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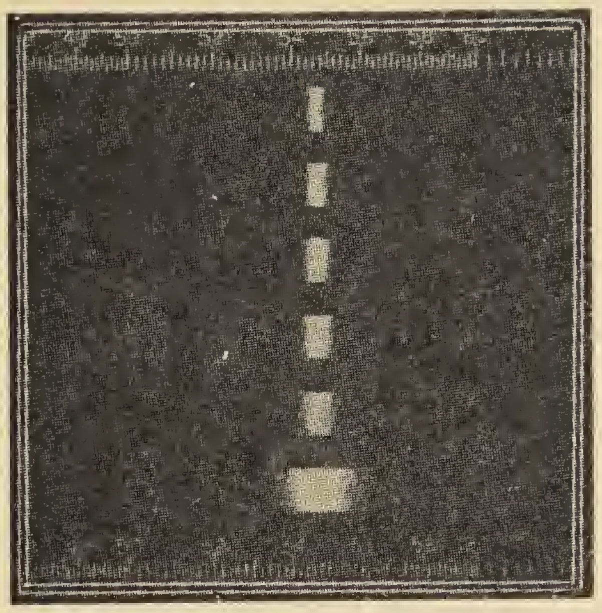
*Sir C V Raman, "The Theory of the Christiansen Experiment," *Proc. Indian Acad. Sci.*, 1949, **A29**, 381–390. See also, 1949, **A30**, 211–215 and 277–283.



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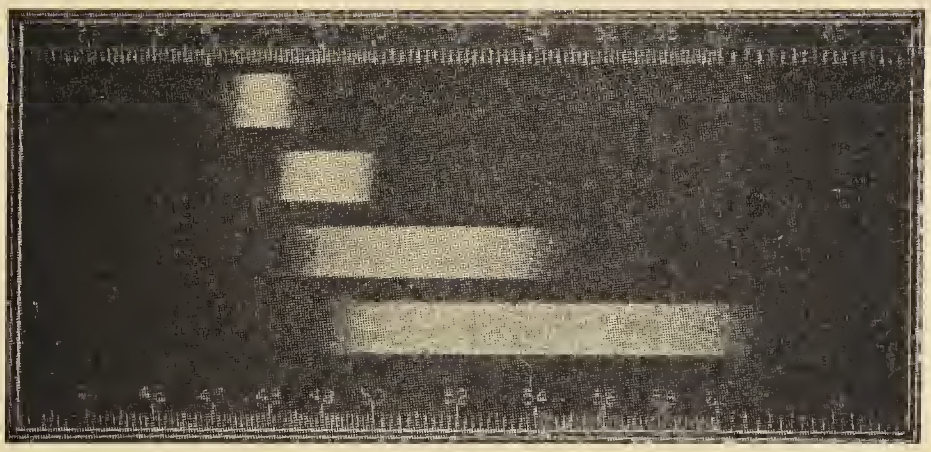

(2)

(3)

**Figures 1–3**

The size of the particles of the powder is also of great importance in determining the spectral character of the transmitted light. This effect is illustrated in the series of four spectrograms reproduced in figure 2. These were obtained with powdered glass the particles of which had been graded by sieving and elutriation into four groups having respectively as average diameters 300 $\mu$, 100 $\mu$, 18 $\mu$ and 9 $\mu$, the thickness of the layer traversed being the same, viz., 3 mm. The composition of the benzene-carbon disulphide mixture covering the powder was also adjusted to be as nearly as possible the same in the four cases. From the figure it will be seen that spectral width of the transmitted light becomes very great when the particle size is small. The widening is also totally unsymmetrical; the spectrum stretches further and further towards the red, while its short wavelength limit remains unaltered.

The series of spectrograms reproduced as figure 3 exhibits another effect of interest. They were obtained by focussing the light of a carbon arc on the slit of a spectrograph after passage through a cell containing hexamine powder suspended in a considerable excess of a benzene-carbon disulphide mixture. In the first of the series, the powder was distributed more or less uniformly throughout the entire volume of liquid. The subsequent spectrograms were recorded at short intervals of time following each other as the powder settled down in the cell, finally leaving the region traversed by the light beam nearly free of suspended powder except for the finest particles of all. The progressive increase in the spectral width of the transmitted light is particularly conspicuous in the last few spectrograms. The unsymmetrical character of this broadening is also strikingly evident. The effects noticed are a consequence of the diminishing quantity of suspended solid which is effective as well of the increasing fineness of its particles. The former of the two effects can be demonstrated separately by comparing the character of the transmitted light when the powder has all settled down to the bottom of the cell with that observed when the powder is distributed uniformly throughout the volume of the liquid.

The difference in the dispersive powers of the solid and the liquid also plays a decisive role in the Christiansen experiment. This becomes particularly obvious when this difference is very small. The series of spectrograms reproduced in figure 4 shows the spectral character of the light transmitted through different thickness of potassium chloride powder immersed in tetrachloroethane (symmetric), to which a few drops of carbon tetrachloride had been added. Practically the whole of the spectrum appears in the transmitted light when the thickness of the layer is a millimetre or two. A thickness of nearly a centimetre is necessary before any concentration of intensity in the region of equality of refractive indices of the solid and the liquid becomes noticeable. Even so, the transmission extends to the extreme limit of sensitiveness of the photographic plate in the red, while on the other hand, there is a complete cut-off on the violet side. Very different results are obtained when, instead of tetrachloroethane, either toluene or an acetone-carbon disulphide mixture is employed. These liquids have a much higher dispersive power than potassium chloride.

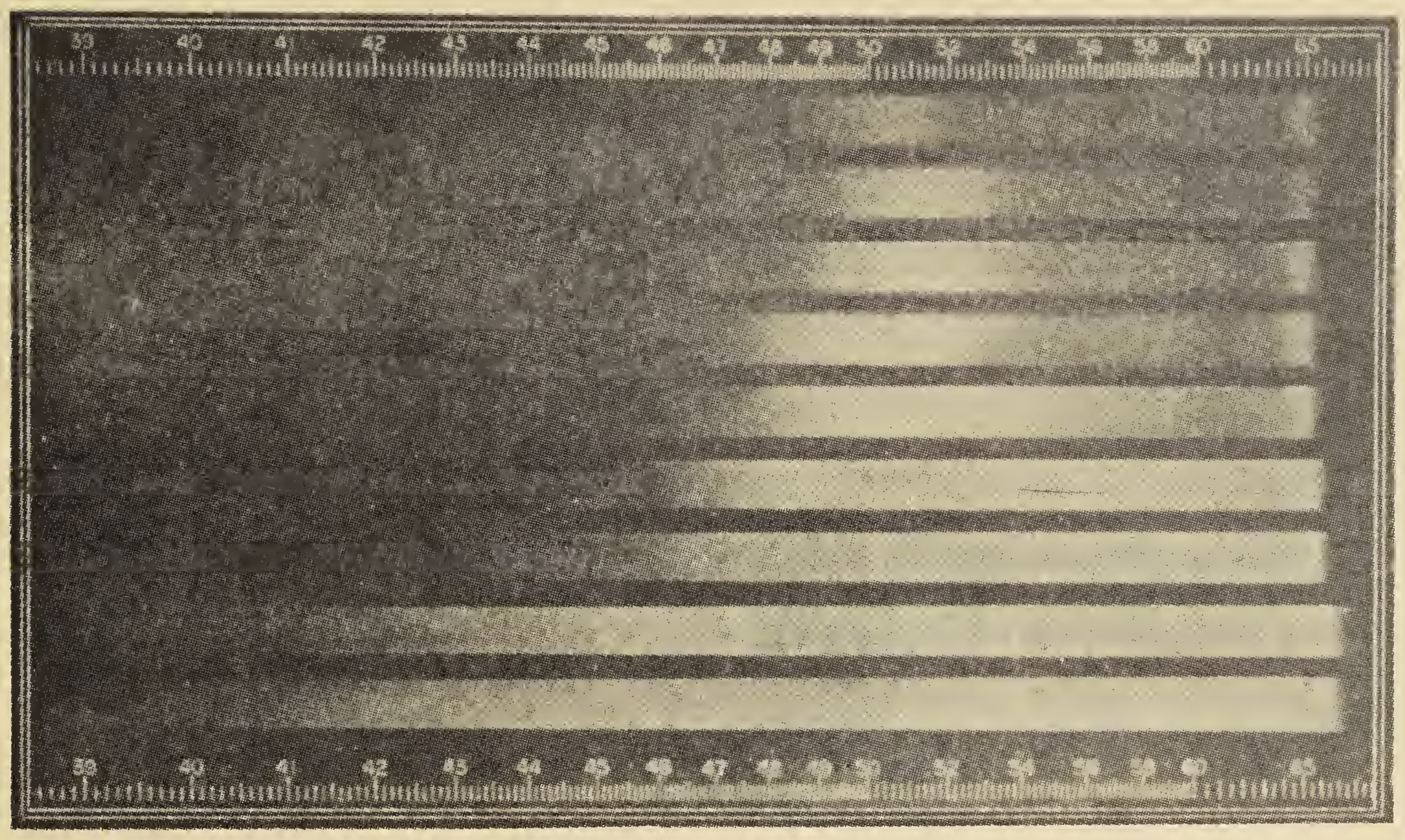

**Figure 4**

On a superficial view, one may be tempted to believe that the optical behaviour of a Christiansen filter is a matter of geometrical optics, the part of the spectrum at which there is equality of refractive index coming through without deviation, while the rest of the light is diffused as a result of multiple reflections and refractions. Such an explanation of the action of the filter is, however, not only inadequate but definitely misleading as can be seen from the facts set out and illustrated in this article. In a paper published recently and referred to above, an attempt has been made to deal with the subject from the standpoint of wave-optics. The expressions developed in that paper for the extinction coefficient of a Christiansen filter afford at least a general explanation of the facts of observation set forth in the present article. It appears not unlikely however that a fresh approach from the standpoint of the electromagnetic theory of light may be necessary to give a more complete account of the observed phenomena.

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# The structure and optical behaviour of some natural and synthetic fibres

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## 1. Introduction

Cotton, silk and wool which are the fibres commonly used in the textile industry have recently found competitors in various synthetic products, viz., rayon, nylon and the rest. The technological importance of a knowledge of the properties of all such fibres has naturally resulted in a great deal of attention being devoted to the investigation of their molecular structure. The principal tools employed in such studies have been the polarisation microscope, the X-ray diffraction-camera and the electron microscope. In the present paper we describe a different method of study, which judging from the results so far obtained and now presented, appears to be very promising. The technique employed is to record the *optical diffraction pattern* of a single fibre with arrangements generally similar to those used in an X-ray diffraction camera, but with the difference that visible light is used instead of X-rays. The incident light beam may consist of either unpolarised light or of light polarised with its vibration direction making any desired angle with the fibre axis. It is also possible to place an analyser in the path of the light diffracted by the fibre, with its vibration direction making any desired angle with the fibre axis. A great variety of patterns exhibiting observable differences may thus be recorded even in the case of a single fibre. The two most interesting and characteristic patterns are those obtained respectively with unpolarised light and with the fibre placed between crossed polaroids with its axis bisecting the angle between their vibration directions.

## 2. Experimental technique

A narrow pencil of sunlight emerging through an aperture of 1·5 mm diameter is focussed by means of a photographic lens placed 124 cm away from it to a small

brilliant image in the plane of the photographic film which is 40 cm from the lens. The fibre itself is placed in the path of the condensed beam of light 12 cm away from the plane of the recording film; a short length of the fibre is thus completely bathed in the pencil of incident light. Two stops in the path of the incident beam placed respectively before and after the lens ensure that in the absence of the fibre, the field is completely dark except in the immediate vicinity of the focussed image of the source. The fibre itself is attached to a metal frame, and by manipulating a pair of screws can be held perfectly straight without any undue tension being exerted on it. An exposure of the order of a few seconds is adequate to record the principal feature in the diffraction pattern, namely a bright straight streak running transversely to the direction of the fibre. Longer exposures of the order of a few minutes are needed to record the fainter outlying features of the pattern which reveal the inner structure of the fibre. When recording them, it is useful to place a small blackened stop immediately in front of the photographic film to prevent its fogging by over-exposure of the central spot. Replacing the photographic film by a ground glass plate and viewing the latter through a polaroid held in front of the eye, one may look for evidence of polarisation in the diffracted light in various parts of the field. By inserting a polariser in front of the lens, the light falling on the fibre may itself be polarised in any chosen direction of vibration, viz., parallel or perpendicular to the direction of the fibre or making an angle of 45° with it. The state of polarisation of the diffracted light may then be observed on the screen through a suitably held analyser. By placing the analyser immediately after the fibre in a crossed position with respect to the polariser, the depolarised component of the diffracted radiations may be recorded photographically. It is usually convenient to set the vibration directions of the polariser and analyser at an angle of 45° with the fibre axis on either side. One is then not troubled with the scattering of light by the material of the analysing polaroid.

Various improvements naturally suggest themselves, as for example, to use instead of a flat film a cylindrical camera with the diffracting fibre centred along its axis and providing arrangements for rotating the fibre or moving it up and down along its length. Such a cylindrical camera with appropriate arrangements for collimating the light beam should enable the complete diffraction pattern to be recorded. A further improvement which seems likely to prove useful is to employ, instead of sunlight, truly monochromatic light of sufficient intensity obtained with a pointolite mercury arc and a colour filter. We intend to return to the subject after effecting various improvements in technique, but have thought it desirable not to delay the publication of the present preliminary report.

## 3. Some theoretical considerations

It will be useful here to recall briefly the nature of the diffraction pattern of a dielectric cylinder in the ideal case when it consists of homogeneous and isotropic

material and has a uniform circular cross-section. If the diameter of the cylinder is sufficiently large in relation to the wavelength, the diffraction pattern may be considered roughly as consisting of two patterns which are superposed on each other; the first part is the pattern produced by a narrow obstacle with parallel edges which by Babinet's principle is the same as the diffraction pattern of a narrow slit; the second part arises from light which finds entry into the cylinder and emerges therefrom after suffering refractions and possibly also one or more internal reflections. Interferences would necessarily arise between the radiations diffracted in any given direction but which have traversed different optical paths. Monochromatic light would, in general, have to be used to fully reveal the characters of these interferences. It is obvious, however, that in all cases the diffracted light would appear as a fan of rays lying in a plane strictly transverse to the direction of the fibre, the rest of the field remaining perfectly dark.

If in the ideal case under consideration the cylinder though homogeneous and uniform is of birefringent material, the same results as those stated above would still be valid except that the distribution of intensity as well as the phase of the vibration in the fan of diffracted rays in various directions would depend upon the direction of the electric vector in the incident light beam. Hence, if the incident light be polarised, the fan of diffracted rays would in general be elliptically polarised and would therefore not be extinguished when observed through an analyser. Exceptions, however, would arise in special cases, as for example if the vibration direction of the incident light is parallel or perpendicular to the fibre-axis and coincides with an axis of the optical indicatrix of the material of the cylinder. In such cases we may expect the fan of rays to be more or less completely extinguished when the polariser and analyser are crossed.

The foregoing statements are very elegantly illustrated by the photographs reproduced as figures 11 and 12 in plate VI. Figure 12 was obtained by enclosing fragments of glass wool orientated at random between two glass plates and viewing a small distant aperture illuminated by the light of a sodium vapour lamp with the plates held close to the eye of the observer or the lens of a camera. Each piece of glass wool gives rise to a streak of diffracted light running through the field in a direction transverse to its length. A great many radial streaks accordingly appear in the field of view. Owing to the fact that all the fibres are of nearly equal diameter, the individual streaks conspire to build up a diffraction pattern consisting of a bright central disk surrounded by a system of rings of gradually diminishing intensity. The photograph shows at least two such rings clearly with a few faint and rather irregular ones further out. The entire pattern disappears when polaroids in a crossed position are placed one on either side of the plates containing the diffracting material. Such disappearance is a consequence of the fact that the material of the glass wool is optically isotropic.

If, instead of glass wool, a quantity of shredded rayon or cocoon silk is held between the two glass plates with the individual fibres orientated at random, we observe a diffraction pattern of the same general character as that reproduced in

figure 12 with glass wool. But when the combination is placed between crossed polaroids, the pattern of rings seen surrounding the source vanishes; nevertheless, a notable amount of diffracted light in the form of bright streaks continues to be visible in the field and appears as a cross with bright and dark arms. This effect is illustrated for the case of rayon in figure 11, plate VI. The bright arms of the cross in the figure bisect the angles between the vibration directions of the polaroids. Cocoon silk exhibits similar effects, the origin of which is clearly ascribable to the birefringence of the fibres. Surprisingly enough, however, the bright cross illustrated in figure 11 is not observed with either cotton or wool. That these fibres are birefringent is indeed shown by the presence of diffracted light in the field when viewed through crossed polaroids. Why the cross is not shown by either cotton or wool in these circumstances will be explained later.

Returning now to the elementary considerations set forth earlier, it is obvious that even if the dielectric cylinder consists of optically non-homogeneous material, the fan of diffracted rays would be confined to a sheet transverse to the direction of the fibre provided that both the dimensions and the structure of the cylinder are invariable along its length. Any departure from such uniformity may be expected to reveal itself by the appearance of diffracted streams of light in various other directions. From the character of the diffraction pattern observed, we can draw inferences regarding the nature of the non-uniformities giving rise to them.

## 4. Experimental results

(i) *Glass wool*

As already indicated, glass wool gives with the arrangements described in section 2 an intense straight streak running transversely to the direction of the fibre. However, with prolonged exposures, faint bands running nearly parallel to the course of the streak but exhibiting irregularities in their intensity are recorded photographically. These features indicate that though the material is in the main, isotropic and homogeneous, variations either in composition or diameter or of both exist of sufficient magnitude to give rise to observable effects. This report being a preliminary one, we have not thought it worthwhile to pursue the matter further or to reproduce any of the photographs obtained with glass wool.

(ii) *Cocoon silk*

Figures 9 and 10 in plate V reproduce photographs obtained with cocoon silk, figure 9 being recorded with incident unpolarised light and figure 10 with crossed polaroids set at 45° to the direction of the fibre. The former was taken with 30 seconds exposure and the latter with about 3 minutes. The birefringence of the silk fibre is thus clearly exhibited. The diffraction pattern, however, is not essentially different from that expected for the ideal case of a cylinder of uniform cross-

section and structure. Actually, the portion of the fibre bathed by the incident light was seen to be visibly uniform. It should be mentioned that the streak of diffracted radiation exhibited brilliant colours which, in general, were very different when observed respectively with unpolarised light and between crossed polaroids.

(iii) *Rayon*

The diffraction patterns of rayon are markedly different from those of cocoon silk (see figures 7 and 8 in plate IV). The central bright streak is bordered on either side by a series of bands running parallel to it, so much so that in heavily exposed photographs, the diffraction pattern appears as a broad band with fainter bands lying further out. In order to exhibit clearly the central bright streak in figure 7, the negative had to be heavily printed so that the fainter outlying bands were completely blotted out. The intensity of the bands varies notably along their length. This feature is even more conspicuous in the photograph recorded between crossed polaroids (figure 8).

(iv) *Chrysotile* (*asbestos*)

The finest possible fibre was isolated from a lustrous specimen of the mineral and its diffraction pattern was recorded (figures 5 and 6 in plate III). As seen visually, the brightest part of the diffraction pattern is the straight bright streak. When recorded with adequate exposures, however, it is evident that the pattern consists of a great many streaks of diffracted radiation running at various angles to the principal streak, their intensity decreasing with increasing distance from the centre of the pattern. With the exposures given, the central part of the pattern was heavily recorded while the outer streaks appear only faintly. Thus, it was not possible to make a print for reproduction which would exhibit both the inner and the outer streaks clearly. However, the general nature of the pattern can be seen from figure 5. The pattern as recorded with the crossed polaroids shows the central streak very clearly, while the marginal parts were hardly visible, presumably due to their faintness. We note that in figure 6 the central streak appears curiously curved along its length.

(v) *Cotton*

By far the most interesting patterns recorded in the present investigation are those of single cotton fibres (plate I). Figure 1 shows the pattern as observed in unpolarised light and figure 2 as observed between crossed polaroids; the former was recorded with about 30 seconds and the latter about 4 minutes exposure. The central bright streak which is clearly seen in figure 1 has almost completely vanished in figure 2. The radiating streaks of diffracted light which are very conspicuous in figure 1 are much less evident in figure 2: the latter seems to consist principally of wavy bands running approximately in a parallel direction.

(vi) *Wool*

The patterns of wool recorded respectively with unpolarised light and with crossed polaroids are reproduced in figures 3 and 4 in plate II. They are very similar to each other except that the central streak which is very clearly seen in figure 3 has completely disappeared in figure 4. In the latter case, however, the general intensity is smaller and correspondingly longer exposures are needed for recording the observed pattern. We may mention here some interesting observations made by us visually. With the incident light polarised and its vibration direction parallel to the fibre, the central streak is almost completely extinguished while the outer parts of the pattern are visible though enfeebled in intensity when the pattern is viewed through a crossed polaroid. Much the same result is observed between crossed polaroids when the vibration direction of the incident beam is perpendicular to the fibre. On the other hand, when the vibration direction of the incident light is inclined at 45° to the fibre direction, the appearance of the field is complementary in the two cases when the analysing polaroid is respectively in the parallel and in the crossed position. In the former case, the central streak is brightly visible while the surrounding field is much enfeebled; in the latter case, the central streak disappears while the marginal parts brighten up. A striking feature in figures 3 and 4 to which attention may be specially drawn is the lack of symmetry of the pattern on the two sides of the principal streak. Such an effect is not noticeable in the case of the other fibres studied.

From a comparison of the diffraction patterns for the individual fibres, it will be readily understood why aggregates of rayon and silk exhibit the phenomenon illustrated in figure 11 of plate VI, whereas cotton and wool do not. In all the four cases, the diffraction disk and rings in the central part of the field (illustrated in figure 12 of the same plate for the case of glass wool) disappear when the material is placed between crossed polaroids. The residual light in the field then consists of the diffracted radiations emerging after passage through the birefringent material of the fibres. In the cases of rayon and of silk, this radiation is mostly or entirely concentrated in the diametral streak; the intensity of this is negligible when the fibre axis is parallel to the vibration direction of either the polariser or the analyser, but is a maximum when it bisects the angle between them. The net result due to an aggregate of fibres orientated in various directions is thus the phenomenon of the cross illustrated in figure 11. On the other hand, in the cases of cotton and wool, the complex patterns seen in the field contain much of the diffracted light, and this is not extinguished when viewed between crossed polaroids, whatever be the setting of the fibres with respect to the vibration-directions of the polariser and analyser. It is not surprising that in these circumstances, cotton and wool aggregates do not exhibit the phenomenon under consideration.

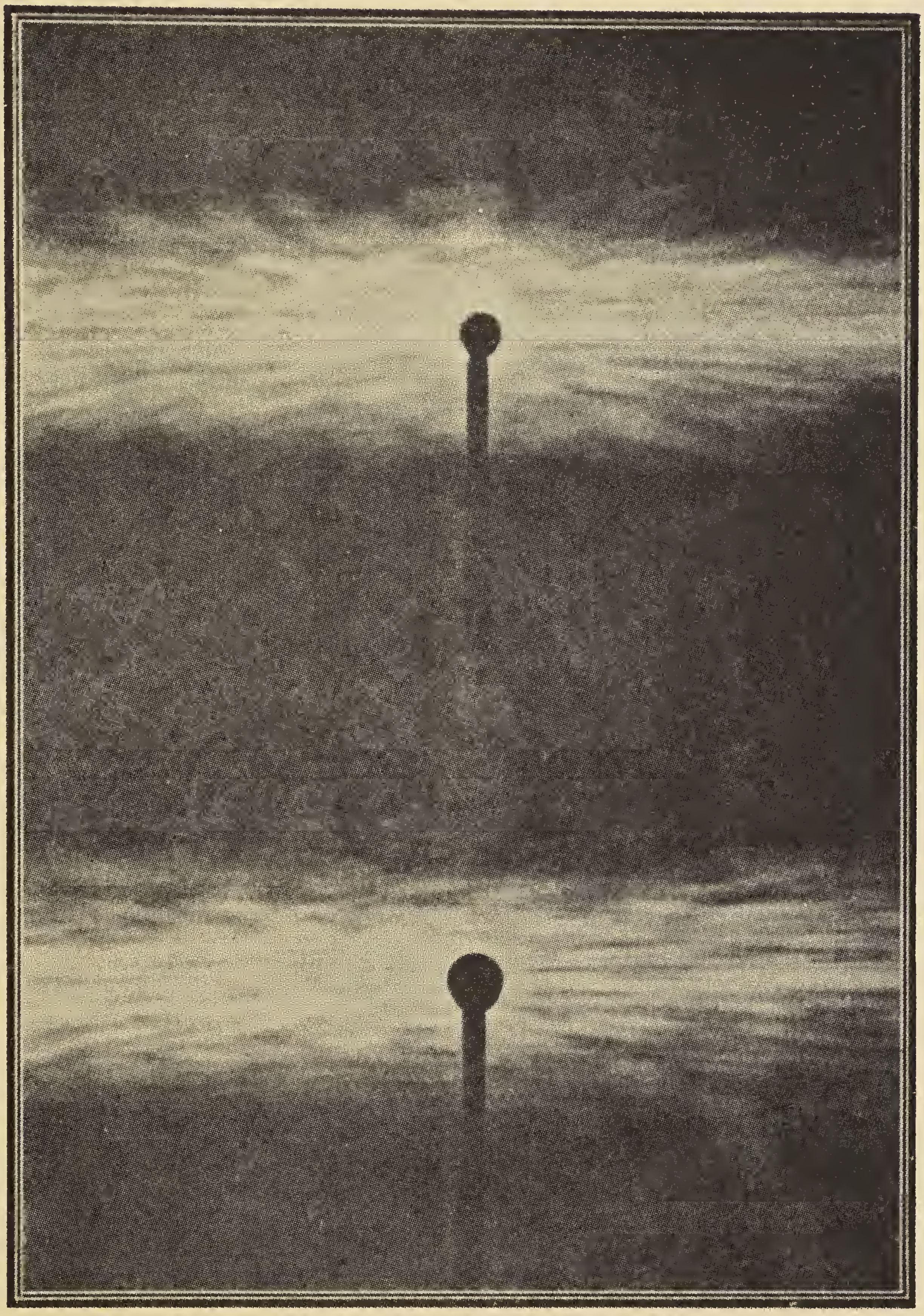
## 5. Interpretation of the results

We do not propose in this preliminary report to discuss in any detail the features observed and recorded in the plates accompanying the paper. Such a discussion could be better undertaken when the techniques have been further improved and the results are expressed in quantitative terms. We shall content ourselves here with some remarks of a general nature.

The appearance of features other than the central diametral streak in the diffraction pattern of a fibre is connected, as already remarked, with variations in the structure of the material along the length of the fibre; perfect uniformity of structure would result in the absence of such features, while *per contra* they would be very pronounced when the structure varies conspicuously from point to point along the fibre. This inference is supported in the cases under study by evidence from other sources. It is well known, for example, that silk is a crystalline fibre, its X-ray pattern exhibiting well-defined spots. The uniformity of structure thus indicated is also evident from its optical diffraction pattern which is of the simplest type. *Per contra*, in the case of cotton, the X-ray diffraction pattern shows very clearly that the cellulose groups comprising the fibre are variously inclined to the fibre axis; the characteristic X-ray reflections appear spread out as arcs instead of as sharp spots. The optical diffraction pattern reproduced in figure 1, plate I, also indicates the presence of diffracting units inclined to the length of the fibre. That the diffraction pattern does not vanish when viewed between crossed polaroids but continues to be visible with certain significant changes indicates that these diffracting units are themselves birefringent. Thus, while supporting the indications furnished by the X-ray data, the optical diffraction pattern of the cotton fibre gives us a fuller insight into its structure.

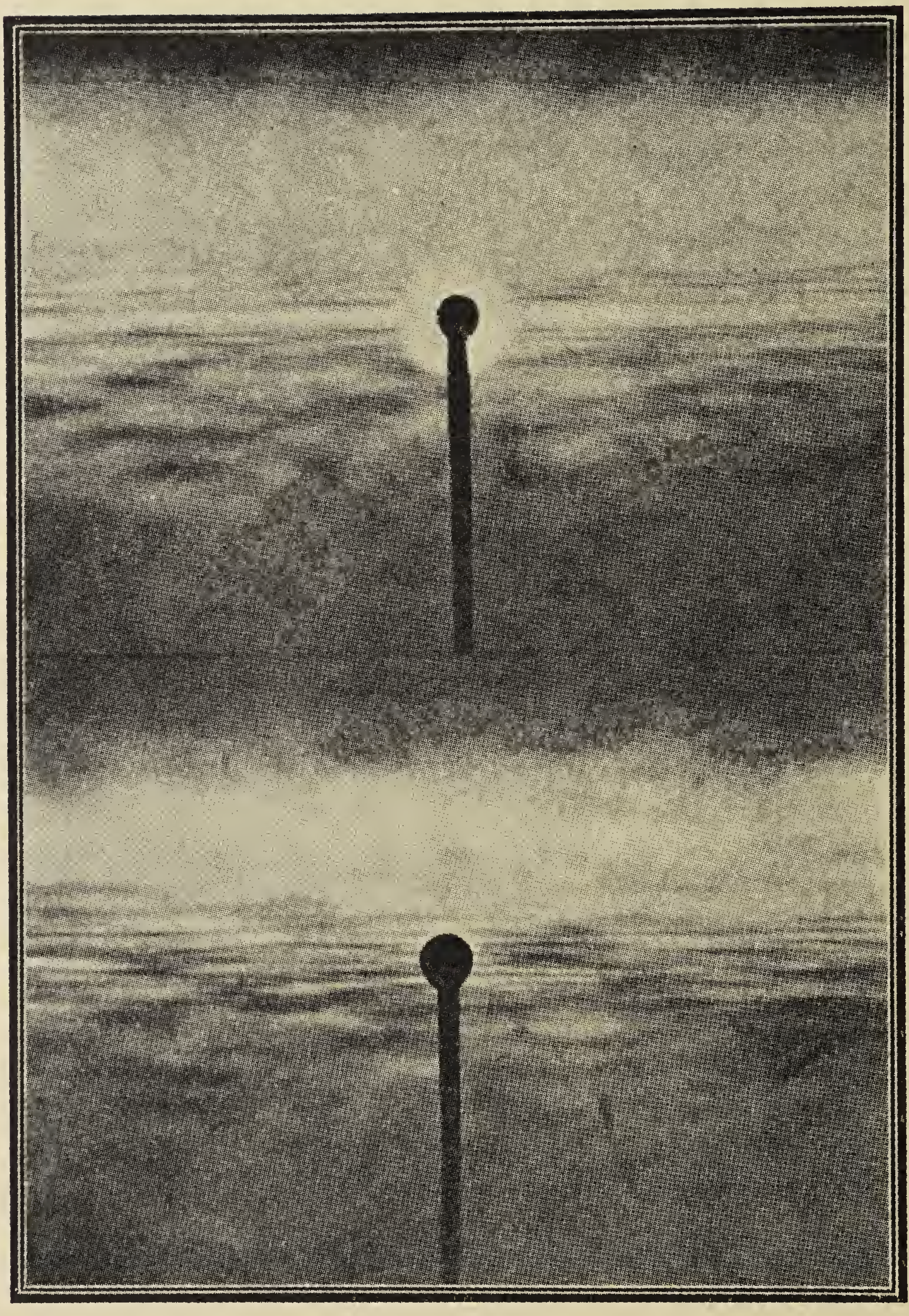
## 6. Summary

When a narrow pencil of light which comes to a focus in the plane of a photographic film traverses a single fibre set transversely to it in its path, the light diffracted by the fibre in various directions records itself on the film when adequate exposures are given. Photographs thus obtained with single fibres of cotton, wool, silk, rayon and chrysotile are reproduced in the paper. The birefringence of the fibres exhibits itself by the non-disappearance of the pattern when two polaroids crossed with respect to each other are placed one on either side of the fibre, the axis of the latter bisecting the angle between their vibration directions. The diffraction patterns indicate the inner structure of the fibres and are strikingly different for the different materials.




**Figures 1 and 2.** Diffraction pattern of cotton fibre. **1.** With unpolarised light. **2.** Between crossed polaroids.

**Plate I**

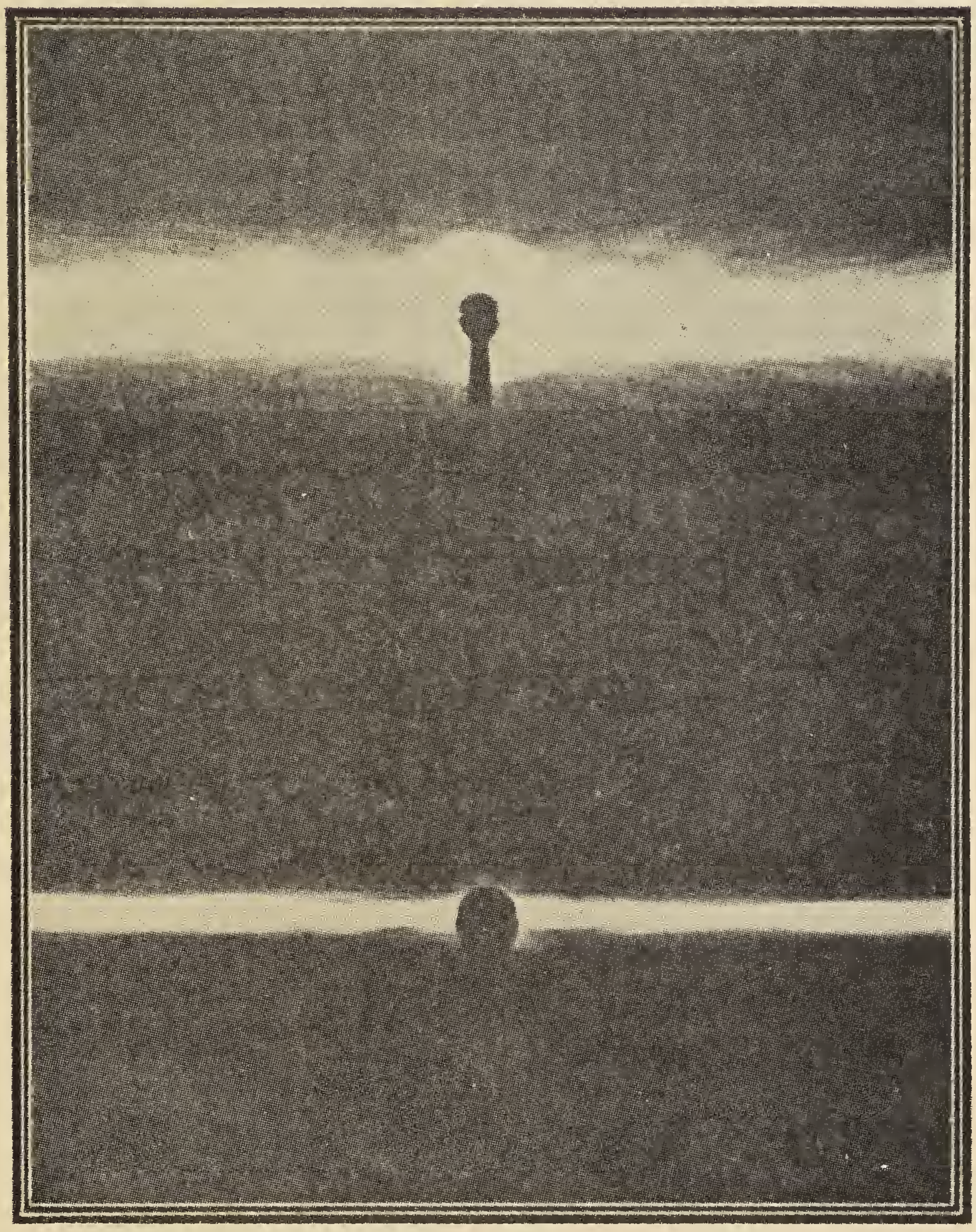


(3)

(4)

**Figures 3 and 4.** Diffraction pattern of wool fibre. **3.** With unpolarised light. **4.** Between crossed polaroids.

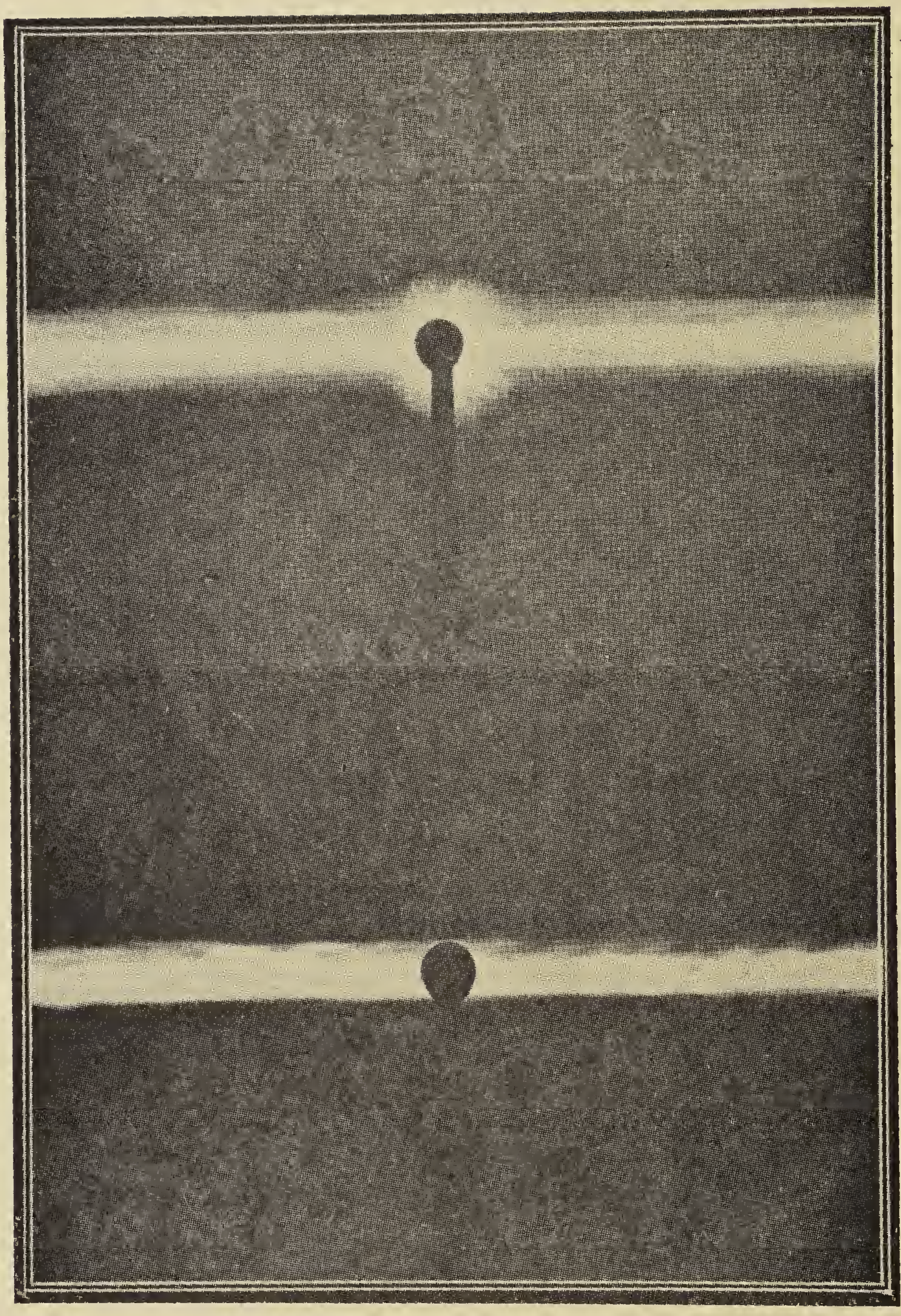
**Plate II**


(5)

(6)

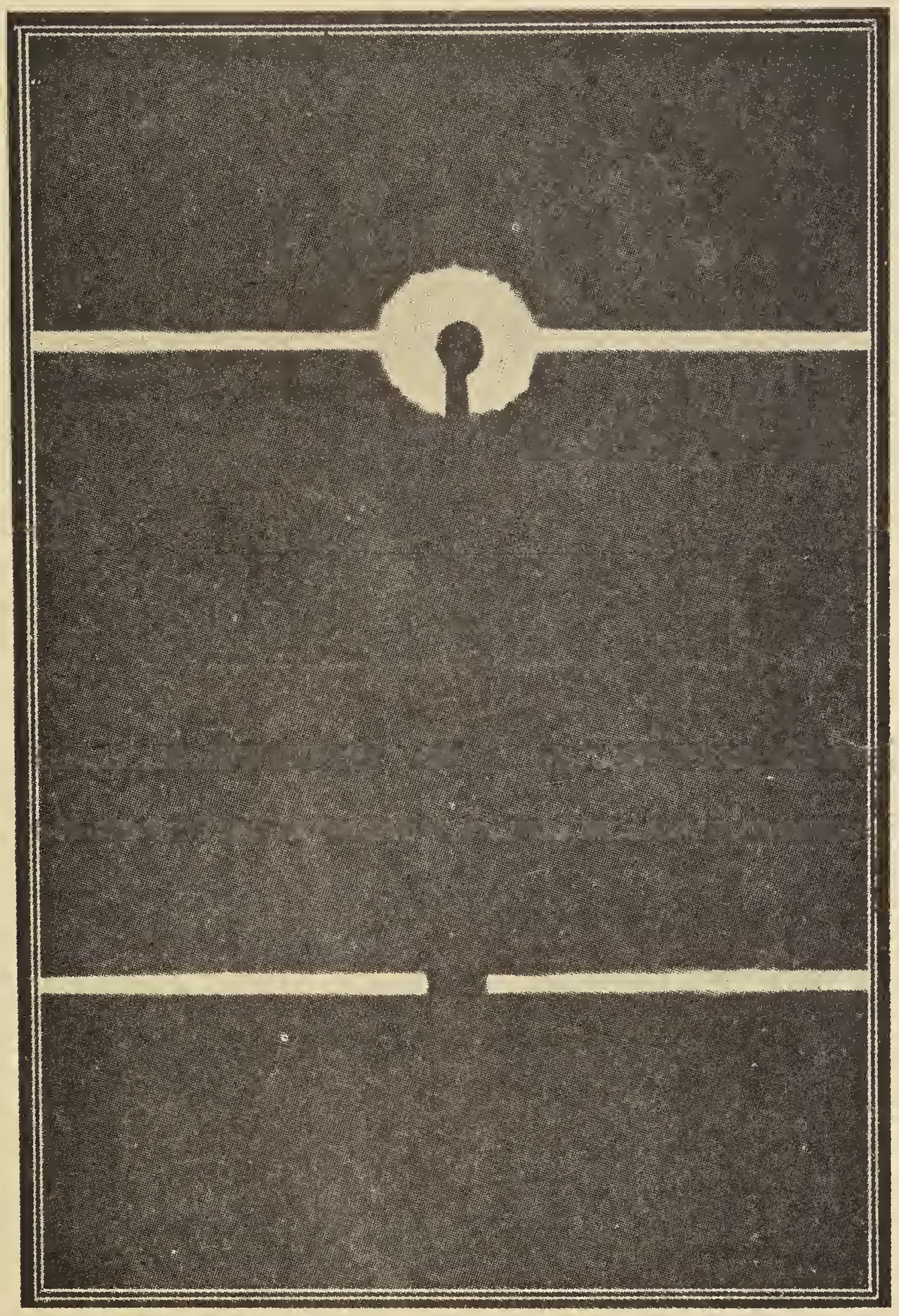
**Figures 5 and 6.** Diffraction pattern of chrysotile (asbestos) fibre. **5.** With unpolarised light. **6.** Between crossed polaroids.

**Plate III**


(7)
(8)

**Figures 7 and 8.** Diffraction pattern of rayon fibre. **7.** With unpolarised light. **8.** Between crossed polaroids.

**Plate IV**

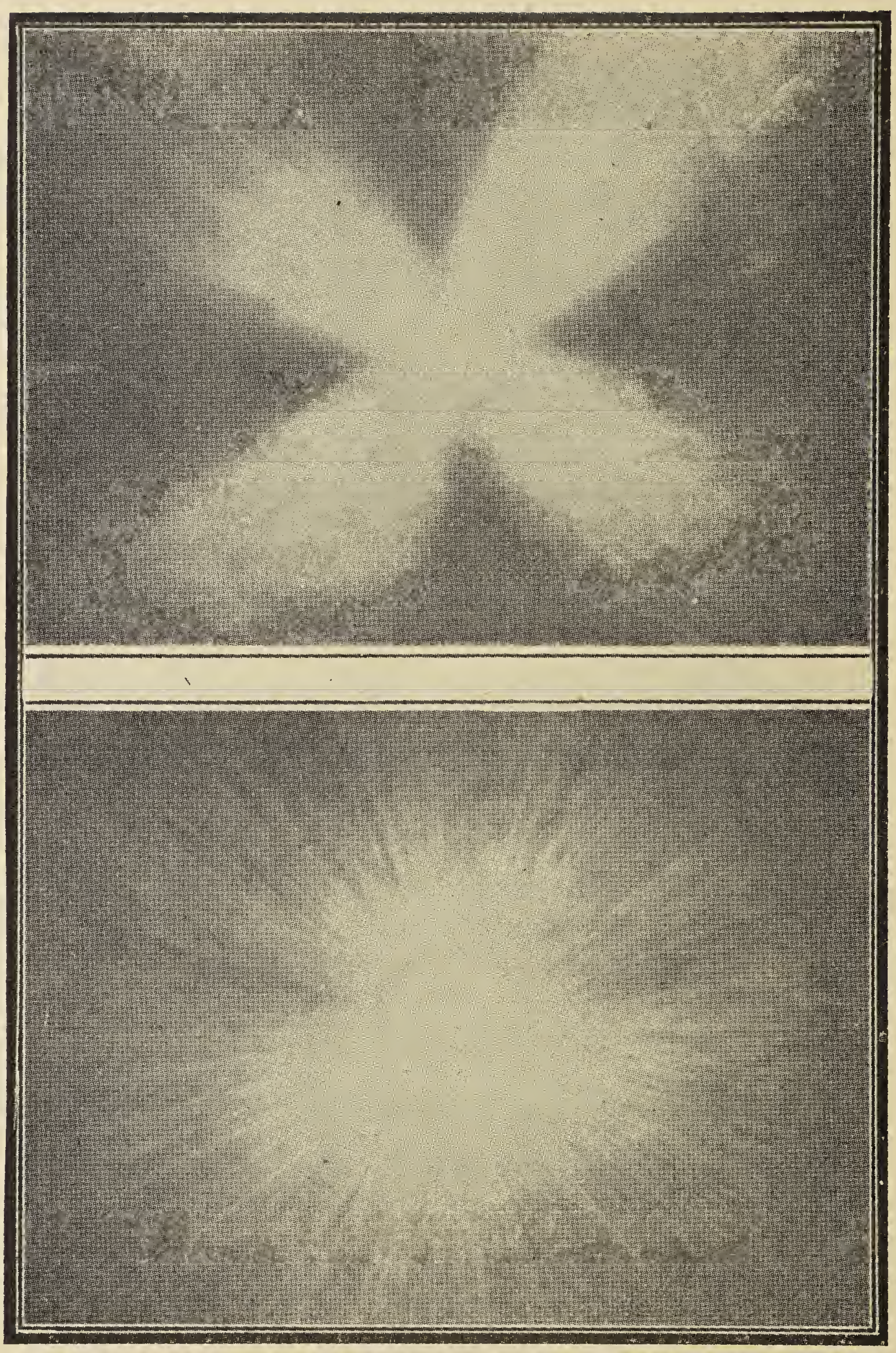


(9)

(10)

**Figures 9 and 10.** Diffraction pattern of cocoon silk fibre. **9.** With unpolarised light. **10.** Between crossed polaroids.

**Plate V**



(11)

(12)

**Figures 11 and 12.** Diffraction patterns of fibre aggregates. **11.** Rayon between crossed polaroids. **12.** Glass-wool in unpolarised light.

**Plate VI**

## References

Detailed accounts of the structure of fibres of cotton, silk, wool and rayon as determined by the known methods are to be found in several recent treatises, amongst others, those listed below:

E Ott *Cellulose*, Chap. III, Interscience Publishers, 1946.

P H Hermans *Physics and Chemistry of Cellulose Fibres*, Second Part, Elsevier, 1949.

G L Clark *Applied X-rays*, Chap. XXIII, McGraw-Hill, 1940.

A Frey-Wyssling *Sub-Microscopic Morphology of Protoplasm and Its Derivatives*, pp. 193–203, Elsevier, 1948.

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# The theory of the propagation of light in polycrystalline media

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## 1. Introduction

Many common minerals occur in nature as polycrystalline aggregates. For example, quartz, calcite and gypsum appear in massive form respectively as quartzite, marble and alabaster, and on examining thin sections of these materials under the polarisation microscope, it is found that they consist of great numbers of crystallites variously orientated and firmly adherent to each other so as to form a coherent solid. The size, shape and manner of orientation of the crystallites may differ enormously in individual cases. Some minerals are indeed cryptocrystalline, in other words, the particles are so small that they cannot be identified by the usual polariscopic methods and require the aid of X-ray analysis to enable their true nature to be determined.

The foregoing is by way of stressing the importance alike to the mineralogist and to the physicist of a study of the physical properties of polycrystalline aggregates. It is obvious that the optical properties of the single crystal and especially its birefringence and pleochroism (if any) would play a dominant role in determining the optical characters of the polycrystalline aggregate. Considering the matter from the standpoint of geometrical optics, it is evident that when light enters a polycrystalline aggregate, it would suffer reflection at the intercrystalline boundaries. The stronger the birefringence, the greater would be the coefficient of reflection at these boundaries and hence the more quickly would the incident light be returned back towards the source. The brilliant whiteness of pure marble is thus a recognisable consequence of the strong birefringence of calcite. On the other hand, if the birefringence be feeble as in the case of quartz and gypsum, the incident light would penetrate far more deeply into the aggregate. Ultimately, all the light would necessarily be turned back provided that a sufficient thickness of the material be available and that no absorption intervenes. If the thickness of the material be insufficient, a part of the light would diffuse—a phenomenon readily

observed with various materials. Light so emerging would be depolarised even if the light incident on the plate were fully polarised in the first instance.

Various considerations indicate that a purely geometric theory is inadequate to cover all the optical phenomena actually exhibited by polycrystalline media. It will suffice to mention here the question of the influence of the size of the crystallites. Geometric considerations would suggest that the smaller the crystallites, the more numerous would be the reflections and refractions at the intercrystalline boundaries, and hence the more rapidly would the incident light be diffused and extinguished in its passage through the medium. Experience however suggests that the contrary may actually be the case, and that the more fine-grained the material is, the more deeply would the light penetrate into it. Various minerals, e.g., alabaster and jade, which exhibit marked translucency, are usually fine-grained; the finer the grain, the more deeply does light penetrate into them. This suggests that the optical problems presented by polycrystalline aggregates require to be considered from the standpoint of the wave-theory of light. That indeed is the object of the present paper.

## 2. A simplified model

To obtain some results of physical interest and also with a view to simplify the mathematics, we shall here restrict ourselves to the case of a feebly birefringent material and consider the case in which light is incident normally on a plate with parallel faces; this is assumed to be sufficiently thick to include a great many individual crystallites but not so thick that the incident light is completely extinguished before it can emerge at the rear face. We may disregard the geometric course of the individual rays of light and view the matter purely from the wave-theoretical standpoint. Owing to the varying orientation of the individual crystallites, the waves of light entering the plate would be retarded to different extents in passing through them. To enable the resulting total retardation to be evaluated, we use a simplified model and assume the plate to be an assembly of a great number of small cubical blocks each having a common edge-length $\Delta$ and completely filling up the available space. Each block is assumed to be a single crystallite, and the three edges of each cube to be parallel to the three optic directions for which the refractive indices are $\mu_1$, $\mu_2$ and $\mu_3$ respectively. To introduce the idea of varying orientation and to take account of its influence on the propagation of light through the material, we assume the incident light-beam to be plane-polarised with its vibration direction parallel to one set of edges of the cubical blocks; on the other hand, the operative refractive index of any one block may be either $\mu_1$ or $\mu_2$ or $\mu_3$, the respective probabilities for these being $p_1$, $p_2$, $p_3$. The case where the three probabilities are equal would correspond to a random orientation of the crystallites in the present restricted sense of that term. More generally, by giving appropriate values to $p_1, p_2, p_3$ such

that their sum remains equal to unity, we obtain a representation of a polycrystalline aggregate with any desired measure of preferred orientation along the particular direction under consideration. If, for example, we put $p_1 = 1$ while $p_2$ and $p_3$ are zero, it would mean that all the particles of the aggregate have a common refractive index for the particular direction of vibration, though the indices may be different in the perpendicular direction.

On the assumptions stated, the incident plane-polarised disturbance would remain plane-polarised in its passage through the plate, though subject to phase retardations of varying extents. The situation would no doubt be different for any actual polycrystalline material, since the incident plane-vibration would be transformed to an elliptic vibration and the parameters describing the ellipticity would alter as the disturbance passes from crystallite to crystallite. While it would no doubt be possible to deal mathematically with this general case, a very considerable simplification is effected by our present assumptions, and as we shall see, the usefulness of the results obtained is not affected thereby.

A further question needing consideration is the effect of the reflections which would occur at the boundary between every two successive blocks. This would obviously diminish the amplitude of the transmitted disturbance. As a first approximation, we may assume such diminution to be the same over all the individual elementary areas $\Delta^2$ on the rear surface of the plate and represent it by a numerical factor of appropriate magnitude. In other words, we ignore the variation of *amplitude* over the different elementary areas $\Delta^2$ on the rear face of the plate and consider only the variations of *phase*. The latter are in reality of much greater importance for the determination of the final observable result.

## 3. Mathematical formulation

Let us suppose that the wave-train before entry into the plate is represented by

$$y = \exp\left[\frac{2\pi i}{\lambda}(ct - Z)\right] \tag{1}$$

and that there are $n$ cells along the direction of the thickness of the plate. We shall first consider a typical case in which the wave has passed through $k_1$ cells of refractive index $\mu_1$, $k_2$ cells of refractive index $\mu_2$ and $k_3$ cells of refractive index $\mu_3$ before emerging from the plate. The numbers $k_1$, $k_2$ and $k_3$ can all vary from zero to $n$ subject to the relation

$$k_1 + k_2 + k_3 = n. \tag{2}$$

The optical path retardation of the emergent wave would then be equal to $(k_1\mu_1 + k_2\mu_2 + k_3\mu_3)\Delta$.

Now the number of ways in which $k_1$, $k_2$ and $k_3$ cells can be orientated along a

row of $n$ cells so as to have refractive indices $\mu_1$, $\mu_2$ and $\mu_3$ is obviously

$$\frac{n!}{k_1!k_2!k_3!},$$

and the probability of occurrence of each one of these cases is $p_1^{k_1}p_2^{k_2}p_3^{k_3}$. Hence the proportion of the total area of the rear surface of the plate from which a wave represented by

$$\exp\left[\frac{2\pi i}{\lambda}(ct - Z - \overline{k_1\mu_1 + k_2\mu_2 + k_3\mu_3}\Delta)\right] \tag{3}$$

emerges is equal to

$$\frac{n!}{k_1!k_2!k_3!}p_1^{k_1}p_2^{k_2}p_3^{k_3}. \tag{4}$$

The emergent wave-train can now be obtained by summation of waves of the type (3) with their appropriate amplitudes and phases for all possible integral values of $k_1$, $k_2$ and $k_3$ satisfying the relation (2). We therefore have for the emergent wave

$$y = P\sum_{k_1+k_2+k_3=n}\frac{n!}{k_1!k_2!k_3!}p_1^{k_1}p_2^{k_2}p_3^{k_3}\exp\left[\frac{2\pi i}{\lambda}(ct - Z - \overline{k_1\mu_1 + k_2\mu_2 + k_3\mu_3}\Delta)\right] \tag{5}$$

where $P$ is a factor which is introduced to take into account the loss in intensity of the light due to reflections at the intercrystalline boundaries.

In view of the multinomial theorem, equation (5) may be rewritten as

$$y = P\exp\left[\frac{2\pi i}{\lambda}(ct - Z)\right]\left[p_1\exp\left(\frac{-2\pi i\mu_1\Delta}{\lambda}\right) + p_2\exp\left(\frac{-2\pi i\mu_2\Delta}{\lambda}\right) + p_3\exp\left(\frac{-2\pi i\mu_3\Delta}{\lambda}\right)\right]^n. \tag{6}$$

The average refractive index of the medium is clearly

$$\mu = (p_1\mu_1 + p_2\mu_2 + p_3\mu_3). \tag{7}$$

If therefore we set $\nu_1 = (\mu_2 - \mu_3)$; $\nu_2 = (\mu_3 - \mu_1)$; and $\nu_3 = (\mu_1 - \mu_2)$, we can then express $\mu_1$, $\mu_2$ and $\mu_3$ as

$$\begin{aligned}\mu_1 &= \mu + (p_2\nu_3 - p_3\nu_2)\\ \mu_2 &= \mu + (p_3\nu_1 - p_1\nu_3)\\ \mu_3 &= \mu + (p_1\nu_2 - p_2\nu_1).\end{aligned} \tag{8}$$

Further, the thickness of the plate is given by $d = n\Delta$. Hence substituting the

relations (8) in (6) and expanding the exponential terms in power series of their arguments, one obtains

$$y = P\exp\left[\frac{2\pi i}{\lambda}(ct - Z - \mu d)\right] \times \left\{1 - \frac{2\pi^2\Delta^2}{\lambda^2}\sum p_1(p_2\nu_3 - p_3\nu_2)^2\right\}^n. \quad (9)$$

As the birefringence is assumed to be small, we have ignored terms of third and higher powers of $(\mu_1 - \mu_2)$, $(\mu_2 - \mu_3)$ and $(\mu_3 - \mu_1)$ in (9). Also by means of a small simplification, it can be verified that

$$\sum p_1(p_2\nu_3 - p_3\nu_2)^2 = \sum p_2 p_3 \nu_1^2.$$

Hence, we can rewrite (9) as

$$y = P\exp\left[\frac{2\pi i}{\lambda}(ct - Z - \mu d)\right]\left\{1 - \frac{1}{n} \times \frac{2\pi^2\Delta d}{\lambda^2}\sum p_2 p_3 \nu_1^2\right\}^n$$

$$= PR\exp\left[\frac{2\pi i}{\lambda}(ct - Z - \mu d)\right],$$

where

$$R = \exp\left(\frac{-2\pi^2\Delta d}{\lambda^2}\sum p_2 p_3 \nu_1^2\right) \quad \text{as } n \text{ is large.}$$

The ratio of the intensity of the transmitted light to that of the incident radiation is therefore given by

$$\frac{I}{I_0} = P^2R^2 = P^2\exp\left[\frac{-4\pi^2\Delta d}{\lambda^2}\sum p_2 p_3(\mu_2 - \mu_3)^2\right]. \quad (10)$$

If the three optic axes of any cube have the same probability of being orientated in the direction of the incident light, then $p_1 = p_2 = p_3 = \frac{1}{3}$ and (10) reduces to

$$\frac{I}{I_0} = P^2\exp\left[\frac{-8\pi^2\Delta d}{9\lambda^2}\left(\sum \mu_1^2 - \sum \mu_2\mu_3\right)\right]. \quad (11)$$

## 4. Significance of the results

The physical meaning of the result stated in (10) is that the plane-polarised waves incident on the front of the plate emerge from the rear face of the plate also as a plane-polarised vibration but with an attenuated amplitude determined by an exponential factor involving four variables, namely the size of the particles, the thickness of the plate, the wavelength of the light and a quantity which is a measure of the birefringence of the material, since it vanishes when the three indices $\mu_1$, $\mu_2$, $\mu_3$ are all equal. The appearance of $\lambda^2$ in the denominator indicates that white light entering the plate would emerge enfeebled but with the longest

waves predominant, in other words, much reddened in colour. Since the wavelength $\lambda$ is a small quantity, the actual intensity of the emerging light would be negligible if both $\Delta$ and $d$ are large. The individual crystallites have, in fact, to be quite small and the total thickness traversed should be moderate if any observable fraction of the light is to emerge as a coherent optical beam. We have already assumed the birefringence to be small and the need for such assumption is reinforced by our final result which indicates that unless the three indices $\mu_1$, $\mu_2$, $\mu_3$ differ from each other by quantities which are small fractions of their absolute magnitudes, no light can emerge from the rear of the plate.

We may illustrate the foregoing remarks by the case of a plate of alabaster 1 mm thick taking $\lambda$ and $\Delta$ equal to 5896 A.U. and 1 $\mu$ respectively; the three indices for gypsum are $\mu_1 = 1{\cdot}520$, $\mu_2 = 1{\cdot}523$ and $\mu_3 = 1{\cdot}530$. The percentage of transmission then comes out as 13·5% but increases to 37% and 82% if $\Delta = 0{\cdot}5\,\mu$ and $0{\cdot}1\,\mu$ respectively. Thus, the formula indicates that a plate of alabaster approaches practically complete transparency as the crystallites of which it is composed approach colloidal dimensions.

The case of preferred orientations is also of interest in view of the known optical behaviour of chalcedony and of certain forms of gypsum. Taking now the general formula (10), if we put $p_1 = 1$ while $p_2$ and $p_3$ are both zero, the formula indicates that the transmission becomes complete. In other words, if the crystallites are so orientated that all of them have a common refractive index for the direction of vibration of the incident light, then we have a complete transmission of the incident light wave. But the position would be totally different for a perpendicular direction of vibration if the refractive indices for the latter direction differ from crystallite to crystallite. The general formula (10) would then show only a partial transmission depending upon the actual values of the probabilities and the refractive indices for that direction. Since the latter transmission would depend upon the thickness $d$, it would follow that if the light incident upon the plate be unpolarised, the state of polarisation of the emerging light would vary with the thickness of the plate. Formula (7) also indicates that the effective refractive index of the medium would be different for the two directions of vibration under consideration. Light which is plane-polarised in any arbitrary azimuth when incident on the plate, would emerge as elliptically polarised light, the parameters describing such ellipticity varying with the thickness of the plate—a phenomena readily capable of experimental verification with materials of the nature under consideration.

## 5. Some further remarks

The question naturally arises as to where the energy goes which disappears from the incident light according to (10). The answer to this is not far to seek. Since the reduction of amplitude is a consequence of the random variations of phase over

the elementary areas of the rear face of the plate, the missing light would appear as diffracted radiation spread out in various directions surrounding the direction of the incident beam. The angular dimensions of the diffraction halo would obviously be comparable with the ratio between the wavelength $\lambda$ of the light and the linear dimension $\Delta$ of the crystallites which we have assumed the material to be composed of. A diffusion halo of this type can indeed readily be observed on viewing a bright source of light through a thin plate of alabaster surrounding the sharply defined image of the source itself. According to the theory developed above, both the light emerging from the rear surface of the plate and the light appearing in the diffusion halo would be perfectly polarised if the incident light be itself plane-polarised. These results are consequences of the special assumption regarding the orientation of the crystallites which we have made. We may now ask ourselves whether they would continue to be true if the particles are orientated truly at random. The answer to this question is most readily ascertained by making a few actual observations with a plate of alabaster sufficiently thin to give a true transmission. It is then observed that the true transmission is completely polarised while the diffraction halo seen overlying it is imperfectly polarised. A more complete mathematical theory which takes account of the ellipticity resulting from the passage of light through an arbitrarily orientated crystal block would no doubt yield results in agreement with these facts of observation. It is clear, however, that the present theory suffices to indicate the state of polarisation of the transmitted light correctly and also its intensity, at least as regards the order of magnitude. But the theory fails to indicate the state of polarisation of the diffracted light accurately, since it ignores the ellipticity produced by the passage of light through a birefringent crystal in an arbitrary orientation; such ellipticity would obviously result in diverting some of the incident energy into the perpendicular component of vibration as diffracted radiation.

## Summary

A formula based on wave-theoretical considerations is deduced which gives the coefficient of extinction of plane-polarised light traversing a polycrystalline aggregate in terms of the wavelength of the light, the size of the particles and their birefringence. The general formula covers the case where the particles have preferred orientation expressible by three different probability numbers for three mutually perpendicular directions, and the special case of isotropic orientation is readily derivable therefrom. The significance of the results is discussed in relation to the facts of observation.

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# A generalised theory of the Christiansen experiment

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## 1. Introduction

In a paper[1] published in these *Proceedings* nearly six years ago, the theory of the well known Christiansen experiment was discussed on a wave-optical basis. The expression derived in that paper for the transmission coefficient of a Christiansen light filter showed it to be an exponential function involving five variables, namely the wavelength of the light, the thickness of the cell, the size of the individual particles of the powder, the difference between the refractive indices of the powder and the surrounding liquid and finally also the proportions of the volume of the cell occupied respectively by the liquid and by the particles of the powder. In the present paper, it is proposed to deal with the more general case in which the particles of the powder are *birefringent* and hence their refractive index varies with the orientation of the crystallites within the cell. The mathematical treatment adopted is on much the same lines as that followed by us in discussing the theory of the propagation of light in polycrystalline media.[2] The only difference, in fact, is that some of the cubical elements of volume each of edgelength $\Delta$ which we imagine the cell to consist of must now be considered as being filled either by the liquid of refractive index $\mu_l$ or by the crystallites. These latter are assumed to be of cubical shape and to have their edges parallel to the three optic directions for which the refractive indices are $\mu_1$, $\mu_2$ and $\mu_3$ respectively. We also assume the incident light beam to be plane-polarised with its vibration direction parallel to one set of edges of the cubical blocks and that the operative refractive index of any one block may be either $\mu_1$, $\mu_2$ or $\mu_3$ with equal probabilities if it is a crystallite or $\mu_l$ if it is filled with the liquid.

## 2. Mathematical formulation

We shall denote by $p$ and $q$ the respective probabilities of a cubical block having any one of the three principal refractive indices $\mu_1$, $\mu_2$, $\mu_3$ of the crystallites and of the refractive index $\mu_l$ of the liquid. Then $3p$ and $q$ would represent the proportion

of solid and liquid elements in the Christiansen filter and therefore

$$3p + q = 1 \tag{1}$$

As before, we consider a typical case in which the incident wave-train which we shall represent by

$$y = \exp\left[\frac{2\pi i}{\lambda}(ct - Z)\right], \tag{2}$$

encounters in its passage through the Christiansen cell $(n - m)$ elementary cells of solid blocks and $m$ cells of liquid elements. The probability of the occurrence of this event is obviously

$$\frac{n!}{(n-m)!m!}(3p)^{n-m}q^m. \tag{3}$$

If further in any specification of the state of orientation of the crystallites inside the filter, $k_1$, $k_2$ and $k_3$ of the $(n-m)$ cubical blocks considered above have refractive indices $\mu_1$, $\mu_2$ and $\mu_3$ respectively, then the optical path retardation of the emergent wave for this configuration is $(k_1\mu_1 + k_2\mu_2 + k_3\mu_3)\,\Delta + m\mu_l\Delta$. We have in addition

$$(k_1 + k_2 + k_3) = (n - m). \tag{4}$$

Also, the probability of occurrence of a state in which $k_1$, $k_2$ and $k_3$ cells in a row of $(n-m)$ cells can be orientated so as to have refractive indices $\mu_1$, $\mu_2$ and $\mu_3$ is

$$\frac{(n-m!)}{k_1!k_2!k_3!}\left(\frac{1}{3}\right)^{n-m}. \tag{5}$$

Combining (3) and (5) we find that the proportion of area of the rear surface of the filter from which an emergent wave described by

$$\exp\left[\frac{2\pi i}{\lambda}(ct - Z - \overline{k_1\mu_1 + k_2\mu_2 + k_3\mu_3 + m\mu_l\Delta})\right] \tag{6}$$

proceeds is equal to

$$\frac{n!}{k_1!k_2!k_2!m!}p^{n-m}q^m. \tag{7}$$

The disturbance emerging from the Christiansen cell can now be obtained by a superposition of all the different wave functions of the type (6) multiplied by suitable weight factors of which (7) is a typical example. Hence,

$$y = \sum_{k_1,k_2,k_3,m} \frac{n!}{k_1!k_2!k_3!m!}p^{n-m}q^m \exp\frac{2\pi i}{\lambda}$$

$$\times\,(ct - Z - \overline{k_1\mu_1 + k_2\mu_2 + k_3\mu_3 + m\mu_l\Delta})$$

$$= \exp\left[\frac{2\pi i}{\lambda}(ct - Z)\right]\left\{p\left(\exp\left(\frac{-2\pi i}{\lambda}\mu_1\Delta\right) + \exp\left(\frac{-2\pi i}{\lambda}\mu_2\Delta\right)\right.\right.$$

$$\left.\left. + \exp\left(\frac{-2\pi i}{\lambda}\mu_3\Delta\right)\right) + q\exp\left(\frac{-2\pi i}{\lambda}\mu_l\Delta\right)\right\}^n. \tag{8}$$

The average refractive index of the medium is now

$$\mu = p(\mu_1 + \mu_2 + \mu_3) + q\mu_l. \tag{9}$$

If the birefringence of the crystalline particles is small and if further the three refractive indices of the solid powder do not differ much from the index of the liquid, then an approximation for (8) for large values of $n$ can be effected by proceeding exactly as in the preceding paper. Denoting by $d$ the total thickness of the cell, and neglecting terms of order higher than two in the differences between the various refractive indices, we can rewrite (8) as

$$y = \exp\left[\frac{2\pi i}{\lambda}(ct - Z - \mu d)\right]\left[1 - \frac{2\pi^2\Delta^2}{\lambda^2}\right.$$

$$\left.\times\left\{\sum_{r=1}^{3} p_r(\mu_r - \mu)^2 + q(\mu_l - \mu)^2\right\}\right]^n. \tag{10a}$$

Now if $\mu_1, \mu_2 \dots \mu_n$ are $n$ quantities having the respective probabilities of occurrence $p_1, p_2 \dots p_n$ in any observation, then

$$\sum_{\substack{r,s\\ r<s}} p_r p_s(\mu_r - \mu_s)^2 = \tfrac{1}{2}\sum_{r,s=1}^{n} p_r p_s(\mu_r - \mu_s)^2$$

$$= \sum_s p_s\left(\sum_r p_r\mu_r^2\right) - \left(\sum_r p_r\mu_r\right)^2$$

$$= \sum_r p_r\mu_r^2 - \mu^2$$

$$= \sum_r p_r(\mu_r - \mu)^2,$$

where $\mu$ is the average of the $n$ quantities $\mu_1, \mu_2, \dots \mu_n$.
Applying the above result to (10) we find that for large values of $n$

$$y = R\exp\left[\frac{2\pi i}{\lambda}(ct - Z - \mu d)\right] \tag{10b}$$

where

$$R = \exp\left(\frac{-2\pi^2\Delta d}{\lambda^2}\right)\{p^2\sum(\mu_2 - \mu_3)^2 + pq\sum(\mu_1 - \mu_l)^2\}. \tag{11}$$

The ratio of the intensity of the transmitted light to that of the incident radiation is therefore given by

$$\frac{I}{I_0}=R^2=\exp\left(\frac{-4\pi^2\Delta d}{\lambda^2}\right)\{p^2\textstyle\sum(\mu_2-\mu_3)^2+pq\sum(\mu_1-\mu_l)^2\}. \qquad (12)$$

Formula (12) expresses the extinction coefficient of light in its passage through the Christiansen cell in terms of several variables, namely, the wavelength of the light employed, the thickness of the cell, the size of the crystalline particles, the birefringence of the same and the three differences between the refractive index and the three principal indices of the birefringent material, and finally the proportion of the liquid and solid elements in the cell. By giving suitable values to $p$, $q$, $\mu_1$, $\mu_2$ and $\mu_3$, several interesting cases of special importance can be deduced from (12). Thus we observe that the case of the polycrystalline aggregate considered earlier follows readily from (12) if $q=0$ and $p=\frac{1}{3}$. Again, by writing $\mu_1=\mu_2=\mu_3$; $p=(\sigma/3)$ and $q=(1-\sigma)$ which corresponds to the case of a Christiansen cell composed of isotropic particles mixed in a liquid in the proportion $\sigma$: $(1-\sigma)$, we obtain the result

$$I=I_0\exp\left[\frac{-4\sigma(1-\sigma)\pi^2\Delta d}{\lambda^2}(\mu_1-\mu_l)^2\right] \qquad (13)$$

derived earlier by one of us[1] on different theoretical grounds. By writing $\mu_1=\mu_2=\mu_3$; $p=\frac{1}{6}$ and $q=\frac{1}{2}$ in (12) or $\sigma=\frac{1}{2}$ in (13) we get the expression

$$I=I_0\exp\left[\frac{-\pi^2\Delta d}{\lambda^2}(\mu_1-\mu_l)^2\right] \qquad (14)$$

for transmission by a cell composed of isotropic particles and a liquid of nearly the same refractive index mixed in equal proportions.

## 3. Some further remarks

Considering once again the general formula (12), we may draw attention to certain features which we may expect to observe in the Christiansen experiment with birefringent powders differing from those noticeable when isotropic powders are employed. In the latter case, the transmission would be complete for the particular wavelength for which the solid and liquid have equal refractive indices and would fall off rapidly on either side of such wavelength. The light not transmitted by the cell would appear as a diffusion halo surrounding the direction of the optical image of the source as seen through the cell. Brilliant chromatic effects are accordingly to be expected and are indeed observed with isotropic powders in the experiment.

Formula (12) shows clearly that the effect of the birefringence of the powder is

to diminish the intensity of the transmitted light for all wavelengths, and we cannot therefore, expect any observable transmission through the cell with strongly birefringent powders, unless the size of the particles be very small and the thickness of the cell be reduced to a minimum. In such cases the colour of the transmitted light would be determined predominantly by the factor $1/\lambda^2$ appearing in the argument of the exponential. Hence, it would be reddish in colour and the chromatic effects observed with isotropic powders would be absent. In these circumstances, the liquid in the cell serves only to secure optical continuity between the discrete particles contained in the cell. On the other hand, if the birefringence be small, one may expect to observe chromatic effects similar to those observed with isotropic powders. It would be necessary, however, to work with fairly fine powders and moderate cell thickness for noticeable transmission to occur. The formula also shows that the maximum transmission in these circumstances would be exhibited for those wavelengths for which the refractive index of the liquid is most nearly equal to a species of average of the three indices of the crystal. But this is not the average index in the ordinary sense of the word.

Another important consequence of the formula is that even if the birefringence of the powder be not very small, chromatic effects would be observable when the proportion of the volume in the cell occupied by the powder is sufficiently small. In the usual form of the Christiansen experiment, the particles are allowed to settle down and form a compact aggregate at the bottom of the cell. To observe the effects now contemplated, the contents of the cell should be stirred up; alternatively the particles should be so small that they remain suspended for a long time within the liquid. In other words, dilute suspensions of strongly birefringent powders may be expected to give brilliant chromatic effects in a Christiansen cell. However, our theory could hardly be expected to give more than a qualitative indication of the phenomena then noticeable.

Finally, we come to the question of the state of polarisation of the transmitted light as also of the diffracted light when observed with birefringent powders. The present theory indicates that if the light incident on the cell be plane-polarised, both the transmitted and the diffracted light should also be perfectly plane-polarised. So far as the transmitted light is concerned, there can be no doubt that the theoretical result is correct. For, any ellipticity consequent on the passage of light through an arbitrarily orientated crystallite would give rise to a component perpendicular to the original vibration direction in the diffracted light. But such components cannot appear in the light transmitted by the cell in the true optical sense. Indeed, provided the birefringence is small and the size of the particles and the thickness of the cell are moderate we may expect also to find that the diffraction halo is itself strongly polarised. In other circumstances, however, especially when the particles are strongly birefringent, the diffracted light would exhibit a marked imperfection of polarisation by reason of the ellipticity effects which have dropped out of consideration in the present treatment of the problem.

## Summary

A formula is derived for the transmission coefficient of a Christiansen cell containing particles of a birefringent material whose interstices are filled up by a liquid of suitably adjusted refractive index. The consequences of the formula and especially the influence of the birefringence on the spectral character of the transmitted light are discussed.

## References

1. Raman Sir C V *Proc. Indian Acad. Sci.*, 1949, **A29**, 381.
2. Raman Sir C V and Viswanathan, K S *Proc. Indian Acad. Sci.*, 1955, **A41**, 37.

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# The Christiansen experiment with birefringent powders

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## 1. Introduction

In the well-known experiment due to Christiansen which has been applied in the construction of monochromatic light-filters, an isotropic transparent solid, e.g., optical glass, is powdered and placed inside a flat-sided glass cell and the latter is then filled up by a liquid whose refractive index is adjusted to equality with that of the solid for some particular wavelength in the spectrum. Christiansen himself tried using powdered quartz in the experiment, and found that it did not prove a success. The light entered the cell and was diffused by the powder-liquid mixture, but the source of light could not itself be seen through the mixture. Considered from the standpoint of geometrical optics, this result is not surprising, since the refractive index of the particles of a birefringent powder would depend on their orientation within the cell and hence would vary from particle to particle.

However, geometrical optics does not correctly describe what is actually observed in the Christiansen experiment even with optically isotropic powders,[1,2] and hence there is no reason to believe that it would be any more successful in the case of birefringent powders. In another paper[3] appearing in the present issue of these *Proceedings*, a general theory has been developed which indicates that chromatic effects analogous to those observed with isotropic powders should also be capable of appearing with birefringent powders in appropriate circumstances. The manner in which the birefringence would modify the observed effects has also been discussed in that paper.

It is proposed in what follows to place on record some observations which we have made broadly confirming the indications of the general theory. It has been found that provided the birefringence is fairly small and the material is in a finely subdivided state, it is possible to obtain a true transmission exhibiting brilliant colours. It is also found that the light so transmitted is not greatly inferior in the degree of its monochromatism to that observed with isotropic material in similar circumstances. Another noteworthy feature is that if the light incident on the cell is plane-polarised, the light transmitted by it is also completely plane-polarised. This might seem paradoxical when it is recalled that the light has had to pass through considerable thicknesses of a birefringent material; nevertheless the

observation is in strict accord with the theory. On the other hand, the diffusion halo appearing in directions surrounding the regularly transmitted light exhibits imperfect polarisation to an extent depending on the fineness of the subdivision of the material. The colours exhibited by the diffusion halo are also found to be markedly different for the components of the light vibration respectively parallel and perpendicular to that in the incident light.

## 2. Some general observations

Besides the factors which determine the transmission coefficient with isotropic powders, an additional factor appears in the present case, namely the magnitude of the birefringence of the material. The importance of this relatively to the differences in the refractive indices of the solid and liquid varies with the proportion of the volumes occupied respectively by the powder and the liquid in the cell. By varying this proportion, we may pass from one extreme case of a polycrystalline aggregate in which the liquid serves merely to secure optical continuity between the particles of the powder to the other extreme case in which the liquid occupies the whole volume except for the particles of solid held in suspension. The most interesting cases are however those in which the two components are present in nearly the same proportions. To observe chromatic effects in such cases, it is necessary to use material which is not too highly birefringent and which is in a fine state of subdivision. The latter condition is most conveniently secured by using a substance which is commercially obtainable in the state of a fine powder and hence does not need any further preparation. We shall content ourselves here by mentioning three such substances which we have found to work very well in the experiment.

Barium sulphate can be used with success in a cell from five to ten millimetres thick with carbon disulphide as the liquid filling it up. The addition of a few drops of benzene shifts the transmission from the yellow towards the violet end of the spectrum. Precipitated calcium sulphate in the form of gypsum also gives good results, the appropriate liquid in this case being monochlorobenzene; a drop or two of carbon disulphide shifts the transmission towards the red, while the addition of a few drops of benzene shifts it towards the violet. Magnesium fluoride also serves admirably; the appropriate liquid to use in this case is acetone, the addition to which of benzene causes the transmission to appear first at the violet end of the spectrum and shifts it step by step toward the red.

## 3. Polarisation phenomena

In all the three cases mentioned, beautiful chromatic haloes are observed surrounding the direction of the transmitted light. The colour of the halo varies

with the direction of observation and also alters when the spectral region of transmission is shifted. Viewing a bright and well-defined light source through the cell held before the eye of the observer, with one polaroid inserted between the source and the cell and a second polaroid between the cell and the observer's eye, striking polarisation phenomena may be observed. In all cases, the image of the light source is itself completely extinguished when the two polaroids are crossed. But the diffusion halo seen surrounding the light-source shows imperfect polarisation. The magnitude of this imperfection differs very much in the case of the three substances mentioned above. With magnesium fluoride the halo disappears almost completely when the polaroids are crossed. With barium sulphate its extinction is less complete, and the diffusion halo remains observable in directions adjacent to the source and gives indications of a bright cross with its arms bisecting the angle between the vibration directions of polariser and analyser. The extinction of the diffused light is least perfect in the case of calcium sulphate and the halo remains observable over the whole area of the field even with the polaroids crossed.

## 4. Observations with powdered quartz

The following method was adopted to prepare quartz in a state of fine subdivision but uncontaminated by extraneous material. A transparent piece of crystalline quartz was heated and dropped into cold distilled water. The fragments into which it broke as the result of this treatment were heated in a silica dish and then again dropped into cold distilled water. Repetition of this procedure reduced the substance to a state of powder and the material thus obtained was then ground up very fine between two quartz crystals with flat faces and finally separated into four grades by elutriation in distilled water. The two finest grades thus obtained were those which remained in suspension in a tall beaker of distilled water for ten and twenty minutes respectively.

The optical effects exhibited by the four grades of quartz when placed in a cell and filled up with a mixture of benzyl alcohol and carbon disulphide were found to be very different. The first or roughest grade gave only a diffusion halo without any regular transmission even with a cell only one millimetre thick. With the second grade of powder and the same thickness of cell, a very weak transmission can be glimpsed, the bright diffusion halo overlying it making the observation rather difficult. On the other hand, the two finer grades when allowed to settle down in a cell two millimetres thick show brilliantly coloured transmitted images of the source. With a cell only one millimetre thick, the transmission is even more brilliant, but its colour is then less saturated.

In all cases, if a transmission is obtained at all, it is completely extinguished when the cell is placed between two crossed polaroids. On the other hand, the appearance of the diffusion halo as well as its state of polarisation shows

remarkable variations with the grade of powder and the thickness of cell employed. With the coarser grades of material, the halo is found to be completely depolarised. On the other hand, with the finer grades the halo shows a very marked degree of polarisation. Not merely the brightness of the halo but also the distribution of colour in it is strikingly different when the polaroids are respectively parallel and crossed. This difference is best described by the statement that when the polaroids are crossed, the halo exhibits colours similar to those of the transmitted light (which is extinguished in the same circumstances); *per contra*, with the polaroids parallel, the colour of the halo is complementary to that of the transmitted light. These changes were most striking when observed with the cells of smaller thickness. For, with such cells, the colour of the halo over its whole area is markedly different from that of the transmitted light and exhibits its complementary character most clearly. The changes produced by the rotation of the analyser are therefore particularly striking.

## 5. Observations with dilute suspensions

With the two finer grades of quartz powder, it is possible to use much thicker cells with success for observing the transmitted light, if they are filled with an excess of liquid in which the powder is held as a dilute supension. By varying the thickness of the cell and the quantity of material held suspended in it, one can either increase or decrease the saturation of the colours observed in the transmitted light and in the diffusion halo. It is worthy of note that in such cases the diffusion halo exhibits a colour complementary to that of the transmitted light even in directions adjacent to the latter. This indeed is what optical theory indicates should be the case for a dilute suspension. On the other hand, multiple scattering comes into play in the case of dense aggregates, and the colour of the halo is in consequence almost indistinguishable from that of the transmitted light in closely adjoining directions and only further out changes by insensible gradations to the complementary tint.

It appears worthwhile also to record some observations made with strongly birefringent powders, calcium carbonate in the form of precipitated chalk being a typical example. It is not possible to obtain any transmission with this material in the usual form of the Christiansen experiment. But interesting effects may be observed even with cells of considerable thickness if they are filled with carbon disulphide into which a little precipitated chalk is put in and stirred so that the liquid appears as a milky-white suspension. A bright source of light can be seen through such a suspension and exhibits a deep red colour, as indeed it should according to the theory. Here again the transmitted light is completely extinguished if the cell is placed between crossed polaroids. On the other hand, the light diffused by the cell is depolarised, but if the suspension is very dilute, the cell exhibits a bright cross between crossed polaroids. If benzene is added to the

suspension of chalk in carbon disulphide, thereby bringing the index of the liquid mixture nearer to the mean index of calcium carbonate, the colour of the transmitted light is shifted towards shorter wavelengths in the spectrum.

## 6. Description of the figures in plate I

By way of illustration of the foregoing observations, some photographs have been reproduced in plate I accompanying this paper. The following remarks are explantory notes on the same.

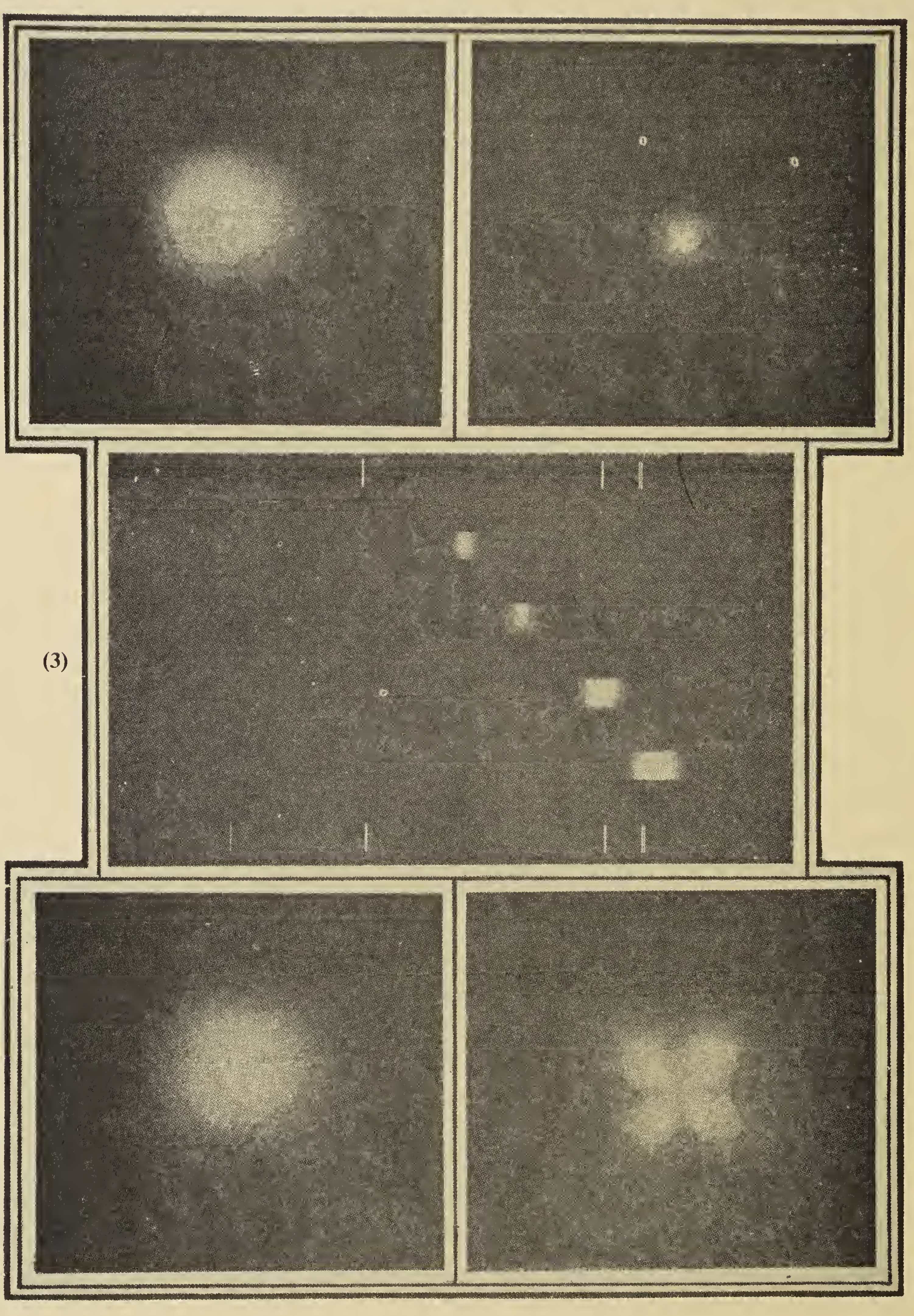
Figure 1 shows the appearance of a small brilliant light-source as viewed through a cell five millimetres thick containing barium sulphate in a finely subdivided state. This had settled down at the bottom of a mixture of carbon disulphide and benzene so adjusted as to transmit the green part of the spectrum. The picture clearly shows the transmitted light and the diffusion halo surrounding it.

Figure 2 shows the same experiment with the cell placed between crossed polaroids and photographed with a much longer exposure. The source itself is extinguished in these circumstances. The halo also disappears except at its brightest part near the centre which exhibits the bright cross whose arms bisect the angle between the vibration directions of the polariser and analyser.

Figure 3 reproduces the spectra of the light transmitted by the cell containing barium sulphate in the same circumstances. The colour of the transmitted light was shifted in steps from orange-yellow to blue by the successive additions of benzene to carbon disulphide. The lines of the mercury arc spectrum are reproduced to indicate the positions of the transmission band.

Figure 4 is a photograph of a small bright source of light seen through a cell two millimetres thick containing the finest grade of quartz powder which had settled down at the bottom of the cell. Benzyl alcohol to which a few drops of carbon disulphide had been added was the liquid used and the mixture transmitted the yellowish-green part of the spectrum. The photograph itself was rather overexposed with the result that the transmission and the halo are not seen clearly distinguished from one another. The streaky nature of the halo is clearly shown.

Figure 5 is a photograph of a bright source of light seen through a thin film of nitrobenzene containing fine particles of lithium carbonate in suspension and held between crossed polaroids. The dark arms of the cross seen in the diffusion-halo are parallel to the vibration directions of the polariser and analyser respectively.



(1) (2) (3) (4) (5)

**Figures 1–5.** Christiansen experiment with birefringent powders.

**Plate I**

## 7. Summary

Optical effects analogous to those exhibited by isotropic materials in a Christiansen cell are also observable with birefringent materials in a fine state of subdivision. While the transmitted light is fully polarised, the diffusion halo is depolarised in part and exhibits colours between parallel and crossed polaroids which are complementary to each other.

## References

1. C V Raman and M R Bhat, *Curr. Sci.*, 1953, **22**, 31.
2. M R Bhat, *Proc. Indian Acad. Sci.*, 1953, **A38**, 67.
3. C V Raman and K S Viswanathan, *Proc. Indian Acad. Sci.*, 1955, **A41**, 55.

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# The optical behaviour of polycrystalline solids

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(Memoir No. 89 from the Raman Research Institute, Bangalore-6)

ABSTRACT

The paper discusses the influence of the factors which determine the penetration of light into a polycrystalline solid, viz. the birefringence of the crystallites, their size and shape and their relative orientations. The results of the theoretical discussions are compared with the facts of observation. Five X-ray diagrams are also reproduced which exhibit the relation between the structure and optical behaviour of polycrystalline media in particular cases of interest.

## 1. Introduction

As examples of polycrystalline solids, which by reason of the desirable properties they possess have been extensively used for the fashioning of works of art, may be mentioned especially the following minerals: marble, alabaster, chalcedony and jade. Visual observations supplemented, where necessary, by examination of thin slices under the polarisation microscope and the use of X-ray diffraction techniques reveal that individual specimens of these minerals may be widely different in regard to the size and shape of the crystallites of which they are composed and the manner in which these crystallites are disposed within the solid. Alongside of these differences and obviously related to them are noticed some striking differences in the optical characters of the specimens. As the external appearance of a finished work of art is influenced a good deal by such differences, it is not without interest to discuss in some detail the nature of the relationship between the structure and the optical behaviour of polycrystalline solids in general.

In all the cases mentioned, the basic substance is a transparent crystalline solid and accordingly the aggregate could also have been expected to be free from colour. Though this is so in some instances, it is not always the case. Indeed, the presence of some favoured colours often enhances the value of the material very greatly. The nature of the intrusive material responsible for such colour and the manner in which it is diffused through the substance and influences its optical behaviour are questions of considerable interest, but we shall not enter into them here.

It should be mentioned that the structure of polycrystalline solids and its relation to their optical behaviour has been the subject of both theoretical and experimental investigations by the present writer and his collaborators and the results have been reported in detail in some earlier memoirs of this Institute. In the present publication it is proposed to summarise broadly the results which have emerged from these studies and in addition to present some fresh observational material.

The five illustrations accompanying the present paper are X-ray-diffraction photographs of some interesting specimens recorded by Mr A Jayaraman at this Institute. In each case, the structure revealed by the X-ray diagram is found to be very clearly related to the optical characters exhibited by the specimen. We shall return to these diagrams and their significance later in the paper.

## 2. Theoretical considerations

If no extraneous material be present in a polycrystalline solid and its optical heterogeneity arises solely by reason of the varying orientations of the crystallites, the birefringences of the latter would give rise to a diffusion of the light in its passage through the material. At each incidence of the light on an intercrystalline boundary, it would be reflected and refracted and hence all trace of the original direction of propagation of the light would quickly disappear. After the passage of the light through a sufficient number of crystallites, a state of affairs would establish itself in which we have a diffusion in all directions, forwards and backwards as well as laterally. The greater the birefringence of the material, the fewer would be the number of intercrystalline boundaries needed for such a situation to establish itself. We may visualize the resulting phenomena by imagining a pencil of light to fall normally on a restricted area in the plane face of a block of the material. The observable results would then depend upon the various factors involved, namely the thickness of the block, the size of the crystallites and the magnitude of their birefringence. In the absence of any absorbing material and if the thickness of the block be sufficiently great, all of the incident light must necessarily be turned back towards the original source of light and would re-emerge from the same face spread out from a much larger area than that on which it is originally incident. On the other hand, if the thickness of the block be not too great, part of the incident radiation would also emerge from the rear face of the block spread out over a large area. In each case, the observable effects would be profoundly influenced by the size of the crystallites. The smaller the crystallites are, the more quickly would the incident light be turned back towards the source and the less, therefore, would be the depth of the penetration of the light into the medium. The presence of any absorbing material in the solid would necessarily modify the observable results to a very great extent.

## 3. Diffraction phenomena

While the arguments set forth above based on geometrical optics may be adequate when the crystallites are of sufficiently large dimensions and the birefringence of the material is also fairly large, they would cease to be adequate and would fail to describe what is actually observed when the reverse is the case, i.e., when the crystallites are of small dimensions and the birefringence which they exhibit is also small. In the latter circumstances, the matter has necessarily to be considered from the standpoint of the wave theory of light, and the results indicated by such theory are very different indeed from those indicated by geometrical optics.

We may here broadly indicate the lines on which the problem can be handled on the basis of the wave theory. If the birefringence of the material be sufficiently small, the diffusion of light in its passage through it may be considered as arising from the varying retardations in phase of the light waves in traversing the differently oriented crystallites. Provided that the product of the linear dimension of each crystallite and of its birefringence is sufficiently small, we can regard the waves as moving undisturbed through the medium except for the changes of phase occurring over different areas of the wave-front; such changes may be regarded as being additive in depth and hence capable of being summed up over the whole of the path through a slab of the material, provided that the thickness of the latter is not too great. The wave-front emerging at the rear surface of the block would thus exhibit fluctuations in phase over its area. The final observable effect may then be determined by the usual methods of diffraction theory.

The foregoing brief exposition ignores certain difficulties, namely, that inside each crystallite there would be two possible directions of vibration for which the refractive indices are different and these two directions would vary from crystallite to crystallite owing to their random orientations. This difficulty, however, may be circumvented by assuming that the light incident on the block is plane-polarised and that all the crystallites are so orientated that this original direction of vibration in the wave-front is preserved in its passage through the block, the operative refractive index for any one crystallite being one of its three principal values chosen at random. Such an assumption has the great advantage that it greatly simplifies the formulation and solution of the problem while retaining its essential features. Further, by assuming the probabilities of the three different orientations to be different, we can also determine the optical behaviour of polycrystalline solids in which the crystallites are not orientated at random but prefer certain directions. What the solution of the problem gives us is the intensity of the light regularly transmitted through the polycrystalline plate. This appears as an exponential function in which the argument has a negative sign and is directly proportional to the thickness of the plate and to the size of the crystallites and also to a quantity depending on the differences between the three refractive

indices which is a measure of the birefringence; it is also inversely proportional to the square of the wavelength of the light. From the theory it follows that the light emerging from the block would be polarised in the same fashion as the light incident on the first face. The theory also shows that if the crystallites have no preferred orientation, the transmission coefficient is independent of the direction of vibration in the incident light. If, however, the crystallites are preferentially orientated, the intensity of the light emerging from the plate would vary with the circumstances, viz., with the relative probabilities of the three permitted orientations. In particular cases of this kind, e.g., when all the crystallites are orientated along a fibre axis which is also the optical vibration direction, the transmission may be actually complete.

We may summarise the foregoing by stating that on the basis of the assumptions made, the light incident on a plate of polycrystalline material would emerge from it greatly enfeebled in intensity but retaining its primitive state of polarisation. The fraction of the light regularly transmitted for a given thickness of the material would increase with diminishing size of the crystallites and would reach the limiting value of unity if they are sufficiently small. This result is precisely the opposite of that indicated by the geometrical theory according to which diminishing particle size would result in greater opacity. These two contrasting results may be regarded as valid respectively in the limiting cases of a very weak and of a very strong birefringence.

The energy that disappears from the regularly transmitted light would appear as diffracted radiation. Its distribution of intensity in different directions would be determined by the factors entering into the problem, namely, the size of the crystallites, the thickness of the plate, the birefringence of the material and finally the wavelength of the light. It is a consequence of the special assumptions made regarding the orientation of the crystallites that the diffracted radiation would be polarised in the same manner as the incident and emerging pencils. It is not, however, to be expected that this particular result would represent the actual facts of the case. As a consequence of the unrestricted randomness of the orientation of the crystallites, the diffused or diffracted radiation would necessarily be depolarised to a greater or less extent. The greater the birefringence of the material or the thickness of the plate traversed, the more complete would be such depolarisation.

It is a particular consequence of the theory that if unpolarised light be incident on a plate of material having a fibrous structure and of sufficient thickness, the regularly transmitted beam would be completely polarised on emergence, while the rest of the light would appear as a fan of diffracted rays in the plane perpendicular to the fibre axis and polarised predominantly in the perpendicular direction. It has also to be remarked that a fibrous structure of the medium is favourable to the appearance of a regularly transmitted pencil. The same material with crystallites randomly orientated might not exhibit any transmission at all, but only a diffusion of light in various directions surrounding that of the incident

pencil. Finally, we may remark that since these phenomena arise by reason of wave-optical considerations depending on the wavelength of the light, the transmitted light whenever it is observable should exhibit a reddening which increases progressively with the thickness of the material traversed.

## 4. Some illustrations of the theory

Marble which in its purest forms consists of calcite furnishes excellent illustrations of the optical behaviour of a polycrystalline medium consisting of strongly birefringent crystallites. The influence of the grain size on the penetration of light into the material indicated by the theory is readily confirmed by observations with different specimens. A very interesting fact is that the light diffused by marble both backwards and forwards exhibits a surprisingly rich redness even when the marble itself is only of a faintly pinkish colour.

Alabaster which consists of randomly orientated crystallites of gypsum affords good illustrations of the behaviour of a polycrystalline medium in which the crystallites have only a feeble birefringence. The size of the crystallites can of course be readily determined by cutting a thin section and viewing it under a polarising microscope. The relationship between the size of the crystallites and the optical behaviour of the material can be readily confirmed by viewing a bright source of light through a slice of alabaster mounted between microscope cover-slips with a little Canada balsam. Only with the thinnest sections is a true optical transmission observable. This appears surrounded by a diffusion halo exhibiting streaky colours in white light. The depolarisation of the halo and the polarisation of the regularly transmitted light when the incident light is polarised are also readily confirmed by observation.

The X-ray diffraction diagram of alabaster depends very much on the size of the crystallites which it contains. Unless the alabaster is very fine-grained, a collection of spots only vaguely indicative of a ring pattern is obtained. Very different is the picture shown in figure 3 in plate I which shows a readily recognizable ring system, though spotty in character. This was obtained with a very fine museum piece exhibiting a remarkably high degree of translucency.

Figure 2 in plate I is an X-ray diagram recorded with a dish of Chinese origin stated to be of jade, but not really so. The material was colourless and exhibited a high degree of translucency. The extremely small size of the crystallites of which it is composed is evident from the X-ray diagram.

Figure 1 in the same plate is the X-ray diagram obtained by passing a beam of X-rays through a plate of a white porcelain-like material which was cut out from a block of agate and which was identified as $\alpha$-cristobalite by its X-ray pattern. The extreme fineness of the particles of the material is evident from the X-ray diagram. A source of white light seen through the plate is visible, but appears of a deep red colour. The material thus furnishes an excellent example of a polycrystalline

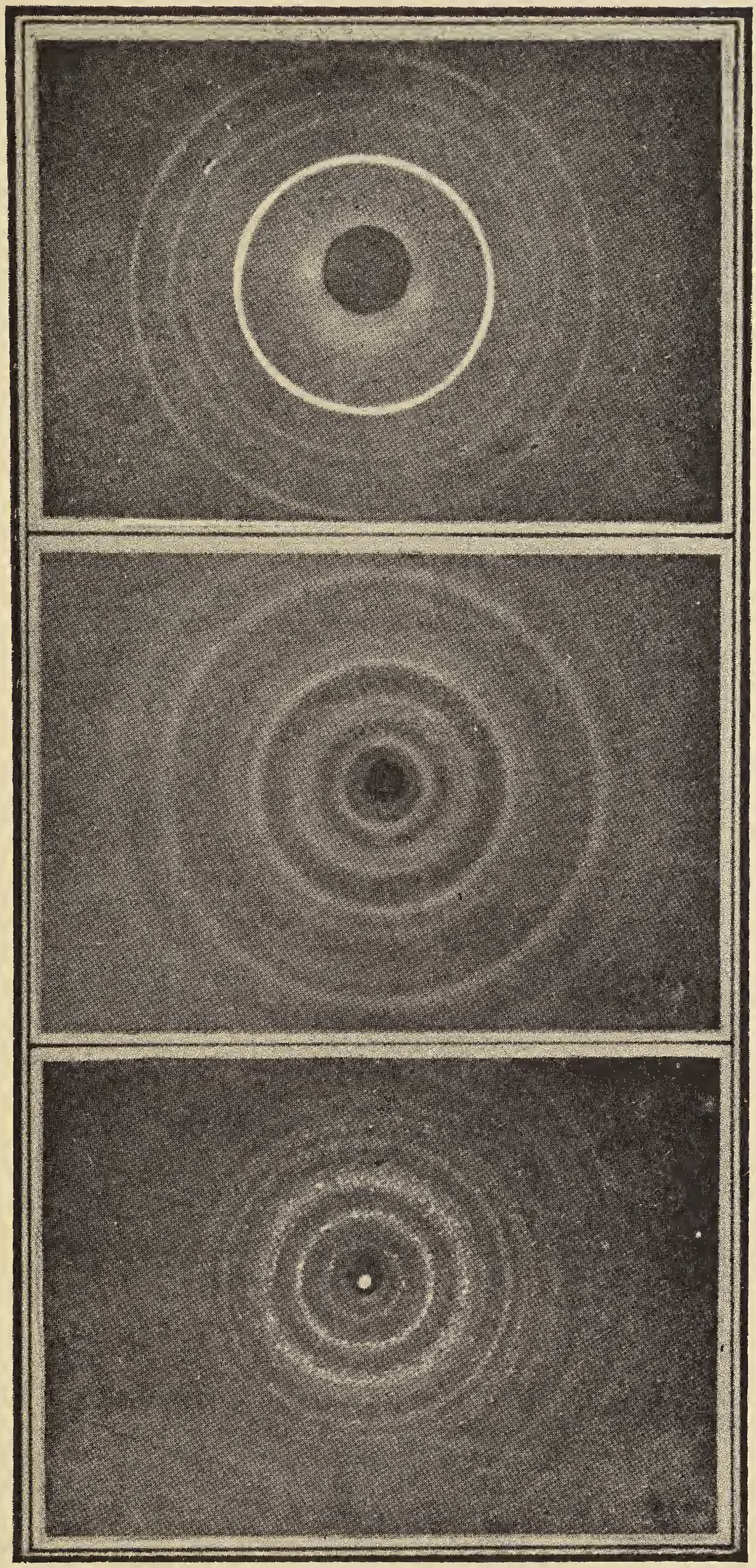
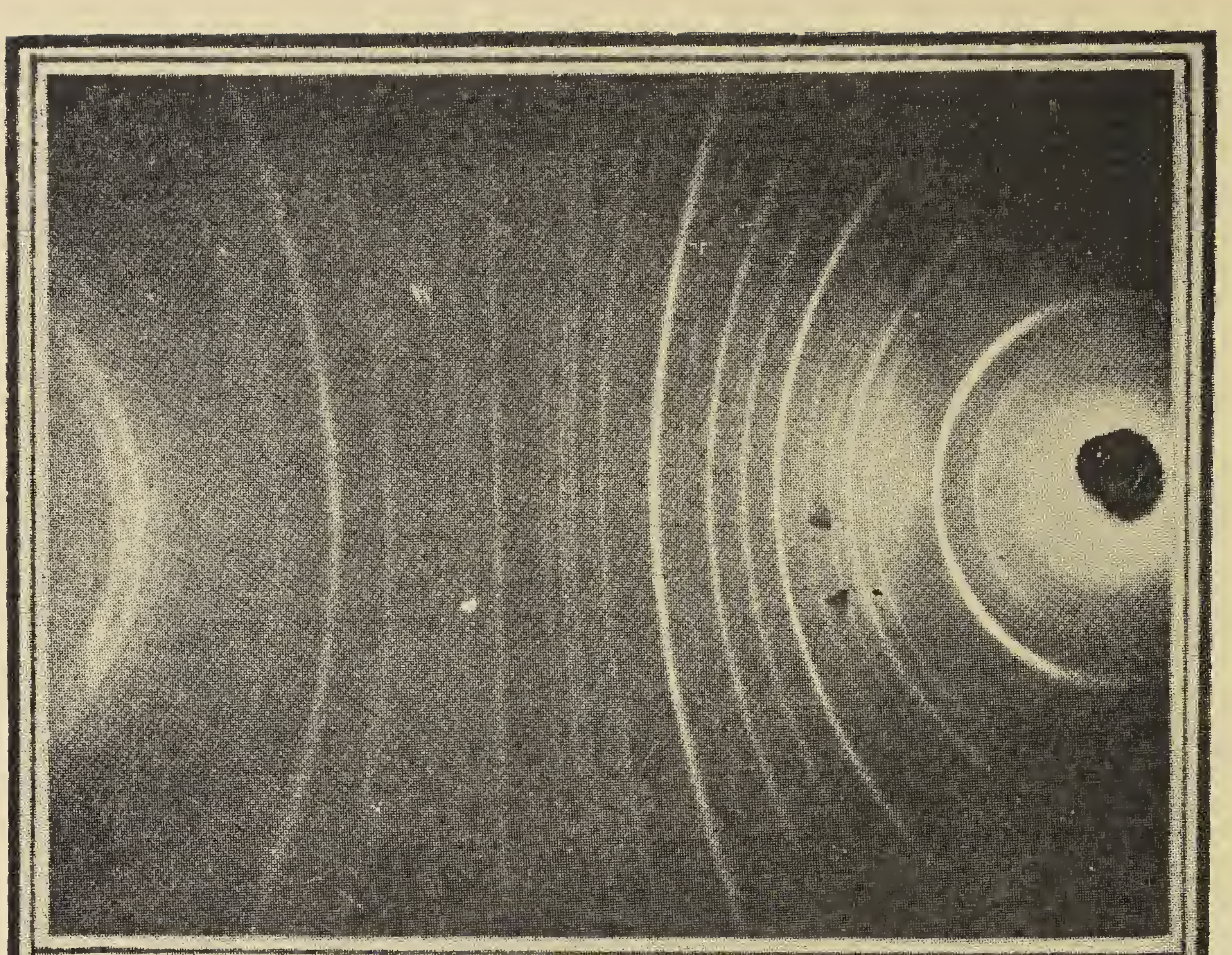

(1)
(2)
(3)

Plate I


(4)

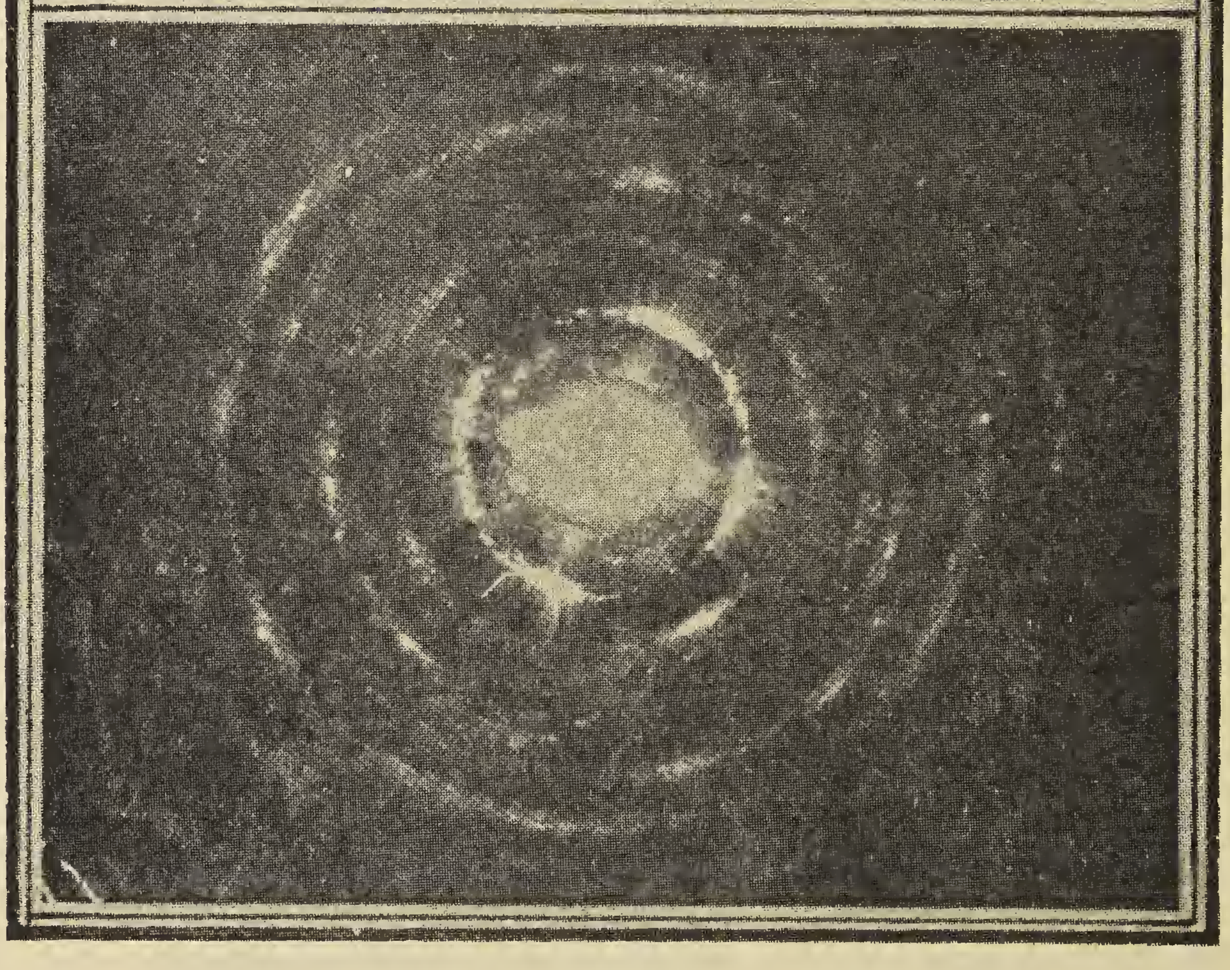

(5)

Plate II

medium consisting of a birefringent material which nevertheless exhibits a true optical transmission by reason of the small size of the crystallites.

The phenomena arising from the propagation of light through a medium with a fibrous structure are beautifully illustrated by observing a source of light through a polished plate of chalcedony. They exhibit in a very conspicuous manner the various features indicated by the theory. It should be emphasized that chalcedony

exhibits large variations in its structure and these correspond to striking differences in optical behaviour. Two extreme cases are illustrated respectively by figures 4 and 5 in plate II. The first is an X-ray diagram recorded with a specimen of agate which exhibited only translucency but no real transmission of light. It will be seen from the X-ray diagram that this material consisted of extremely fine particles. Figure 5, on the other hand, is an X-ray diagram of a specimen of chalcedony showing a coarsely fibrous structure. A source of light could easily be seen in perfect focus through the plate though much dimmed in intensity and reddened in colour. The transmitted light was found to be completely polarised, and this was evidently a consequence of the fibrous structure revealed by the X-ray pattern and also evidenced by the streams of diffracted light seen surrounding the image of the source.

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# Christiaan Huyghens and the wave theory of light

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## 1. Introduction

The world, as we perceive it, is pictured for us by the rays of light which proceed from each point of the objects around us and form images of these objects on the retinae of our eyes—a statement which needs no amendment even when the aid of optical instruments such as the telescope or the microscope is invoked to enlarge our powers of vision. To put it a little differently, the principles of geometrical optics suffice to describe the behaviour of light as commonly experienced, viz., that the rays of light are propagated in straight lines; that the angles of incidence and of reflection are equal; and that in refraction the rays of light are bent according to the law of sines. In his celebrated *Traite de la Lumiere* published in the year 1690, Christiaan Huyghens showed that these facts of experience are consistent with the hypothesis that light is in the nature of wave-motion propagated through space and can indeed be satisfactorily explained on the basis of that hypothesis. The treatise of Huyghens contains much other material of importance; a perusal of it leaves on the mind of the reader the impression that it is a masterpiece of scientific thought and exposition which possesses an enduring value and interest.

In connection with some experimental investigations on the diffraction of light undertaken by the present author—the results of which will soon be published in these *Proceedings*—the need was felt for a careful study of the original ideas of Huyghens. The task was made much easier by using the literal translation of his treatise from the original French into English by Sylvanus P Thompson published by the Chicago University Press in the year 1912. The results of the study were surprising; it emerged that the ideas of Huyghens were not fully or even correctly understood by later writers. This misunderstanding has had some far-reaching consequences. Especially in regard to the so-called "Principle of Huyghens" do we find that later writers have chosen a path for which there is no warrant in the writings of Huyghens. In view of these circumstances, it has appeared desirable to put forward a clear exposition of the ideas of Huyghens and supplement the same by a critical examination of the writings of later authors which claim to be based on those ideas.

## 2. The nature of light

In the first few pages of his book Huyghens set out the considerations which led him to infer that light is in the nature of a movement which spreads into space in all directions from a luminous source. He remarks that the terrestrial sources which are observed to emit light, such as fire or flame evidently contain bodies in rapid motion. Then again, when sunlight is collected by a concave mirror and concentrated on material objects, it has the same effects as fire, viz., it disunites the particles of those objects. It is therefore natural to suppose that light is itself some kind of motion and that the sensation of light is excited when such movement is communicated to the nerves at the back of the human eye. Huyghens also remarks on the extreme speed with which light spreads on every side and on the circumstance that when light comes from different regions, even from those directly opposite each other, the rays traverse one another without hindrance. The facts indicate that light is a movement transmitted through space and not a transport of matter which reaches the eye from the source of light.

To account for the very high velocity of propagation of light—known from the observations of Römer on the eclipses of Jupiter's satellites—and the fact that light can pass through empty space, Huyghens proposed a physical picture of the aetherial medium which could explain its power to transmit waves with such high velocity. He suggested that the aether of space consists of an immense number of extremely small and extremely hard spherical particles in close contact ~~with~~ each other. Experiments on the percussion of elastic solid spheres on each other show that a medium of the nature postulated could propagate waves simultaneously in all directions with high velocity and in such manner that waves travelling in different directions at one and the same time would not hinder each other's progress. Huyghens further recognized that every luminous object would necessarily contain an immense number of centres emitting light and that from each of such centres thousands of waves might emerge in the smallest imaginable time; he pointed out that these considerations would make it easier to understand, why in spite of the enfeeblement of the individual waves by their spread through immense distances, the light of the distant stars continues to be perceptible to human eyes.

## 3. The rectilinear propagation of light

The mechanical model of the aether proposed by Huyghens to account for the propagation of waves of light through it also enabled him to give a simple and satisfactory explanation of why light travels out from the original source in straight lines. Considering the medium which transmits light to be composed of an immense number of very small and very hard spherical particles in close contact with each other, it follows that each of these particles when it is displaced

from its position by the passage over it of the parent wave sent out from the original source would itself function as the source of a wave which spreads out from it in all directions. The particular or partial waves of this nature recognized by Huyghens in his argument would be as numerous as the total number of the individual particles of aether contained in a sphere drawn with the source of the primary wave as centre and the distance to which it has travelled out as the radius. It is evident, also, that these partial waves, though present in enormous numbers within the volume of the sphere, would individually be of excessively feeble force. In theory, they are assumed to travel out from their respective centres in all directions but actually, by reason of their excessive feebleness the effect of all the partial waves may be totally ignored except of those which arrive together at the same instant, in other words, *simultaneously*, at the point of observation. For, in the latter case, their effects would be superposed and would add up to give an observable result. Such simultaneity in arrival and consequent superposition of effects would be possible only in the case of those partial waves which originate at points in the medium between the primary source of light and the point of observation which lie on the straight line joining them. The summation of the partial waves originating at such points and reaching the point of observation at the same instant would produce the luminous effect there observed. Thus the light which reaches the point of observation from the original source may be considered as having travelled out along the straight line joining the two points.

## 4. The reflection and refraction of light

In the second and third chapters of his treatise, Huyghens considered the phenomena which arise when light travelling in one medium reaches the boundary which separates it from another. In considering those phenomena, various physical questions arise and are discussed by him, as for example, why some materials are transparent to light and others are opaque, why the velocity of light in a material medium should differ from that in empty space and what the actual configuration of the boundary of the separation between two media is or can be.

The explanation of reflection and refraction in terms of the wave hypothesis is based on an idea which Huyghens found himself compelled to introduce, viz., *that the elements of the area* of the boundary of separation between the two media act as sources of partial waves which travel out respectively into the two media. The partial waves returned from the boundary into the first medium build up the regularly reflected wave, while those travelling into the second medium give the refracted wave.

A simple geometric construction based on the foregoing ideas enabled Huyghens to explain the familiar law of reflection of light. Likewise, assuming the second medium to be transparent and that the velocity of light in it differs from

that in the first, a similar geometric construction led him to the result that the sine of the angle of incidence and the sine of the angle of refraction bear to each other a constant ratio which is the same as the ratio of the velocities of light in the two media. The phenomena of the total internal reflection of light also found a satisfactory explanation. Huyghens showed further that his construction leads directly to Fermat's principle of minimum time for the passage of light from one point to another when the two points are in different media.

## 5. The wave-optics of Huyghens

Before proceeding to comment on the writing of later authors on the work of Huyghens, we may usefully here summarise the basic concepts of his theory. Huyghens put forward and sought to establish the proposition that when a wave of light diverges from its source, every small portion of the wave is capable of propagating itself independently with the same velocity as the rest of it; in an isotropic medium, the direction of such propagation is the wave-normal and hence this is also the direction of the ray in the sense of geometrical optics. The same idea forms the basis of Huyghens' explanation of the reflection and refraction of light. When the elements of area of an advancing wave-front reach the boundary between two media, each such element gives rise, respectively in the two media, to the elements of area in the reflected and refracted waves. These latter advance normally to themselves in such a direction that they can join up and form continuous wave-fronts. The geometric constructions employed by Huyghens enable these requirements to be satisfied. The propagation of light in an inhomogeneous medium considered in the fourth chapter of Huyghens' treatise can also be very simply dealt with on the same basis. The elements of area of the wave-front in such a medium advance normally to themselves with the velocity appropriate to their positions in the medium. As they advance, they join up to form new wave-fronts which are orthogonal to the path of the light-rays in the medium.

Later writers have criticised the arguments employed by Huyghens in his treatise. One remark which is often made is that the theory of Huyghens would result in his wave-fronts moving backwards as well as forwards and that he had given no explanation for the absence of backward propagation. But this criticism is not justified and is itself based on a misunderstanding. Huyghens was concerned with the behaviour of an *advancing* wave-front in a homogeneous medium. The partial waves which in his theory give the observed light intensity by their superposition are those which diverge from points lying on the straight line between the source and the observer; in order to reach the observer simultaneously they should all move *away* from the source and *towards* the point of observation, in other words move *forwards* towards the observer. The possibility of backward propagation is thus ruled out completely.

Another criticism which has frequently been advanced is that the theory of Huyghens is based on an arbitrary assumption, viz., that only along the envelope of his partial waves would there be any observable intensity of light. This criticism is also based on a misunderstanding. It should be remembered that Huyghens was unaware that the waves of light are periodic disturbances having a definite wavelength. He assumed that light consists of *individual* waves which diverge in all directions from the original source and the partial waves contemplated in his theory would therefore also be of the same nature. The build-up of a finite intensity from the superposition of a very large number of such waves, each of which is extremely feeble, would accordingly be possible only if they arrive *simultaneously* at the point of observation. The diagram appearing in the first chapter of Huyghens' treatise is intended to assist the reader to appreciate the arguments set out in the text; viz., at *each* point on the wave-front a great number of partial waves arrive *simultaneously* and build up the intensity at that point, while the entire wave may be itself considered as made up of a great number of elementary areas at which the light-intensity has thus been built up. In the later chapters in which Huyghens' theories of reflection and refraction and of the propagation of light in an inhomogeneous medium are expounded, the diagrams are intended to exhibit how the complete wave-front arising from these processes is built up out of its elementary parts or areas. Here again, the final result is an individual wave, and it may therefore be correctly described as the envelope of the partial waves which co-operate in building it up.

## 6. The partial waves of Huyghens

Since the concept of partial waves introduced by Huyghens in his treatise has played an important role in physical optics, it is appropriate that we consider it here in some detail. Though the words appear in several chapters of his treatise, it should be remarked that they do not have the same significance in each case. In the first chapter which seeks to explain the rectilinear propagation of light, the partial waves arise as a consequence of the assumed discrete structure of the luminiferous medium; each particle in the medium is regarded as a source of such waves. In the second and third chapters, the partial waves are assumed to arise when the primary wave reaches the boundary separating the two media with different properties. The elements of area of the boundary are here regarded as the source of partial waves. Since they travel with different velocities, they are distinct from each other in the two media. In the fourth chapter which deals with the propagation of light in inhomogeneous media, the partial waves are assumed to diverge from the elements of area of the advancing wave-front in such a medium.

If the luminiferous medium were empty space, the assumption that it consists of discrete particles which can function as emitters of partial waves would be difficult to justify. In the case of material media, however, there is good reason for

assuming that the discrete atoms of which they are composed could function as sources of secondary or partial waves. Even so, however, these partial waves would reinforce each other in the direction of propagation of the primary wave and merge with it, while in other directions they would interfere and cancel out each other's effects. Thus, they would, in all cases, cease to be observable. Accordingly, the notion of partial waves can, in such circumstances, be regarded only as hypothetical or virtual and not as an observable or physical reality. The same remarks would also be applicable in regard to the propagation of light in a medium which is inhomogeneous. Indeed, as already remarked, this particular case could be dealt with in a very simple manner without making any use of the concept of partial waves. Thus, finally, we are left with the phenomena arising from the incidence of light on the boundary between two material media. Huyghens' construction explains the geometric laws of reflection and refraction in so natural and convincing a fashion that it is difficult to resist the conclusion that his concept of partial waves is well-grounded and is a physical reality in these particular cases.

## 7. The so-called principle of Huyghens

It will be evident from what has been said above that the ideas of Huyghens were not correctly understood or appreciated by later writers. It is not surprising therefore that the whole of the vast literature which was subsequently published and which claims to base itself on the ideas of Huyghens, in reality proceeds on a different basis altogether. This is evident from the fact that the mathematicians whose objective was to develop a "rigorous formulation of the principle of Huyghens" concerned themselves with precisely the case in which Huyghens' concept of partial waves has no physical meaning or justification, namely the undisturbed propagation of waves from a source situated in a structureless and uniform continuum.

The well-known formula developed by Kirchhoff is an illustration of the foregoing remarks. Here, the disturbance due to the source at the point of observation is expressed as an integral taken over the area of a closed surface within which the point of observation is included but not the source. Each elementary area of the surface appears in the formula as a source from waves diverge with amplitudes which vary with the direction of emission. The line joining the source and the point of observation is also the direction of maximum amplitude for the waves radiated by the element of area which lies on that line *between* them, and of zero amplitude for an element of area which also lies on the same line but on the *opposite side.* Kirchhoff's formula as actually developed refers to the case of sound-waves, and the attempts made to extend it to the case of light have not met with success. But our present concern is not with the mathematics of the formula but with the physics of the subject. The association of

the formula with the name of Huyghens—honoured as the founder of the wave-theory of light—has naturally disposed whole generations of physicists to look upon it with favour. It has, however, been made clear by the foregoing remarks that Kirchhoff's approach to the subject is quite different from that of Huyghens. We have, therefore, to ask ourselves: Is Kirchhoff's formula really meaningful? Has it any claim to validity or acceptance considered from the standpoint of optical theory? We shall proceed to consider these questions.

As has already been remarked, one of Huyghens' striking successes is his explanation of the geometric laws of reflection and refraction. His concept of partial waves takes its clearest and most acceptable form in this case, viz., that each element of area of the physical boundary acts as a source of partial waves. Since these move with different velocities in the two media, they should be considered as distinct. In other words, the partial waves in each medium are hemispherical, and it becomes a meaningful physical problem to determine the dependence of the amplitude of the waves with direction on the surface of these hemispheres. It would presumably be a maximum in the direction of the normal to the boundary and zero in directions parallel to the boundary. On the other hand, the very generality of Kirchhoff's formula indicates that it has no physical validity or significance. For, it is not possible to discover or assign any reason why an element of area set at an arbitrary orientation in a continuous structureless medium should function as a source of secondary waves with specific features related to that orientation. If the concept of partial or secondary waves is at all to be meaningful, the waves should have a physically recognizable origin, e.g., a local discontinuity in physical properties. In its absence, the formula ceases to have any physical content. Kirchhoff's formula thus reveals itself to be a mathematical abstraction which is not relevant or valid in relation to the actual problems of physical optics.

## 8. Summary

The "principle of Huyghens" has played an important role in physical optics and especially in the explanation of the phenomena of the diffraction of light. An examination of Huyghens' own ideas as expounded in his original treatise is therefore of interest and is undertaken in the present memoir. It emerges that the particular form given to the "principle" by mathematical analysts, notably Kirchhoff, does not find any support or warrant in the writings of Huyghens. The concept of "partial waves" introduced by Huyghens in his treatise is based on specific physical considerations. These are totally lacking in relation to the free and uninterrupted propagation of waves in a continuous structureless medium. The latter case is considered in Kirchhoff's theory and it follows that the Kirchhoff formula is a mathematical abstraction which has no relevance or validity in relation to the actual problems of physical optics.

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# The principle of Huyghens and the diffraction of light

SIR C V RAMAN

## 1. Introduction

When we speak of the diffraction of light, we have in mind certain effects which are observed when the free propagation of light is modified or influenced by the presence of obstacles in its path. It is clear that the nature of the obstacles, including especially their optical properties and their configuration in space, would determine these effects. Surprisingly enough, theories of diffraction have found general acceptance in which these factors receive very inadequate consideration. This situation is connected with the historical development of the subject and has arisen out of a misunderstanding of the ideas originally put forward by Huyghens in his celebrated Treatise on Light. A precis of the first three chapters of that treatise was given in a recent article in *Curr. Sci.*, and it was shown that the so-called principle of Huyghens as enunciated by later authors and made use by them as a basis for the theory of diffraction finds no warrant or support in the treatise. Huyghens did indeed introduce the concept of particular or partial waves and made effective use of it. But these partial waves of Huyghens had definite physical origins and the role which they played could therefore be readily understood. In these respects they differed radically from the ideas ascribed to him by later authors.

Theories clothed in the language of mathematical analysis have not infrequently found supporters and gained acceptance even though the physical ideas on which they are based are unsustainable. Kirchhoff's so-called rigorous formulation of the principle of Huyghens is a case of this kind. A statement often made and generally believed is that the Kirchhoff theory describes the experimental facts of the diffraction of light in a satisfactory manner. This belief has undoubtedly contributed to an uncritical acceptance of the ideas on which that theory is based. It is one of the objects of the present communication to show that it is indeed possible to make Huyghens' concept of partial waves the basis for a treatment of diffraction problems. This leads to results which are in agreement with the facts of experiment but are quite different from those indicated by the Kirchhoff theory. It follows that the latter theory is unsustainable and must accordingly be laid aside.

## 2. The wave-optics of Huyghens

Huyghens sought in his treatise to explain the three most familiar facts of geometrical optics on the basis of wave principles, viz., that the rays of light are propagated in straight lines; that the angles of incidence and reflection are equal; and that in refraction the ray is bent according to the law of sines. His explanations rest on the assumptions which he made regarding the structure of the luminiferous medium and the nature of light waves. His arguments led him to infer that in a homogeneous medium, each little piece of the primary wave emerging from a source of light is capable of travelling in a direction normal to itself more or less independently and that the primary wave-front is the locus or surface at which all the little pieces of which it is made up arrive together at the same instant. The same idea underlies Huyghens' explanation of the laws of reflection and refraction. Each piece of the original wave-front on reaching the boundary between two media is unable to continue on its original course by reason of the velocity of light being different in them. Accordingly it takes fresh paths, one in each of the two media, the direction of travel being such that the pieces of the original wave-front which are diverted from their path can all join up together again to form new wave-fronts in each medium. The latter requirement leads immediately to the equality of the angles of incidence and reflection in the first medium and to the law of sines for refraction into the second medium. This explanation was put into geometric form by Huyghens and is both simple and convincing. Regarded as a physical theory, it is highly successful, since it demonstrates that the refractive indices of the two media are in the inverse ratio of the velocities of light in them.

Examining the ideas of Huyghens in detail, it becomes apparent that his explanation of the rectilinear propagation of light cannot possibly serve as a starting point for a theory of diffraction. On the other hand, his theory of reflection and refraction does offer itself as a basis. For, it makes use of the idea that each element of area of the boundary between two media on which light is incident is a source of partial or secondary waves in the two media. Conceptually, these waves can diverge from each element in various directions, but the requirement imposed by the theory of Huyghens that the disturbances originating at the different elements of area should arrive simultaneously at a common wave-front fixes the actual direction of their movement. If, instead of considering light waves as impulses, we take account of their periodicity and also of the possibility of interferences between the secondary or partial waves having their origin at the different elements of area on the boundary, the restriction of the observable effect to precisely defined directions ceases to exist. In other words, the diffraction of light becomes a possibility.

That the diffraction of light stands in the closest relation to the phenomena of reflection and refraction is also otherwise obvious. As remarked earlier an obstacle of some kind in the path of a light-wave is a *sine-qua-non* for the

manifestation of diffraction effects. A discontinuity in optical properties in the region traversed by the light represents such an obstacle, and if it exists over a sufficiently extended area, it would necessarily give rise to reflection and refraction.

## 3. The law of the secondary wave

A theory of diffraction which bases itself on the original ideas of Huyghens has accordingly to consider the secondary waves having their origin at the elements of area of a boundary between two media of different refractive indices on which light is incident. There would clearly be two sets of such secondary waves travelling out respectively into the two media. The velocity of travel and the amplitude of the disturbance in the two sets being different, they must be considered as completely distinct from each other. If both media are isotropic, the configuration of the secondary waves in each medium would be hemispheres. It is evident also that the particular circumstances of the case, viz., the refractive indices of the two media, the angle of incidence of the primary waves on the boundary and the state of polarisation of the incident light would determine the manner in which the energy of the incident radiation would be divided up between the reflected and refracted wave trains. These same circumstances would also determine the amplitude of the disturbance in the secondary waves sent out respectively into the two media.

A question of importance needing an answer is the manner of dependence on the angle of diffraction of the amplitude of the disturbance in the secondary waves. Considerations of an elementary nature enable us to deduce this. The projection of an element of area dS of the boundary on the surface of the enclosing hemisphere would be dS cos $\phi$, $\phi$ being the angle of diffraction measured from the direction of the normal to the reflecting or refracting surface. This projected area would be a measure of the contribution which the element dS would make to the luminous effect observed in the direction $\phi$. This would accordingly be a maximum in the direction of the normal ($\phi = 0$) and zero along the plane of the boundary ($\phi = \pi/2$). Hence in the expression for the amplitude of the effect due to each individual element, $\cos \phi$ would appear as a multiplying factor. At a sufficiently great distance from the diffracting surface, the angle of diffraction $\phi$ may be assumed to be the same for all its elements of area. It follows that when the expression for the intensity in the diffraction pattern is evaluated by a consideration of the interferences between the effects of the elementary areas, $\cos^2 \phi$ would appear in it as a multiplying factor.

The foregoing results are obviously of very general validity in respect of the diffraction patterns of the Fraunhofer class observed in various circumstances. All that is required is that the diffraction arises by reason of the limitation of the area of a plane surface at which light is reflected or refracted or through which it is

transmitted; in the case of reflection, the material may be either a dielectric or a metal. It is not necessary that the surface should be continuous or that it should have uniform reflecting or transmitting power over the entire area. It might, for example, consist of several parallel strips, thus forming a plane diffraction grating. Further, since refraction at the boundary between two media which differ only infinitesimally in refractive index is equivalent to a simple transmission, it follows that the result would also be applicable to diffraction patterns of the Fraunhofer class arising from the passage of light through apertures in opaque screens.

## 4. Verification of the obliquity law

Any elementary treatment of diffraction theory can only be expected to be valid when the linear dimensions of the diffracting aperture are large compared with the wavelength of the light. As the angular spread of the diffraction pattern would in these circumstances be small, an experimental test of the law of the secondary wave might seem impracticable. Fortunately, however, this is not the case. For, the angle of diffraction $\phi$ is measured from the direction of the normal to the aperture and hence when the incidence of the light on the aperture is oblique, $\phi$ may be large enough for the factor $\cos^2\phi$ to vary rapidly over the area of the diffraction pattern. Further, at such settings the diffraction patterns are spread out over a fairly wide angular range even when the dimensions of the aperture are many times larger than the wavelength. In these circumstances, the effect of the $\cos^2\phi$ factor on the distribution of the intensity in the pattern becomes conspicuous and can indeed easily be observed and measured.

We may illustrate these remarks by considering a simple case, viz., a diffracting aperture which is a plane strip bounded by parallel straight edges. As is well known, when the effects due to the infinitesimal elements of such an aperture are summed up, the expression obtained for the intensity in its Fraunhofer pattern includes a factor of the form $\sin^2\zeta/\zeta^2$. This factor has a maximum value when $\zeta = 0$, and vanishes when $\zeta = \pm\pi, \pm 2\pi, \pm 3\pi$, etc. Since the value of $\sin^2\zeta/\zeta^2$ is unaltered by a reversal of the sign of $\zeta$, the graph of the function when set out with $\zeta$ as the abscissa is a symmetric curve in which the maxima on either side intermediate between the zero values are of equal intensity. The obliquity factor $\cos^2\phi$ appearing in the expression for the intensity would, however, modify this situation to an extent determined by the circumstances of the case.

In the particular case of normal incidence of the light on the aperture, $\zeta = \pi\alpha\sin\phi/\lambda$, $\alpha$ being the width of the aperture, $\lambda$ the wavelength and $\phi$ the angle of diffraction as already defined. More generally, when the light is incident on the aperture at an angle $\theta$ in a plane normal to its edges, $\zeta = \pi\alpha(\sin\phi - \sin\theta)/\lambda$. Differentiating this, we obtain $d\zeta = \pi\alpha/\lambda \cos\phi\, d\phi$. Hence as the incidence is made more oblique and $\cos\phi$ diminishes in value, the angular spread of the pattern determined by the increments of $d\phi$ becomes greater. The bands for which $\phi$ is

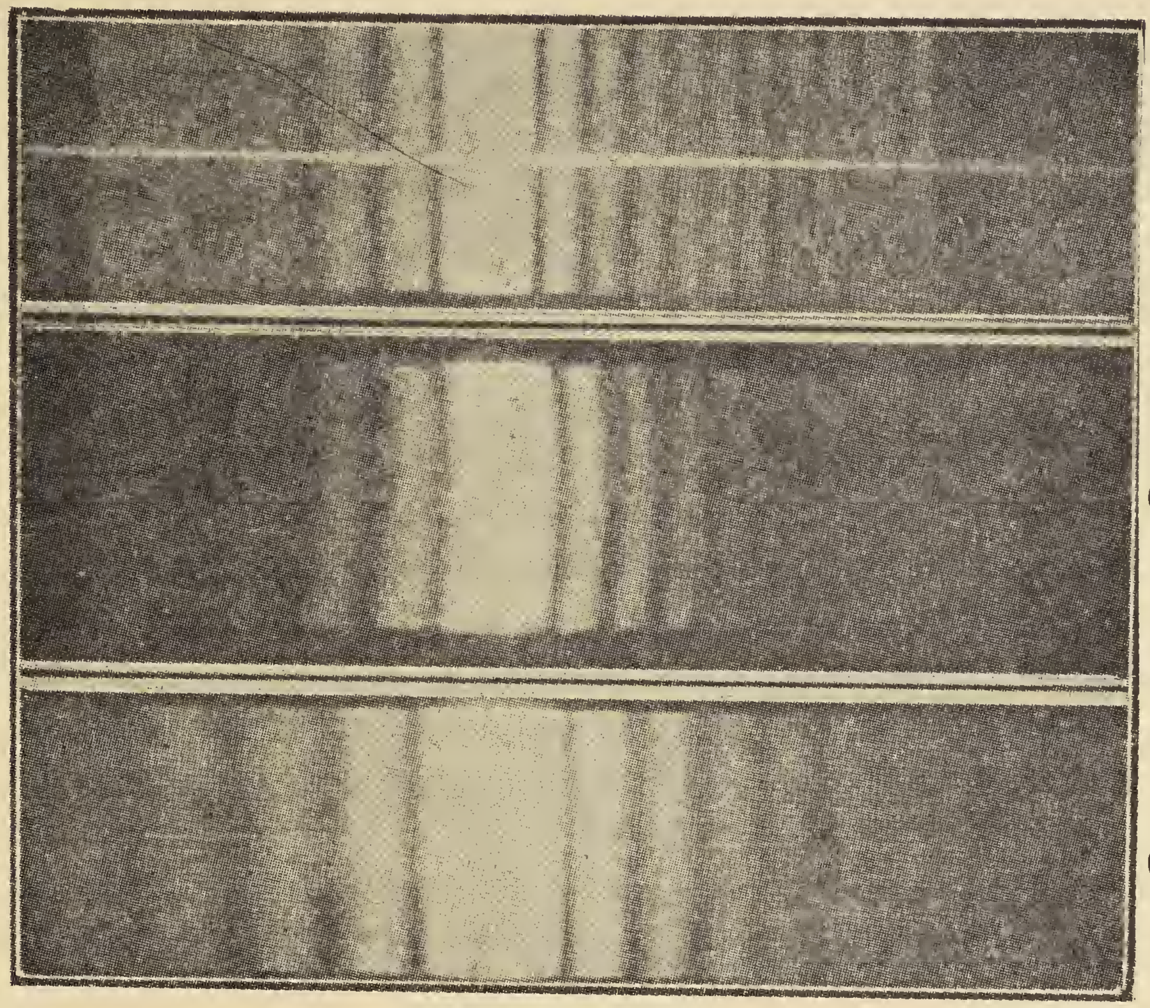


**Figure 1.** Diffraction of light by rectilinear apertures.

greater than $\theta$ would also appear more widely spaced than those for which $\phi$ is less than $\theta$. In these circumstances, the obliquity factor $\cos^2 \phi$ would have a very conspicuous influence on the character of the pattern. The bands for which $\phi$ is greater than $\theta$ would be much less intense than those for which $\phi$ is less than $\theta$; indeed as $\phi$ approaches the limiting value $\pi/2$, the intensity in the former cases would become vanishingly small.

## 5. The results of experimental study

The present theory of diffraction and that of Kirchhoff thus differ fundamentally in the observable results which they indicate. This is scarcely a matter for surprise since they approach the diffraction problem from completely different points of view. Whereas the diffracting body or aperture plays the leading role in the present theory, it is not considered at all in the Kirchhoff formulation; the latter is based on the idea that the primary radiation from a source in free space can be represented as an integral in which the elements of area of a surface enclosing the primary source function as sources of secondary waves. The present theory leads to the result that the amplitude of the secondary waves emitted by the elements of the diffracting aperture vanishes in the plane of the aperture and increases progressively as we move away from that plane towards the direction of its

normal. On the other hand, the Kirchhoff formulation indicates that the secondary waves have a maximum amplitude in the forward direction of the incident light rays and zero amplitude in the backward direction. The difference is of such a striking character that it is a simple matter by means of experimental study to decide between the two theories.

In view of the importance of the issue here raised for a correct understanding of the theory of the diffraction of light, an extended series of experimental studies have been carried out by the writer. Diffracting apertures of various sizes ranging from several centimetres down to fractions of a millimetre have been employed. The angles of incidence of the light on the apertures have been varied from normal right up to grazing incidence. The circumstances in which the diffraction manifests itself have also been varied to include various cases, e.g., the reflection of light at a plane surface of a dielectric or metal, the emergence of light after refraction through a transparent medium at various angles, the internal reflection of light within a transparent medium at incidences beyond the critical angle, and the transmission of light through apertures in plane opaque screens. The cases investigated include both simple and multiple apertures and plane diffraction

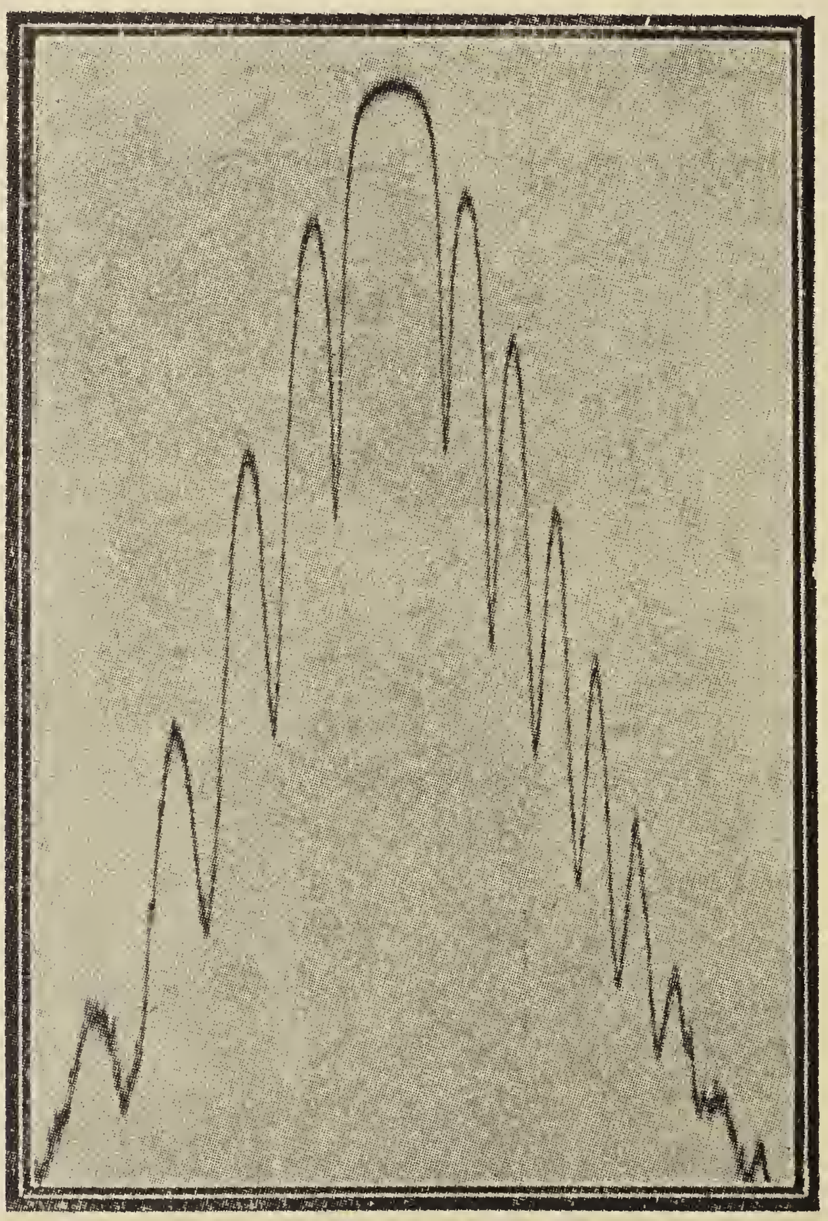

**Figure 2.** Microphotometer record of figure 1(*c*).

gratings prepared by various techniques and operating by reflection as also by transmission. It will suffice here to state that in all the cases investigated, the consequences of the present theoretical approach have been completely vindicated by the facts of observation.

Figure 1 ($a, b$ and $c$) in the text are photographs of the diffraction of light at oblique incidences by a rectilinear aperture obtained by three different techniques. Figure 1($a$) represents the diffraction pattern of the Fraunhofer class obtained with the monochromatic light of a sodium lamp *reflected* by a plane polished surface of glass one millimetre wide at oblique incidences. Figure 1($b$) represents a diffraction pattern observed when light emerges obliquely after *refraction* through a prism of glass, the rear face of which was covered up by an opaque film of silver except for a narrow slit with parallel edges scratched out of it. Figure 1($c$) represents the diffraction pattern *transmitted* obliquely through a rectilinear slit formed by the edges of two razor blades held parallel to each other. It will be seen that all the three photographs show the characteristic features indicated by theory and discussed in the third paragraph of section 4 above. It will be noticed that in each case the intensity of the diffraction bands falls off rapidly to zero on the side where they are broader and the number visible is quite small, while on the other side a great many fringes are seen, the intensity of which falls off very slowly. A microphotometer record of the pattern reproduced as figure 1($c$) appears as figure 2 in the text. The record shows very conspicuously the great difference in the intensities of the corresponding bands on either side of the central maximum.

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# The optics of mirages

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## 1. Introduction

Chapter IV of the celebrated *Traite de la Lumiere* by Christiaan Huyghens published in the year 1690 deals with "the Refraction of the Air". Huyghens discussed the effects which arise by reason of the atmosphere having a refractive index varying with the height above the surface of the earth and also with the physical conditions. The problem was considered by him from two distinct points of view, viz., that light consists of rays travelling out from the source, alternatively of wave-fronts propagated through the medium; he showed that the two approaches give completely identical results. Each small piece of the wave-front in an inhomogeneous but isotropic medium moves forward normally to itself with the velocity appropriate to the refractive index at its position; the pieces thus moving forwards join up to form a continuous wave-front. As the result, we find that the rays of light traverse curved paths which are everywhere normal to the wave-fronts.

Mirages constitute one of the most remarkable effects arising from a variation of refractive index of the atmosphere. A very familiar type of mirage is that seen when the light rays are nearly parallel to the heated surface of the earth; the sky and objects on the horizon then appear to be reflected by the surface or rather by the cushion of hot air in contact with it. The phenomenon is frequently observed over asphalted or concrete roads on sunny days and over firm smooth sands in warm weather. Very striking pictures of such mirages are reproduced in a book by Minnaert.[1] Mirages can also be artificially reproduced in the laboratory. R W Wood[2] in his *Physical Optics* gives photographs of the phenomenon as thus obtained.

On an examination of the explanations usually given for the formation of mirages, it becomes apparent that they are inadequate and indeed unsatisfactory. The authors felt that the subject should be investigated more fully both from the theoretical and the experimental standpoints. The present memoir records the results of such study and throws new light on the nature and origin of mirages.

## 2. The nature of the problem

Consider two media of refractive indices $\mu_1$ and $\mu_2$ which we suppose in the first instance to have a sharp plane boundary of separation that is normal to the $z$-direction. If a plane wave from the first medium be incident on the boundary at a glancing angle $\phi_1$ then, in general, there would exist both a reflected and a refracted wave; the former follows the usual law of reflection, while the direction $\phi_2$ of the latter is given by Snell's law of refraction $\mu_1 \cos\phi_1 = \mu_2 \cos\phi_2$, the intensities of the reflected and refracted beams being given by Fresnel's formulae. If however $\mu_1 \cos\phi_1 > \mu_2$, there exists no refracted beam, so that the incident energy is totally reflected.

In the present paper, we are interested in the case in which the change from the refractive index $\mu_1$ to the refractive index $\mu_2$ does not occur abruptly at a sharp boundary, but takes place continuously and progressively over a certain transition layer—the refractive index being constant along planes normal to the $z$-direction. In this case, general considerations based on wave-optics show that the refracted wave in the second medium should still satisfy Snell's law of refraction $\mu_1 \cos\phi_1 = \mu_2 \cos\phi_2$ while, if the refractive index does not change appreciably within distances of the order of the wavelength, there would normally be no reflected wave in the first medium, the entire energy going into the refracted wave. However, an exception to this statement must clearly arise when $\mu_1 \cos\phi_1 > \mu_2$. Since in these circumstances, there can be no refracted wave in the second medium, a reflected wave must be formed, the reflection being in fact total. Such a situation occurs in the phenomenon of the mirage. Though it is clear that the reflected wave in the first medium must be a plane wave following the usual laws of reflection, the nature of the disturbance within the transition layer can only be ascertained by a rigorous analytical treatment from wave-theoretical considerations. This has been done by Epstein[3] choosing a particular profile for the refractive index variation and the solution obtained is expressed in terms of hypergeometric functions; however, the only application of Epstein's work which he has made and is relevant to the present problem is to remark that there is total reflection in the conditions mentioned, the angle of incidence being equal to the angle of reflection.

## 3. Some elementary considerations

The theory of the mirage which is usually accepted purports to base itself on geometrical optics. Any incident ray initially making an angle $\phi_1$ with the plane of the stratifications would be progressively deviated from its course so that when it reaches a limiting layer of refractive index $\mu_l = \mu_1 \cos\phi_1$, it would be tangential to that layer. It is then usually stated that the ray continues to curve round, but it must be noted that on the basis of geometrical optics the ray should really

continue parallel to the stratifications along the limiting layer. This would happen to every one of the rays in the incident beam. As a consequence, the entire energy of the incident radiation would be concentrated in an infinitely thin layer having the refractive index $\mu_l$. If we take two adjacent rays a finite distance apart, the part of the wave-front between them would on entry into the transition layer swing round and at the same time contract in its extension and ultimately become reduced to a point travelling along the limiting layer of refractive index $\mu_l$.

It is evident that the foregoing situation deduced from elementary considerations presents some fundamental difficulties. In the first place, as we have already noted, considerations of a general nature indicate that the energy reaching the limiting layer which cannot penetrate further must of necessity be returned back through the transitional region into the first medium as a reflected wave. Then, again, as has been stressed by Planck[4] in his *Theory of Light*, the equivalence of the results of geometrical optics with the exact results of wave theory is subject to certain limitations. One of them is that the amplitude of the wave-function must be only slowly variable in space. This condition is clearly violated in the present case, for according to what has been said above, the disturbance would be infinitely great at the limiting layer of refractive index $\mu_l$ and zero immediately below it. It follows that while the actual facts may bear some resemblance in their general features to this elementary picture, the real situation would nevertheless be rather different; firstly, a reflected wave must emerge from the limiting layer; secondly, there should exist a finite intensity below the limiting layer, while above that layer, the intensity though large would exhibit successive maxima and minima arising from the interference of the incident and reflected waves and progressively diminish as we move from the limiting layer toward the first medium.

It is clear from the foregoing remarks that the situation in the vicinity of the limiting layer would resemble that well known in physical optics to appear in the vicinity of caustic surfaces formed by reflection or refraction of light. The effects observed in such cases are usually described as due to the propogation of a cusped wave-front along the caustic surface. Such a result is clearly capable of experimental test in the present case and has been confirmed by the observations made by us presently to be described.

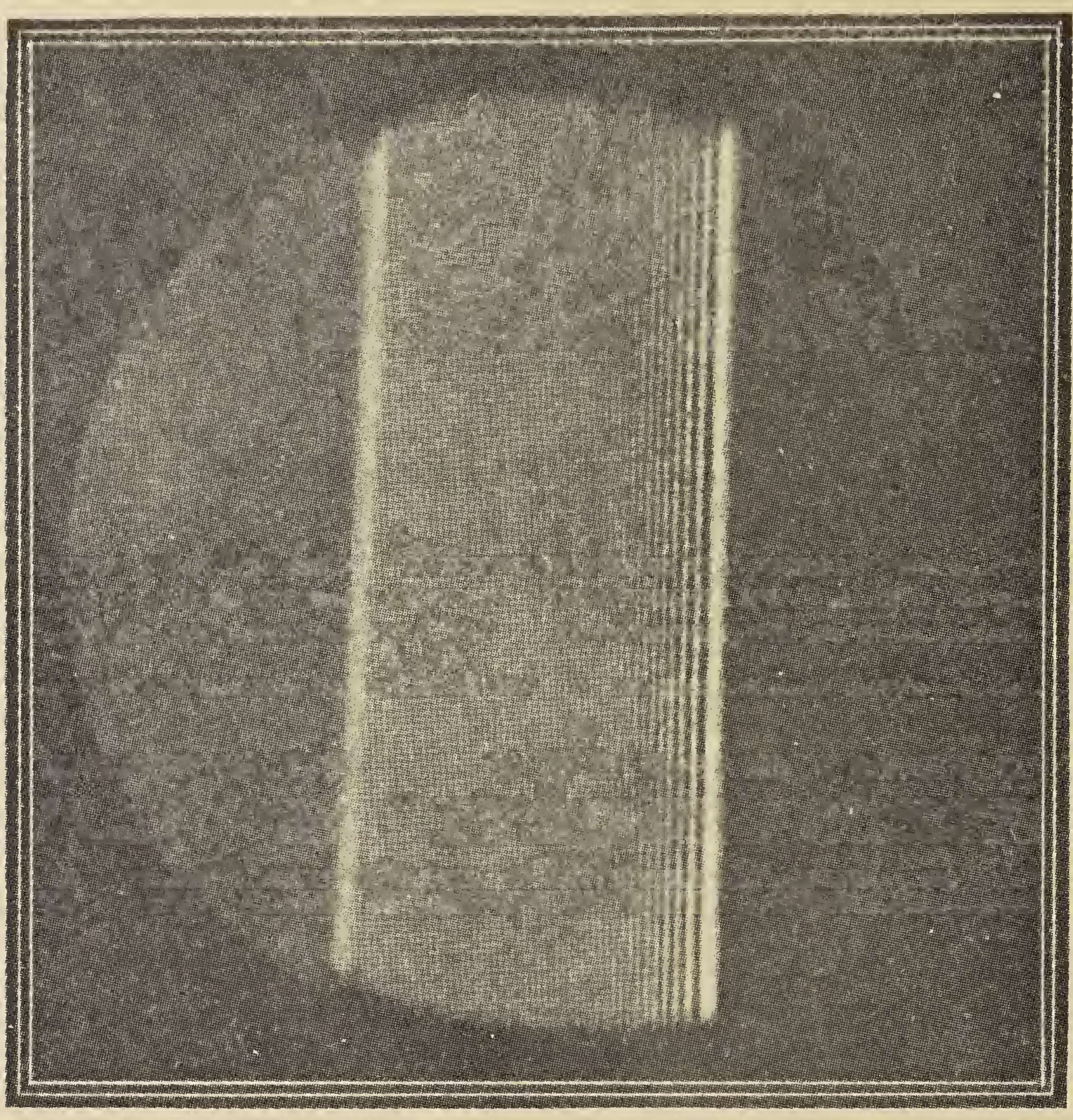
## 4. Observation of the caustic and accompanying interferences

The arrangement first tried for producing the mirage under laboratory conditions was similar to that described by R W Wood. A long steel plate, approximately $1\frac{1}{2}$ metres long, 10 cm in width and 2 cm in thickness was supported horizontally and heated from below by four gas burners, the upper surface being covered with soot. The arrangement did not prove very satisfactory

since the rising of the hot air from the heated surface prevented a sufficiently sharp temperature gradient from being established, while at the same time such phenomena as could be observed were of an exceedingly labile nature owing to the turbulent convection of air near the hot surface. However, the conditions were greatly improved when an electric fan was used to blow away the hot air from above the surface. When this was done, it was possible to see clearly the reflected image of the 'mountain peaks' formed by holding the serrated edge of a cardboard against an illumined ground glass screen at the far end of the plate. The action of the fan in sharpening the temperature gradient was picturesquely revealed by suitably lowering the eye, when a luminous thin cushion of air was seen to form over the heated surface. In order to study the phenomena critically, the object viewed was replaced by an illuminated slit kept parallel to the heated surface and the light diverging from it was rendered parallel by a collimating lens and allowed to fall obliquely on the hot plate. The beam was allowed to cover the whole length of the plate, the angle of incidence being adjusted merely by moving the slit. Further, the plate was heated electrically to ensure a fairly uniform temperature. When the temperature was sufficiently high and the glancing angle sufficiently small, the field at the nearer end of the plate, when examined through a pocket lens, displayed a bright strip of light separated by a dark region from the plane of the hot surface. It was clear, however, that the use of the fan was causing a smearing out of the caustic and preventing phenomena of the nature of interferences from being clearly seen.

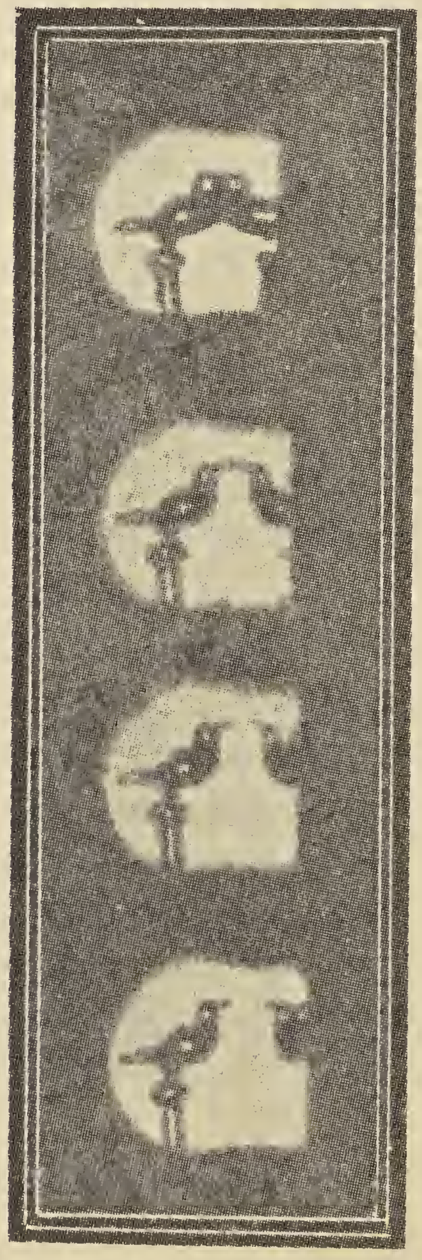
The above difficulty was removed by the artifice of turning the plate edgewise so that the hot surface was now vertical, though its length remained horizontal. The hot air now flowed up in streamlines parallel to the surface of the plate, thus rendering the use of the fan unnecessary. With the slit now vertical, it was possible to observe clearly with the aid of a pocket lens a bright caustic bordered by a large number of interferences of the type referred to in the last section. A photograph of this phenomenon is given in plate I, figure 1. In order to completely arrest the oscillations of the caustic and its fringes, a short exposure (1/100th of a second) was necessary so that the slit had to be brilliantly illuminated. Accordingly, sunlight was used for this purpose and this, in fact, enabled also the introduction of a red filter for increasing the clarity of the fringes.

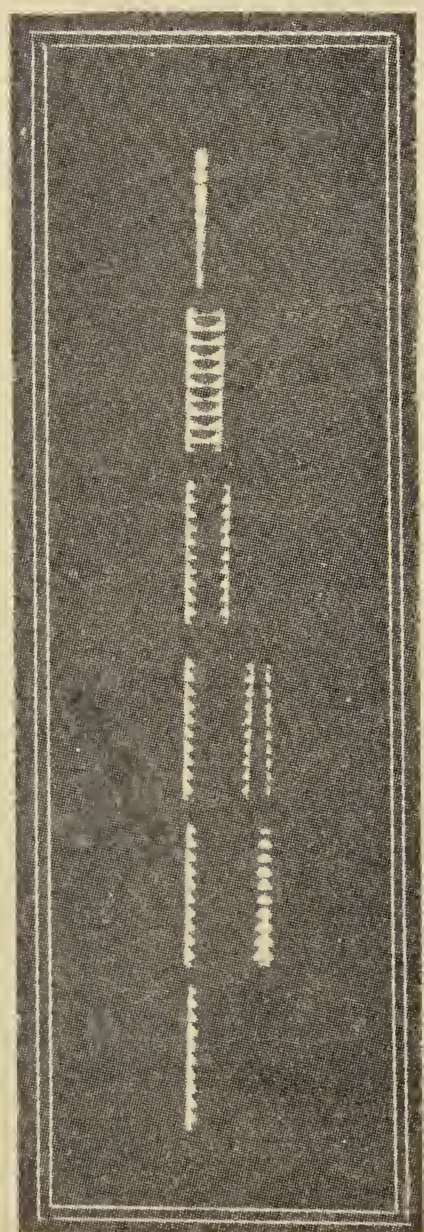
It will be seen that the field in plate I, figure 1 consists of three parts. To the right of the bright caustic (i.e., towards the heated surface) the field is dark, while to its left lies an illumined strip containing a large number of interference fringes whose separation narrows down to a constant value as we move away from the caustic; this is of course due to the increasing inclination between the two branches of the cusped wave-front. Since the heated plate is necessarily of finite extension, the reflected part of the cusped wave-front does not extend to infinity but is terminated. This manifests itself in the field of view by the occurrence of a second edge to the left of which the intensity is considerably less (though not zero), the edge being bordered by some broad fringes.



(1)

(2)





(3)

Plate I

## 5. Relation of the mirage to the cusped wave-front

The actual mirage is observed when the eye is kept at any point which lies on the bright strip of light lying to the left of the caustic, the eye being focussed on the plane containing the object, i.e., at infinity. It is to be expected that two images would then be seen whose positions lie respectively along the directions of the normals drawn from the eye to the two branches of the cusped wave-front leaving the nearer edge of the plate. In fact, the fringes observed to the left of the caustic in plate I, figure 1 may be regarded as due to the interference between the light from two such virtual sources, the progressive narrowing of the spacing of the fringes to the left of the caustic corresponding to the increasing separation of the sources. That the separation of the two images observed depends on the position of the eye or aperture through which the phenomenon is viewed is illustrated in plate I, figure 3. In order to make the nature of the image evident, the serrated edge of a hacksaw blade has been used to form one of the edges of the slit. An aperture was kept in front of the lens and the succession of photographs exhibit the alteration in the phenomena as the aperture is gradually moved to the left. When the aperture is on the bright caustic, a single image is seen, while as it is moved to the left this separates into two images, one of which is a direct (or more properly, a refracted) image, the second being an inverted reflected image, the separation between the two gradually increasing. A remarkable feature of the sequence of phenomena illustrated in plate I, figure 3 is the occurrence of a third erect image close to the reflected image in the fourth and fifth photographs of the sequence; this image starts developing when the aperture has been moved towards the outer edge of the central illumined strip (where broad fringes start appearing in plate I, figure 1), becoming coincident with the reflected image when the aperture is exactly at the edge mentioned. As the aperture is moved further left only the 'direct' image continues to be visible, as is shown in the last photograph of the sequence; this is to be expected since the reflected part of the cusped wave-front is no longer received through the aperture.

The third image mentioned above can be cut off by inserting an opaque screen near the farther end of the heated plate and adjusting it so that its edge protrudes a little beyond the surface of the plate. This shows that the image is due to the ordinary refraction of rays directly entering the edge of the heated layer at the farther end of the plate. The main features of the path of such rays may be deduced from the experimental observations described in the previous paragraph. The terminus of the reflected part of the cusped wave-front corresponds to certain limiting rays entering the region at the farther end of the plate. Rays which are able to enter the edge of the heated layer at a closer distance to the plate than these limiting rays proceed a longer distance before emerging from the heated stratum and also suffer a larger deviation. These rays give rise to the erect third image; in fact, the second refracted wave-front, obtained by drawing the surfaces orthogonal to these rays, meets the termination of the reflected part of the wave-front so as to form a second cusp.

It may be mentioned that an erect third image has been observed and photographed by Hiller[5] in his study of the mirage produced by a long vertical wall. He has attributed the phenomenon to multiple reflection, but it is more probably due to ordinary refraction as in the present case.

Till now we have dealt mainly with the case when the distant object is of negligible angular dimensions. When an object of finite angular dimensions is used, the point on the image which corresponds to any particular point on the object is to be determined as before for each setting of the eye. In this case there will be a distortion of the images because the position of the limiting layer as well as the inclinations of the cusped wave-fronts corresponding to any particular point on the object varies with the position of the object-point. Plate I, figure 2 shows the photographs taken using as the object a small model of a bird made of glass. This was placed near the focal point of the collimating lens. The sequence of photographs show the variation in the appearance of the phenomena as the eye is moved away from the plate. The appearance of a third erect image in addition to the usual reflected image may be discerned in the last two photographs of the sequence.

## 6. Mathematical treatment

Let the $z$-axis be taken normal to the direction of the stratifications of refractive index, the plane of incidence being taken as the $xz$ plane. The disturbance $\psi \exp(iwt)$ at every point must satisfy the wave equation

$$\nabla^2\psi + k^2\mu^2\psi = 0.$$

Since the refractive index is a function only of $z$, we may put $\psi = \exp(ikpx)u(z)$ where $u$ satisfies the one-dimensional wave equation

$$\frac{\mathrm{d}^2u}{\mathrm{d}z^2} + Q^2(z)u = 0, \tag{1}$$

where

$$Q(z) = k\sqrt{\mu^2 - p^2}. \tag{2}$$

We shall take the layer at which the refractive index becomes equal to $p$ to be the plane $z = 0$, and the law of variation of refractive index to be given by

$$\left.\begin{array}{lll} \mu^2 = \mu_1^2 & \text{for} & z \geqslant z_1 \\ \mu^2 = \alpha^2 z + p^2 & \text{for} & z \leqslant z_1 \end{array}\right\}. \tag{3}$$

The general solution of the wave equation (1) for the region $z \geqslant z_1$, can be written

$$\begin{aligned} u = A\{&\exp - ik(\sqrt{\mu_1^2 - p^2 z} + \theta) \\ &+ R\exp ik(\sqrt{\mu_1^2 - p^2 z} + \theta)\}. \end{aligned} \tag{4}$$

If the incident wave be given by $A\exp - ik(\mu_1 \sin\phi_1 \cdot z + \theta)$, the first term will

represent the incident wave with $p=\mu_1 \cos\phi_1$ so that from (3) the plane $z=0$ is the limiting layer; the second term represents a reflected wave with the angle of reflection equal to the angle of incidence, $R$ being the coefficient of reflection. We shall take $R$ to be a real quantity, since this can be ensured by appropriately choosing the phase factor $\theta$, the complex reflection coefficient being $R\exp(2i\theta)$.

Writing $k\alpha = 3\gamma/2$ the wave equation for the region $z \leqslant z_1$ can be written

$$\frac{d^2u}{dz^2}+\left(\frac{3}{2}\gamma\right)^2 zu=0. \tag{5}$$

This is one of the transformations of Bessel's equation, the general solution being expressed in terms of the cylinder functions of order one-third:

$$u=z^{1/2}\,C_{1/3}(\gamma z^{3/2})$$

i.e., with the usual notation for the Bessel functions

$$u=\xi^{1/3}\{A_1J_{1/3}(\xi)+B_1J_{-1/3}(\xi)\} \tag{6}$$

where

$$\xi=\gamma z^{3/2}.$$

For negative values of $z$ the above may be rewritten to avoid the use of imaginary arguments, by putting

$$\begin{aligned}\xi&=i|\xi|=i\gamma|z|^{3/2}\\ u&=\xi^{1/3}\{A_1I_{-1/3}(|\xi|)-B_1I_{1/3}(|\xi|)\}\\ &=\xi^{1/3}\left\{A_1\frac{2}{\pi}\sin\frac{\pi}{3}K_{1/3}(|\xi|)+(A_1-B_1)I_{1/3}(|\xi|)\right\}.\end{aligned}$$

Since $I_{1/3}(|\xi|)$ tends to infinity as $z\to-\infty$, the condition that $u$ remain finite below the limiting layer gives $A_1=B_1$. Hence below the limiting layer $z=0$, we have

$$u=\frac{\sqrt{3}}{\pi}|\xi|^{1/3}A_1K_{1/3}(|\xi|) \tag{7}$$

while above the limiting layer

$$u=A_1\xi^{1/3}\{J_{1/3}(\xi)+J_{-1/3}(\xi)\}. \tag{8}$$

For layers not close to the limiting layer, the asymptotic expansion of (8) for large $z$ may be used:

$$\begin{aligned}u&\sim\left(\frac{3}{2\pi}\right)^{1/2}\xi^{-1/6}A_1\cdot 2\cos\left(\xi-\frac{\pi}{4}\right)\\ &\sim\left(\frac{3}{2\pi}\right)^{1/2}\xi^{-1/6}A_1\left\{\exp-i\left(\xi-\frac{\pi}{4}\right)+\exp i\left(\xi-\frac{\pi}{4}\right)\right\}.\end{aligned} \tag{9}$$

The constants $A_1$ and $\theta$ may be determined by the condition that the solutions (4) and (8) must join smoothly, i.e., by the condition that $u$ and $du/dz$ must be continuous at $z = z_1$. Since the solution $u$ as given by (8) is a real quantity below the plane $z_1$, it is clear from (4) that the reflection coefficient $R$ must be unity, as is to be expected. The constants $A_1$ and $\theta$ may be determined with sufficient accuracy by equating separately the amplitudes and arguments of the periodic functions in (4) and (9) since it is the periodic functions which contribute mainly to $du/dz$. Thus

$$A_1 \sim \left(\frac{2\pi}{3}\right)^{1/2} \xi_1^{1/6} A$$

$$\theta \sim \left(\xi_1 - \frac{\pi}{4}\right) - \mu_1 \sin\phi_1 z_1 \tag{10}$$

where $\xi_1 = \gamma z_1^{3/2}$.

Before proceeding to discuss the solution obtained, we may remark that the function $\xi$ in terms of which the solution is expressed can be given a physical interpretation. Using the fact that $\xi = \int_0^z Q dz$, it may be easily verified that

$$[\xi(z) + kpy] = \int_0^P \mu ds$$

where the right-hand side represents the optical distance from the point $P(x, z)$ to the limiting layer, measured along the limiting ray of geometrical optics. Thus in (9) the solution has been represented as the sum of two disturbances, the surface of constant phase of one of the wave-fronts being taken to be that given by geometrical optics; the other reflected wave-front is symmetrical to the first but has suffered a phase change of $\frac{1}{2}\pi$. The asymptotic solution (9) does not apply near the limiting layer as may also be seen by the fact that it gives an infinite intensity at that layer. The correct solution (8) applicable near the limiting layer may be written in a form similar to (9)

$$u = A_1 \xi^{1/3} \frac{J_{1/3}(\xi) + J_{-1/3}(\xi)}{2\cos(\xi - \frac{1}{4}\pi)} \{\exp - i(\xi - \tfrac{1}{4}\pi) + \exp i(\xi - \tfrac{1}{4}\pi)\}. \tag{11}$$

Thus, the foregoing mathematical treatment provides a justification for the qualitative description given earlier in the paper of the optical field as a cusped wave which moves along the limiting layer.

Using equations (7) and (8) and a table of Bessel functions, the function $(u/A_1)^2$ which is proportional to the intensity has been plotted against $|\xi|^{2/3}$ which is proportional to the distance $z$ from the limiting layer. The graph is shown in figure 1 in the text. It is seen that slightly above the limiting layer there is a large concentration of intensity. Above this again we have a series of interferences which progressively diminish in spacing and intensity, while below the maximum, the intensity rapidly falls to zero.

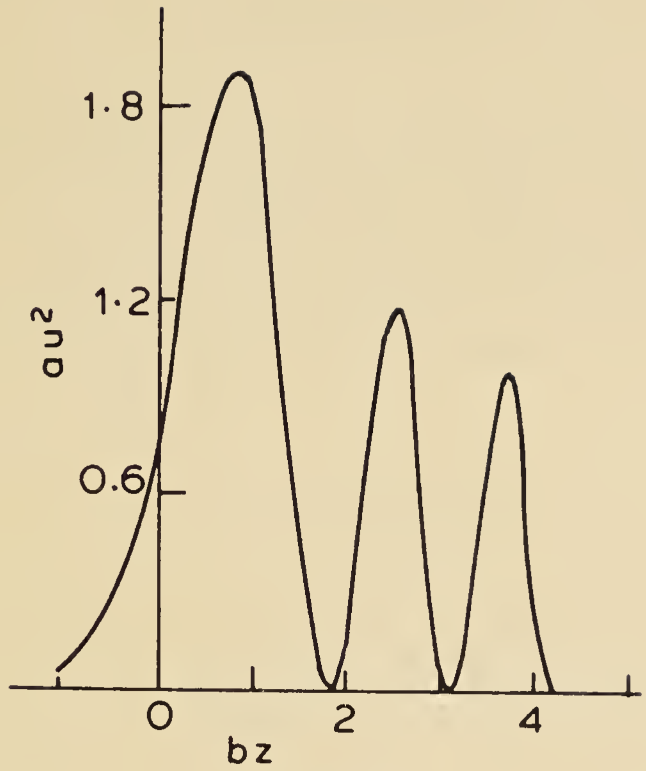



**Figure 1**

It will be seen that the method of solution adopted is similar to that adopted in quantum mechanics where similar mathematical problems arise. In fact, though we have for simplicity considered only the case when $\mu^2$ varies linearly, a similar method[6] can be adopted even in the general case, the solution being again expressible as the sum of two Bessel functions if we neglect the square of the wavelength.

## 7. Summary

Experimental studies reveal that when a collimated pencil of light is incident obliquely on a heated plate in contact with air, the field of observation exhibits a dark region adjacent to the plate into which the incident radiation does not penetrate, followed by a layer in which there is an intense concentration of light and then again by a series of dark and bright bands of progressively diminishing intensity. Photographs of these features have been obtained and are reproduced in the paper. The observed facts find a satisfactory explanation when the problem is considered from the standpoint of wave-theory and more completely on the basis of a formal analytical treatment. The optical characters of the mirage are observed to stand in the closest relation to the wave-optical phenomena referred to, changing with the position of the observer's eye in the field of interference. The explanations of the mirage usually given are thus seen to be inadequate and unsatisfactory.

## 8. References

1. Minnaert M *Light and Colour in the Open Air*, G Bell and Sons, London, 1940, Plate V.
2. Wood R W *Physical Optics*, The Macmillan Co., New York, 1911, Plate III.
3. Epstein S *Proc. Natl. Acad. Sci.*, U.S.A., 1930, **16**, 628.
4. Planck M *Theory of Light*, Macmillan and Co., London, 1932, 204.
5. Hiller W *Phys. Z.*, 1913, **14**, 718; 1914, **15**, 303.
6. Morse and Feshbach *Methods of Theoretical Physics*, McGraw-Hill, 1953, Part II, 1096.

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# The scintillation of the stars

SIR C V RAMAN

## 1. Introduction

The stars in the sky appear to us as mere specks of light having no visible extension. But they exhibit a remarkable feature, viz., a noticeable fluctuation in their observed luminosity. The brightest stars also exhibit flashes of colour when they are located not too high up in the sky. This "twinkling" of the stars is a familiar phenomenon. Its real nature becomes clearer when by the aid of some simple optical device, e.g., a mirror or a lens moved in appropriate fashion, the observer views the image of the star drawn out into a continuous circle of light. This method of observation reveals large and rapid fluctuations of brightness along the track of the moving image of the star. Striking changes in colour are also noticeable in the case of the brighter stars as thus examined. Observation of such stars through a prism which draws out their images into a spectrum of colours further reveals some highly interesting effects.

We are clearly concerned here with an atmospheric phenomenon. In other words, the scintillation arises as a consequence of the passage of the light from a star through the air before it reaches our eyes. But it is by no means easy to understand how such a tenuous medium as the atmosphere could give rise to the observed fluctuations of intensity. It is thus evident that scientific problems of great interest are presented to us by the observed effects. Astronomers are naturally interested in the scintillation of stars by reason of its relationship to the unsatisfactory atmospheric conditions which often interfere with their professional activities. An important aspect of the subject is the location in the atmosphere of the regions in which the disturbed conditions exist giving rise to the observed scintillation. This brings the subject into close relationship with the science of meteorology. Finally, we are concerned with the problem in optical theory of determining how the propagation of the light of a star is modified by its passage through the disturbed layers and gives rise to what is actually observed.

The present communication is not a review article and it is not proposed to survey the published literature or to discuss in detail any particular aspect of the subject. The purpose of the author is to set out a general view of the field as it presents itself to him and in doing so to indicate the basis on which he feels it is possible to reach a clearer understanding of the observed phenomena.

## 2. The thermodynamics of the atmosphere

The light from a star has to traverse the entire atmosphere before it reaches the eye of the observer. The path traversed is the full height of the atmosphere if the star be at the zenith and increases progressively to several times that value as the star goes down in the sky and approaches the horizon. It follows that no attempt to explain the scintillation of stars can claim acceptance which does not take into consideration the actual condition of the atmosphere of the earth at all levels and their influence on the propagation of the light before it reaches the observer.

In the year 1899, the French meteorologist Teisserenc De Bort announced the discovery made by him of the existence of what he called the "Isothermal Layer of the Atmosphere" in its higher levels. The great importance of this finding was appreciated by meteorologists and it is now recognised that the lower part of the atmosphere known as the troposphere and the upper part known as the stratosphere exhibit different structures and play different roles in the thermodynamics of the air. Meteorologists have also given a special name, viz., tropopause, to the layer of transition between the troposphere and the stratosphere. The basic difference between the troposphere and the stratosphere is that the former normally exhibits a progressive fall of temperature with height, whereas the latter can be considered as isothermal, at least in the regions not exceeding some 20 km in height above the surface of the earth. Beyond this height, the density of the atmosphere becomes a small fraction of its value at sea-level.

The question naturally arises how this division of the atmosphere into two parts with a different thermal behaviour arises. The answer may be found in the processes by which the atmosphere periodically gains and loses heat. It is in the troposphere that the energy received as radiation from the sun and absorbed by the surface of the earth is carried upwards into the air as heat by convective processes. The upper limit of the troposphere may, therefore, be taken to be the level at which these convective processes cease to function. In the stratosphere, on the other hand, we are chiefly concerned with the circulation of the atmosphere on a global scale brought about by the unequal heating of the earth's surface in low and in high latitudes.

The explanation of the division of the atmosphere into two parts indicated above receives support from the actual facts of the case. In the first place, it is found that the rate of fall of temperature with the height—termed by the meteorologists as the lapse-rate—has approximately the same mean value at all heights in the troposphere and in all latitudes, viz., 6° C per kilometre. On the other hand, the height of the troposphere is found to depend very markedly on the latitude, being about 16 km at the equator, about 11 km in middle latitudes and dropping to about 6 km at the poles. Further, it has been established by regular soundings of the upper air that there are day-to-day and seasonal variations in the height of the tropopause, these being particularly marked in the middle latitudes. Indeed it is there found sometimes difficult to locate any well-marked

discontinuity between the normal rate of temperature decrease occurring in the troposphere and the approximately isothermal distribution of the stratosphere. It is clear from these findings that the conditions in the layer of transition between the stratosphere and the troposphere are of a dynamic nature and far from being static.

## 3. The optics of the atmosphere

The atmosphere of the earth is a stratified medium in the sense that the refractive index of the air falls off progressively with the height above sea-level in proportion to the diminishing density, but less quickly than the atmospheric pressure owing to the fall in temperature. The figures in table 1 illustrate this for the wavelength $\lambda$ 5896 and the standard USA atmosphere. They justify the remark made in the introduction that the atmosphere is a tenuous medium, which, indeed, is very much in the nature of an understatement with reference to its higher levels.

**Table 1.** Refractive index of dry air

| Height (Km) | Pressure (mb) | Temperature (°C) | Refractive index |
|---|---|---|---|
| 0 | 1013 | +15 | 1·0002765 |
| 5 | 540 | −17 | 1·0001658 |
| 10 | 264 | −50 | 1·0000931 |
| 15 | 120 | −56 | 1·0000435 |
| 20 | 55 | −56 | 1·0000199 |

Accepting the proposition that the scintilation of the stars is a consequence of local variations in the refractive index of the atmosphere, we have to ask ourselves and answer the following questions. How do these variations arise? What is the numerical magnitude of the variations? What is the measure of their extension in space? In what region or regions of the atmosphere do they appear? Finally, are they actually capable of producing the observed effects? We proceed to deal with these questions *seriatim.*

The refractive index of air is determined by its composition and by its pressure and temperature. The content of water-vapour included in it is, in the lower levels of the atmosphere, the variable part of the composition. For the sake of simplicity, we shall, in what follows, not consider the variations of composition explicitly. For a standard atmosphere, the pressure and the temperature are known as functions of the height above sea-level. In dealing with possible variations from these standard values, a certain measure of simplification is introduced by the well known principle that any volume-element of air, whether

it is in a state of rest or of motion, automatically takes up the pressure of its surroundings. Hence, the difference in the refractive indices of the element and its surroundings is determined by their respective absolute temperatures. It follows from this, that local variations of *temperature* play a specially important role in our present problem.

With these guiding principles in mind, we shall proceed to deal with the other questions raised above. The plane waves of light from a star have to travel through three distinct regions before they reach an observer. (1) The stratosphere, (2) the transition layer between the stratosphere and the troposphere, and (3) the troposphere. We may consider these in order.

As shown by the figures in table 1, the refractive index of the air in the stratosphere falls to very low values. Further, this region of the atmosphere may, at least as a first approximation, be considered as isothermal. We are, therefore, justified in assuming that the stratosphere does not play any role in the phenomena of scintillation, though naturally it would contribute sensibly to the refraction and dispersion of the light reaching the earth from stars at a low altitude above the horizon.

As has already been remarked, the region of transition between the stratosphere and the troposphere is one of dynamic change in which air-masses differing in thermal behaviour are continually altering their locations. Hence, this is a region which, *prima facie*, should be capable of giving rise to the phenomenon of scintillation. We shall return to this later and meanwhile turn to the case of the troposphere.

Meteorologists usually subdivide the troposphere into three parts, the lower, the middle and the upper troposphere respectively, and have recognized that each of these divisions has its own special features. The lower troposphere is that most affected by heat transfer between the surface of the earth and the air and it is, therefore, the region of which the structure shows the largest variations due to the periodic heating and cooling of the ground. It is in this part of the troposphere also that the so-called temperature inversions make their appearance in certain circumstances. The middle and upper troposphere, on the other hand, are practically uninfluenced by the diurnal temperature variations and exhibit the normal rate of temperature decrease with height. While they do exhibit seasonal variations in temperature, these variations are brought about by relatively slow processes. Whether in these circumstances, the troposphere can play any role in the production of scintillations may well be questioned. We shall presently proceed to discuss this matter.

## 4. The origin of the scintillations

It is clear that the methods and ideas of geometrical optics cannot possibly lead us to any acceptable explanation of the phenomena of scintillation. Indeed, one

might go further and say that they are entirely out of place in any problem concerning the propagation of light in a medium of variable refractive index. This becomes evident when, adopting the language of wave-optics, we remark that a change of refractive index means an alteration in the rate of change of the phase of the waves as they advance through the medium and hence there arises the possibility of large changes of intensity being produced by interference when overlapping occurs of waves which have traversed slightly different paths in the medium. Considerations of this kind are wholly foreign to the concepts on which geometrical optics is based. On the other hand, they play an essential and highly successful role in explaining the observed phenomena in diverse cases investigated at various times by the present author and his collaborators. We may here mention particularly the phenomena observed when light waves traverse a transparent medium carrying ultrasonic waves.

The foregoing remarks indicate in a general way the lines on which an explanation could be sought of the phenomena of the scintillation of stars. Somewhere on the path of many kilometres which the light from a star has to travel, the wave-fronts pass through a disturbed region in traversing which retardations of phase are suffered which are unequal over the area of the wave-front, while the amplitudes remain unaltered. During the further propagation of the waves, the phase-changes transform themselves into amplitude-changes; in other words, the effect of the unequal retardations of phase higher up in the atmosphere manifest themselves as unequal intensities when the waves reach ground level.

In thus applying the ideas of wave-optics to determine the effect of the passage of light through an atmosphere of varying refractive index, a very great simplification is possible by reason of two characteristic features in the problem, viz., that the refractive index of the medium itself differs but little from unity, and that, further, any possible variations of it would be themselves a small fraction of the difference between the index and unity. From this, it follows that if the waves traverse in succession, two regions in one of which the index is higher and in the other it is lower than the average index of the medium, the phase-changes produced by them would cancel out partly or wholly depending on their actual values. In other words, a medium in a turbulent state and in which the refractive index exhibits random fluctuations would behave in much the same way as one which is quite uniform and has the same average index everywhere. Only in those cases where the phase-changes produced are so large and so distributed that they do not cancel out completely would the waves emerge from the disturbed region exhibiting any observable consequences of their passage through it.

From the figures given in table 1, we may readily deduce the difference in optical path resulting from the passage of light through a column of air one metre thick which is at the same pressure as its surroundings but differs in temperature by 1° C. The results of these calculations are shown in table 2.

**Table 2.** Phase-change in wavelengths

| Height (Km) | Pressure (mb) | Temperature (°C) | Phase-change per metre per degree |
|---|---|---|---|
| 0 | 1013 | +15 | 1·69$\lambda$ |
| 5 | 540 | −17 | 1·02$\lambda$ |
| 10 | 264 | −50 | 0·7$\lambda$ |
| 15 | 120 | −56 | 0·34$\lambda$ |
| 20 | 55 | −56 | 0·15$\lambda$ |

## 5. Location of the disturbed regions

The lowest part of the troposphere lying within the first 5 km above the surface of the earth is the region in which the convective processes set up by alternate heating and cooling of the ground during day and night respectively are most evident. One might be inclined to infer from this that the same region of the atmosphere would be principally responsible for the scintillations of the stars perceived by an observer at ground level. Against this presumption can be urged the following considerations. Actually, we are concerned with the condition of the atmosphere at night time and not during the day. The upwelling of the overheated air from the ground would have reached and passed its maximum before nightfall and the violent changes of temperature consequent thereon would to a large extent have been smoothed out by the adiabatic expansion of the rising air and the cooling resulting therefrom, as also by the mixing up of the masses of air at different temperatures by the process of eddy diffusion. There would, no doubt, be left over some residual differences of temperature, but the individual volume-elements exhibiting such differences might be expected to be of very moderate dimensions. As already explained, the phase-changes suffered by the wave-fronts of the light in passing through these layers of air would be more or less completely cancelled out by a process of averaging. Temperature inversions, if any are present, would not alter this situation, so long as the surfaces of equal average temperature and the surfaces of equal pressure everywhere run parallel to each other, hence also to the surfaces of equal average refractive index.

Similar considerations would apply and even more cogently in the case of the middle and upper parts of the troposphere. The existence of regular stratifications of temperature parallel to the stratifications of pressure in those regions makes it highly improbable that phase-changes of the nature and magnitude necessary for giving rise to scintillations could be produced by the passage of the light waves through those layers.

Thus, we are led by a process of exclusion to conclude that, at least ordinarily, the disturbed region which is responsible for the scintillations perceived by an

observer at ground level lies high up in the atmosphere being in fact the region of transition between the stratosphere and the troposphere. It has already been remarked that this region is essentially dynamic in its origins. Unless the transition between the troposphere and the stratosphere is a sharply-defined geometric plane—and such a situation cannot reasonably be expected to exist or persist—a wave-front passing through it would suffer phase-changes varying from point to point over its area. The figures exhibited in table 2 show that at that level, a transition layer only one metre thick and varying only by 1° C over its area would produce phase-changes of the order of half a wavelength. This would suffice to produce large and readily observable changes of intensity over the area of the wave-front when it has travelled over a sufficiently long path below the tropopause.

It is, however, necessary to remark that disturbed regions of other kinds may also appear in other circumstances and give rise to noteworthy optical effects. Meteorologists are familiar with the idea that a boundary or relatively narrow transition zone must exist between opposing wind currents or contrasting air-masses, and they refer to such boundaries as *fronts*. They are formed when two air-masses meet which differ in temperature and in density. The conditions existing at these boundaries or transition zones would evidently be favourable for large variations of phase to manifest themselves when they are traversed by the waves of light.

## 6. The character of the scintillations

The ideas set forth above when developed in detail lead us to a clear understanding of the entire ensemble of phenomena related to the observed scintillation of the stars. The wave-fronts which emerge from the disturbed regions of the atmosphere and exhibit localised phase-changes may be analysed into groups of plane wave-trains travelling in directions inclined at various angles to the original direction of propagation. The amplitudes of these wave-trains and their inclinations are determined by the magnitude of the phase-changes in the original wave-front and the areas over which they appear. As a consequence, the telescopic image of a star would be spread out over a finite range of angles. The light reaching down to the earth would also exhibit a pattern of interferences due to the overlapping of these wave-trains. The maxima and minima of intensity in this pattern would be the closer together, the larger the angles are between the interfering wave-fronts.

Since the region in the atmosphere at which the optical disturbances originate is at a high level, the interference pattern observed at the surface of the earth would necessarily exhibit a movement parallel to the surface as a result of the rotation of the earth about its axis. The intensity of the light reaching the observer would fluctuate as the interference pattern moves over his eyes. These fluctu-

ations would be the more rapid, the closer together the maxima and minima are in the interference pattern. As has already been remarked, their spacing is determined by the magnitude of the disturbance to regular wave-propagation produced by the atmospheric conditions. A relationship thus emerges between the rapidity of the observed scintillations and the effect of atmospheric conditions on the telescopic appearance of the star.

Colour effects can arise in two different ways. Since the scintillations owe their origin to interference, colour may be expected to manifest itself by reason of the wavelength differences in the spectrum. Effects thus arising would however be inconspicuous unless the interferences are of low order. Colour effects of a different nature are also possible. The light from a star suffers refraction and dispersion in traversing the atmosphere of the earth. The deviations thus arising increase with the zenith distance of the star. The dispersion is however quite small unless the zenith distance exceeds 45° of arc. When the dispersion exceeds a few seconds of arc, the interference patterns for the different parts of the spectrum would cease to coincide at any given instant, but would become identical or nearly so at successive instants determined by the zenith distance of the star and the rotation of the earth. Very striking colour effects would then be observable, especially in the case of the brighter stars.

## Summary

The scintillation of stars is explained as an interference effect which arises in the following manner. Plane waves of light from a star when passing through a disturbed region high up in the atmosphere suffer phase-changes but no changes of amplitude in their wave-fronts. At lower levels, the phase-changes are transformed into amplitude changes, in other words, interference patterns are formed. These patterns move over the surface of the earth by reason of its rotation about the polar axis. The fluctuations of intensity passing over the eye of the observer are perceived by him as scintillations. It is shown that in this way, the entire ensemble of phenomena related to the scintillation of stars receives a satisfactory explanation. Reasons are given for identifying the region in the atmosphere ordinarily responsible for the observed scintillations at ground level to be the tropopause, in other words, the region of transition between the stratosphere and the troposphere.

# LECTURES ON PHYSICAL OPTICS

PART I

(Sayaji Rao Gaekwar Foundation Lectures)

BY

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# Prefatory note

In the month of February 1941, the author visited Baroda and delivered a course of two lectures on 'Light as Wave-motion' and 'Light as Corpuscles' respectively. It was the desire of the Foundation which invited the author to Baroda that the subject matter of these lectures should be developed and written out in the form of a series of six lectures for publication. It was planned that the lectures would deal with the following topics: (I) Interference of Light, (II) Diffraction of Light, (III) Coronae, Haloes and Glories, (IV) Optics of Heterogeneous Media,* (V) Light in Ultrasonic Fields* and (VI) Molecular Scattering of Light.* These topics had been the field of investigation by the author and his collaborators for many years and it was intended that the principal results of those investigations should find a place in the published lectures.

The preoccupations of the author slowed down the writing up of the volume for publication and finally brought it to a stop in the year 1943 after 160 pages had been printed off. Much labour and thought had been devoted to the work and it is believed that it contains material of enduring value and interest. Accordingly, it appeared desirable to release the part already printed as part I of the lectures and thus make it available for perusal by those interested in optical theory and experiment.

*Not published.

# Lecture I

## Interference of light

LIGHT is a phenomenon which we perceive and which plays a fundamental role in human life and activity. Constant experience makes us familiar with various aspects of the behaviour of light. These may broadly be classified under three headings. The first group of experiences relates to the geometric aspects of the propagation of light. Under this heading come the rectilinear path pursued by light from the source to the observer in free space, the casting of shadows by obstacles and the geometric laws of reflection and refraction at the boundary between different substances. The second group of experiences relates to the character of the sensations produced by light, which are three-fold, namely, the brightness of light, the colour and the degree of its saturation. The third group of experiences connects light with the properties of material bodies, namely their capacity to emit, absorb, reflect, refract and scatter light, thereby making themselves visible. The study of the phenomena of light under these headings respectively constitute the three great divisions of optical science, namely, geometrical, physiological and physical optics.

The three categories of optical experience defined above can be brought into intimate relationship with each other only through an understanding of the ultimate nature of the emanation which we perceive as light. Experimental studies enable us to distinguish between those phenomena which are of a subjective or physiological nature and the properties of light that have a definite physical basis. The spectroscope, for example, enables us to separate out the rays of light of different colour and warns us that light which is perceived as yellow visually is not necessarily the same as the light which appears as yellow after spectral analysis. The spectroscope, in fact, enables us to make the first real step in

understanding the physical nature of light. It indicates that there are different kinds of light which are physically different, yet analogous to each other, and that if optics is to be an exact science, we must consider the behaviour and properties of light which is truly monochromatic, in other words appears as a sharp single line in the spectrum. Fortunately, various sources of light are available in which the luminous centres are gaseous atoms, the radiations from which on examination through a spectroscope appear as discrete lines of sufficient intensity to be practically useful. Amongst these, the mercury vapour lamp is by far the most generally useful; it is indeed a veritable Alladin's lamp for the student of optics. Other sources of light are occasionally needed for special purposes. Amongst these may be mentioned specially the zinc amalgam lamp, which gives three lines in the blue region of the spectrum which are highly monochromatic, two of them being fairly intense. The lines of the mercury arc appear sharp and single when examined through an ordinary prismatic spectroscope. But examined through a high resolving-power instrument such as a Fabry–Perot etalon, a Lummer–Gehrcke plate or a reflection echelon grating, the mercury lines exhibit numerous components or satellites, while the lines due to the zinc atoms appear as truly single or monochromatic. In figure 1(a) is illustrated the spectrum of the zinc amalgam lamp as recorded by an ordinary spectroscope. Figure 1(b), 1(c) and 1(d) record respectively the same spectrum as further analysed by a Fabry–Perot etalon, a Lummer–Gehrcke plate and a reflection echelon grating. Each of the lines is marked with its approximate wavelength in Angstrom units ($Å = 10^{-8}$ centimetre), the symbols Hg and Zn denoting that the radiations are due to the mercury and zinc atoms respectively.

*The interference of light*: The geometric theory of light rays forms the basis of applied optics and is extensively and successfully employed in the computation and design of the lens systems used in optical instruments of various kinds. The practical use made of these instruments in the field and in the laboratory also assumes the validity of the laws of geometrical optics, including especially the rectilinear propagation of light in uniform media. The wave-theory starts with a different view of the nature of light, namely that it is wave-motion propagated through space and that monochromatic light has associated with it a definite frequency of oscillation and a definite wavelength, the product of the two being equal to the wave-velocity in the medium. In certain simple cases, namely in the cases of plane and spherical waves in an isotropic medium, the relation between the ray and wave concepts of light is readily stated; the direction of the geometric rays in these cases is identical with the direction of movement of the wave-fronts. The essentially new possibility which the wave-concept involves is that of interference, viz., that when two beams of light originally derived from the same light source are superposed on each other, the light intensity at any point in the field may be either greater or less than the arithmetic sum of the intensities due to

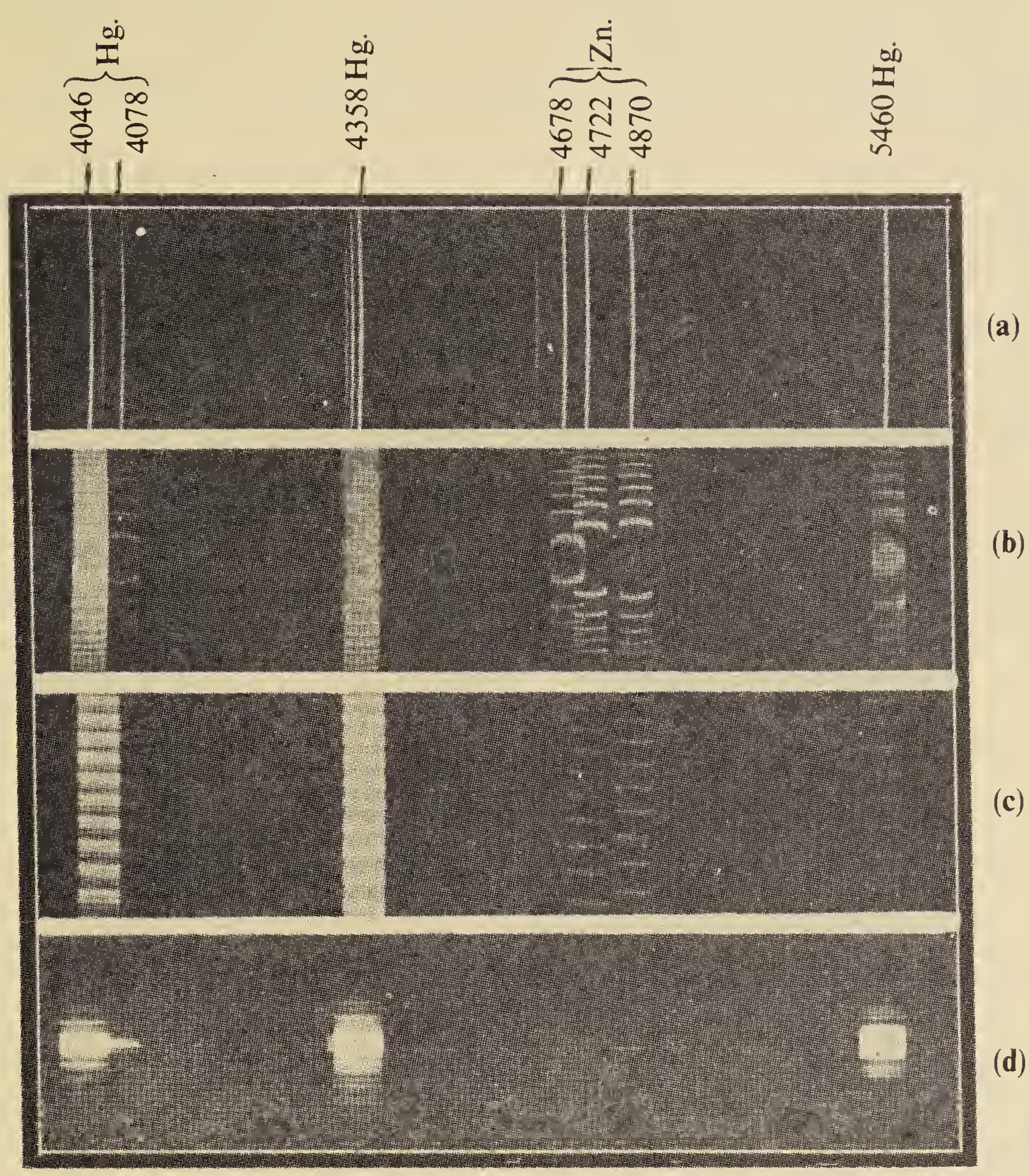


**Figure 1.** Spectrum of zinc amalgam lamp. (**a**) Prism spectrograph. (**b**) Fabry–Perot etalon. (**c**) Lummer–Gehrcke plate. (**d**) Reflection echelon grating.

either separately. That such an effect is actually observed forms the strongest support for the wave-theory of light.

We may illustrate the principle of interference by considering the case of two beams of light which are divided from the same original beam by suitable optical arrangements and which traverse together a limited region of space before again separating. It will be assumed that the light beams consist of polarised light of wavelength $\lambda$ and that the directions of their travel lie in the plane of the paper and cross each other at an angle $2\Psi$ (figure 2). The amplitudes and directions of the light vector in the two wave-trains may be assumed to be identical and to be normal to the plane of the paper. The thin oblique lines in the figure represent the wave-fronts in which the light vector at a particular epoch is a maximum

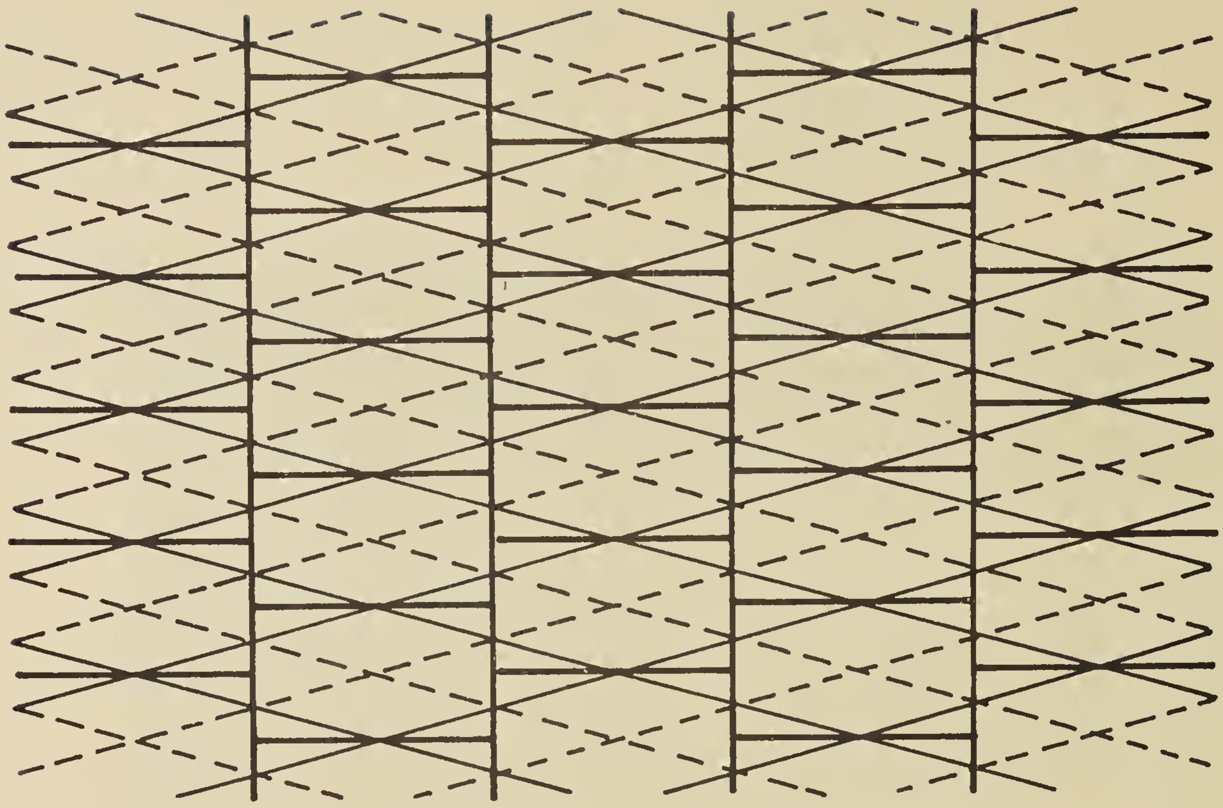

**Figure 2.** Interference of plane wave-trains.

upwards, while the broken lines represent the intermediate planes at which it is a maximum downwards. The thick lines which have been drawn bisecting the obtuse angles between the wave-fronts represent planes in which the light-vectors are in opposite phases and the resultant intensity is, therefore, zero. At the intermediate planes, the intensity would be a maximum. The thick lines bisecting the acute angles between the wave-fronts are the planes along which the resultant light vector at the given epoch is a maximum upwards. The spacings of these two sets of planes are given by the relations $\lambda = 2D \sin \Psi$ and $\lambda = 2d \cos \Psi$ respectively. When $\Psi$ is zero, $D$ is infinite and $d$ is $\lambda/2$, while when $\Psi = \pi/2$, $D$ is $\lambda/2$, and $d$ is infinite. It is evident that the amplitude of the resultant disturbance oscillates as we pass along the horizontal lines in the figure, while its phase is reversed whenever we cross any of the vertical lines of zero intensity. Hence, the horizontal lines cannot be considered as representing true wave-fronts and they do not therefore possess any significance from the standpoint of geometrical optics. This will be further evident when we recollect that when the two wave-trains separate, they pursue their courses independently along their original directions of propagation. The alteration of the energy distribution in the field indicated by the principle of interference is thus not in any way a contradiction of the basic ideas of geometrical optics.

It is evident from figure 2 that the effect of interference between the two sets of plane waves is to produce a stratification of intensity in the medium, the spacing of which is very wide when the inclination between the wave-fronts is small and

diminishes as the inclination increases, reaching the limiting value $\lambda/2$ when the waves travel in opposite directions. When the stratifications of intensity are widely separated, they are readily seen as interference bands in the field. The observation becomes less easy and needs special technique when the spacing of the stratifications narrows down and approaches the limiting value of half the wavelength of light. The optical conditions represented in the figure can be experimentally realised in a variety of ways. The simplest method is to obtain one of the two beams by reflection at the surface of a mirror at the desired angle of incidence, while the other beam is furnished by the incident light itself. It is readily seen from the figures that the character of the resultant disturbance would, in general, be very different when the incident light is polarised with the vibrations respectively parallel and perpendicular to the plane of incidence of the light on the mirror. In the latter case, which is the one discussed above, the light vectors in the interfering wave-trains would be parallel to each other and would differ only in phase, and the alternations in intensity resulting from interference would therefore be most noticeable. On the other hand, when the light vectors lie in the plane of incidence, they would be parallel to the wave-fronts and would therefore be inclined to each other in the two wave-trains. The resulting disturbance would therefore, in general, be elliptically polarised. It is evident that except in the cases of nearly normal or nearly grazing incidence of the light on the mirror, the interferences in this case would result in less conspicuous variations of intensity than when the light vectors are perpendicular to the plane of incidence.

*Interferences of parallel plates*: As remarked above, the spacing of the stratifications of light intensity in an interference field depends on the angle at which the wave-fronts cross; the larger the angle, the less easily noticeable would they be. The optical conditions for observing the results of interference would obviously thus be most favourable when the superposed beams of light are completely coincident in direction, as the intensity of illumination over the entire field would then be enhanced or diminished, and it would be possible also to use an extended source of light. Such a situation arises when a pencil of light is reflected by or transmitted through a plate of transparent material bounded by plane parallel surfaces. In the light reflected by such a plate, a series of successive reflections at its surfaces appear superposed, the first external and internal reflections being the two strongest and nearly equal in intensity. Similarly, the light beam transmitted by the plate has superposed on it the light beams which have suffered an even number of internal reflections within the plate, these being much weaker. It is readily shown that the optical difference of path between the successive superposed beams in either case is $2\mu t \sin\theta$ where $\mu$ is the refractive index and $t$ the thickness of the plate, and $\theta$ is the glancing angle of internal reflection. The reversal of phase which occurs at an external reflection has also to be taken into account. Accordingly, in the light reflected by the plate, we have the minimum intensity if $2\mu t \sin\theta = n\lambda$ where $n$ is an integer and $\lambda$ the wavelength of the light in

vacuum. The same condition gives the maximum intensity for the transmitted light.

The considerations stated above indicate that the intensities of the light reflected and transmitted by a plate should exhibit fluctuations if either the thickness of the plate or the angle at which it is viewed be varied. Interferences of this kind are very readily observed and were indeed historically the first to be noticed and explained on the principles of the wave-theory. If the plate be sufficiently thin, white light may be used for the observations, the alternations of intensity then manifesting themselves as variations in the colour of the reflected or transmitted light. The colours of soap films, for instance, arise in this way. If a flat soap-film be set vertically, the horizontal bands of colour which develop on its surface are the result of the thickness of the film increasing as we go downwards. Colour bands on a soap-film may, however, also arise from a variation of the angle at which its surface is viewed by the eye. This effect is well shown by a spherical soap bubble; if the bubble is blown of a uniform thickness, the colour bands on its surface are due solely to the varying obliquity of observation and appear as concentric circles around the line of sight, irrespective of the direction from which the sphere is viewed. On the other hand, if a bubble be of non-uniform thickness, the distribution of colour depends on the direction from which the bubble is viewed. The colour bands are horizontal circles when the bubble is viewed from above or below. But when the bubble is viewed horizontally, the circles appear deformed or displaced downwards, as the result of the effects of varying thickness and of varying obliquity of observation appearing in combination. A spherical bubble at rest tends to drain downwards. This may however be counteracted and the bubble maintained in a state of uniform thickness by gentle currents of air impinging on its surface.*

The phenomena referred to above are illustrated in figure 3, the two upper pictures being respectively those of a uniform and non-uniform bubble viewed horizontally by transmitted light, while the two lower pictures represent similar bubbles by reflected light. The figures also illustrate other features of interest. It will be noticed that the interferences as seen in transmission and by reflection are complementary in appearance. This is to be expected, as the energy that disappears in reflection appears as transmitted light, and *vice versa*. The dark and bright rings in the transmitted light, therefore, correspond respectively to the bright and dark rings seen by reflection. The contrast between the dark and bright rings by transmitted light is evidently much less than in reflection. This is also to be expected, as the interfering beams are not of comparable intensity in the former case, whereas they are of practically equal intensity in the latter. It will be noticed, also, that the contrast between the dark and bright rings by transmitted light rapidly increases towards the margin of the bubble. This is due to the increased

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*C V Raman and V S Rajagopalan, *Proc. Indian Acad. Sci.*, 1939, **A10**, 317.

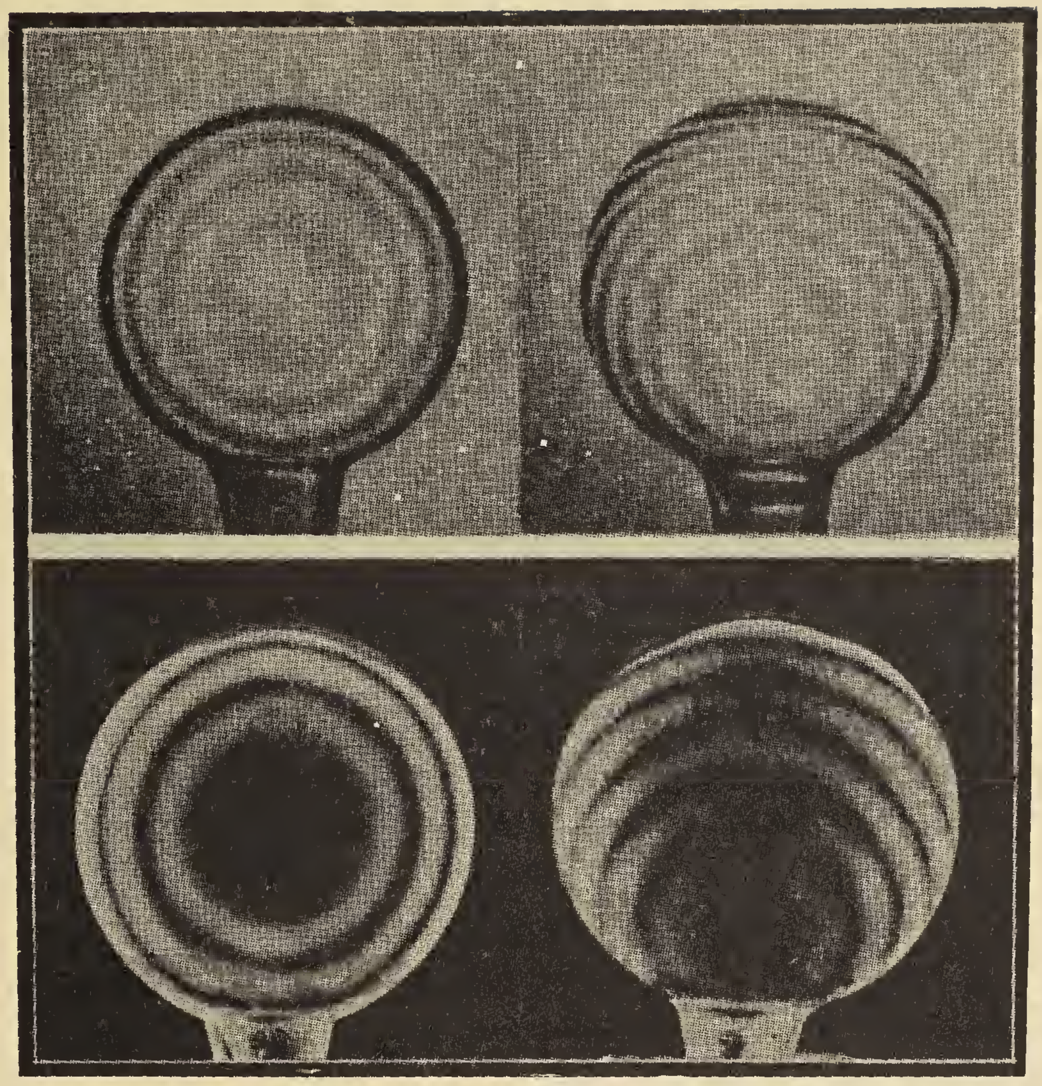

**Figure 3.** Soap bubbles in monochromatic light.

reflecting power at oblique incidences which makes the intensity of the interfering beams much more nearly equal. Very near the margin of the sphere, the dark rings as seen by reflected light are much sharper than the bright rings, while by transmitted light we have the opposite effect. This is due to the influence of multiple reflections within the film which tend to sharpen the interference bands, a principle which is utilised in the Fabry–Perot etalon and the Lummer–Gehrcke plate.

*Haidinger's rings*: The interference colours exhibited by parallel plates in white light naturally cease to be visible when the plate is not very thin. The interferences may, however, be observed with thick plates if we use monochromatic light. The case in which the plate is of uniform thickness is of particular interest, as the fluctuations of intensity would then be solely due to variations of the angle of incidence at which the plate is viewed. If an extended source of monochromatic light is seen by reflection at the surfaces of such a plate, the eye being adjusted for distant vision, a set of dark circles would be seen at infinity in the directions corresponding to the values of $\theta$ for which the formula $2\mu t \sin\theta = n\lambda$ is satisfied, while bright circles would be seen in intermediate directions. It is evident from the formula that the rings would be centred around the direction of the normal to the

plate. As also shown by the formula, they would appear widely separated in the vicinity of the normal, would crowd up in more oblique directions, and open out again in directions nearly parallel to the surface of the plate which correspond to nearly critical incidence of the light within the plate.

The theory of these rings was implicit in the explanation of the colours of thin plates given by Thomas Young in 1809. They were, however, first observed by Haidinger in 1849. It is obvious that to enable them to be seen perfectly in all circumstances, the thickness of the plate should be rigorously constant. In the earliest observation of the rings by Haidinger, this condition was realised by the use of a natural cleavage sheet of mica, the yellow flame of a lamp with salted wick being viewed by reflection at its faces.* The Haidinger rings are of great interest in optics, as they are utilised for the spectroscopic examination of light in the Fabry–Perot etalon and the Lummer–Gehrcke plate, and also furnish the theoretical background for the interferences observed in other instruments; e.g., the Michelson and the Jamin interferometers. In these applications, it is necessary that the plates used should be thick and uniform, and their preparation, therefore, requires a high degree of technical skill. So much emphasis is usually laid on this point that the impression naturally prevails that optically worked plane-parallel plates of glass are essential for the observation of the rings. This impression is, however, not justified. Actually, the rings can be seen in any ordinary plate of glass and it is not necessary that it should be uniform or even plane.† The possibility of observing the rings under these conditions depends upon limiting the area of the plate used to such extent as may be necessary. This may be accomplished using a diffusing screen of smooth board, white in front and blackened behind, with a small circular aperture at its centre. The screen is held very close to and behind the plate of glass with which the rings are to be observed, the white side facing the plate and illuminated by the light of a mercury lamp. The rings are then seen against a dark background by the observer's eye placed behind the aperture in the screen (figure 4). If the plate is both thick and non-uniform, the circular aperture in the screen may be replaced by a fine slit which can be turned round and set parallel to the contour lines of uniform thickness of the plate. A section of the ring-system is then seen in perfect definition along the length of the slit. Figure 5 shows such a system of interferences photographed with a plate of glass 3·5 millimetres thick and so non-uniform that distant objects exhibited by reflection in it two distinct images of varying separation.

If with the arrangements described above, the plate is moved away from the viewing screen, the rings gradually transform to the interferences of the Newtonian type due to the varying thickness of the glass plate. These are located at or near the surface of the plate, while the Haidinger pattern for a flat plate is located at infinity. It is also possible to observe the Haidinger's rings in a curved

*See also T K Chinmayanandam, *Proc. R. Soc. London*, 1918, **A95**, 176.
†C V Raman and V S Rajagopalan, *Philos. Mag.*, 1940, **29**, 508.

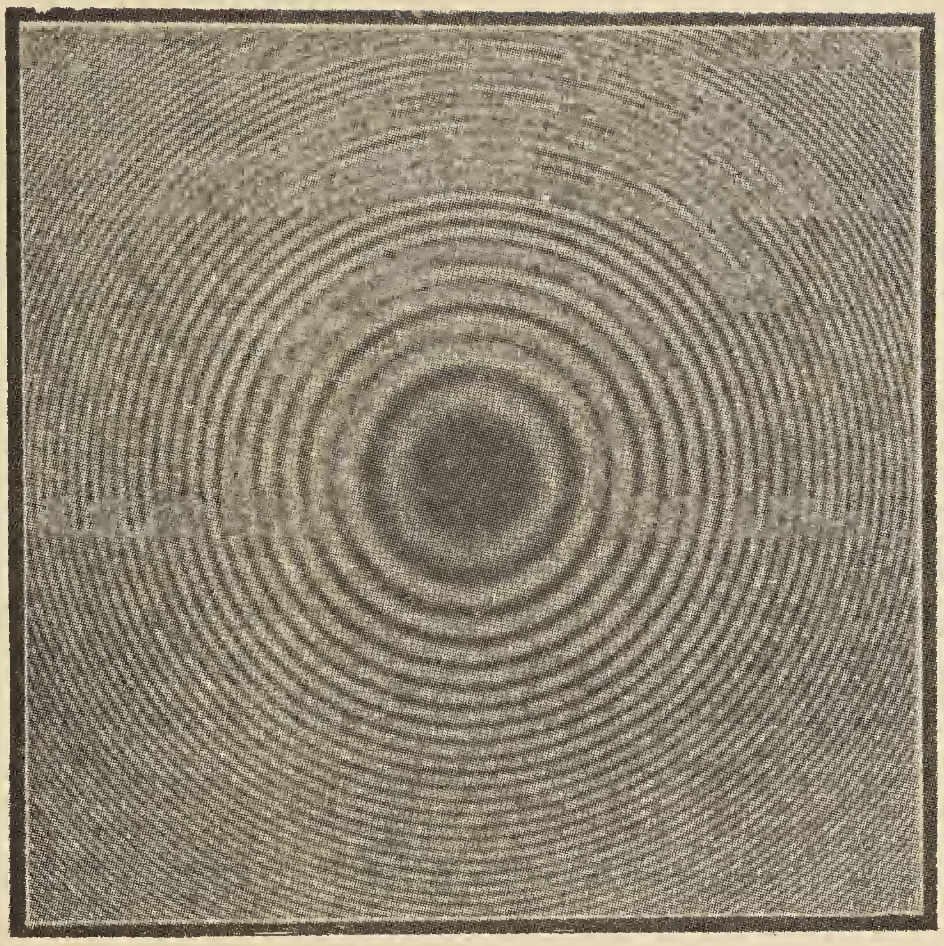

**Figure 4.** Haidinger's rings in a glass plate.

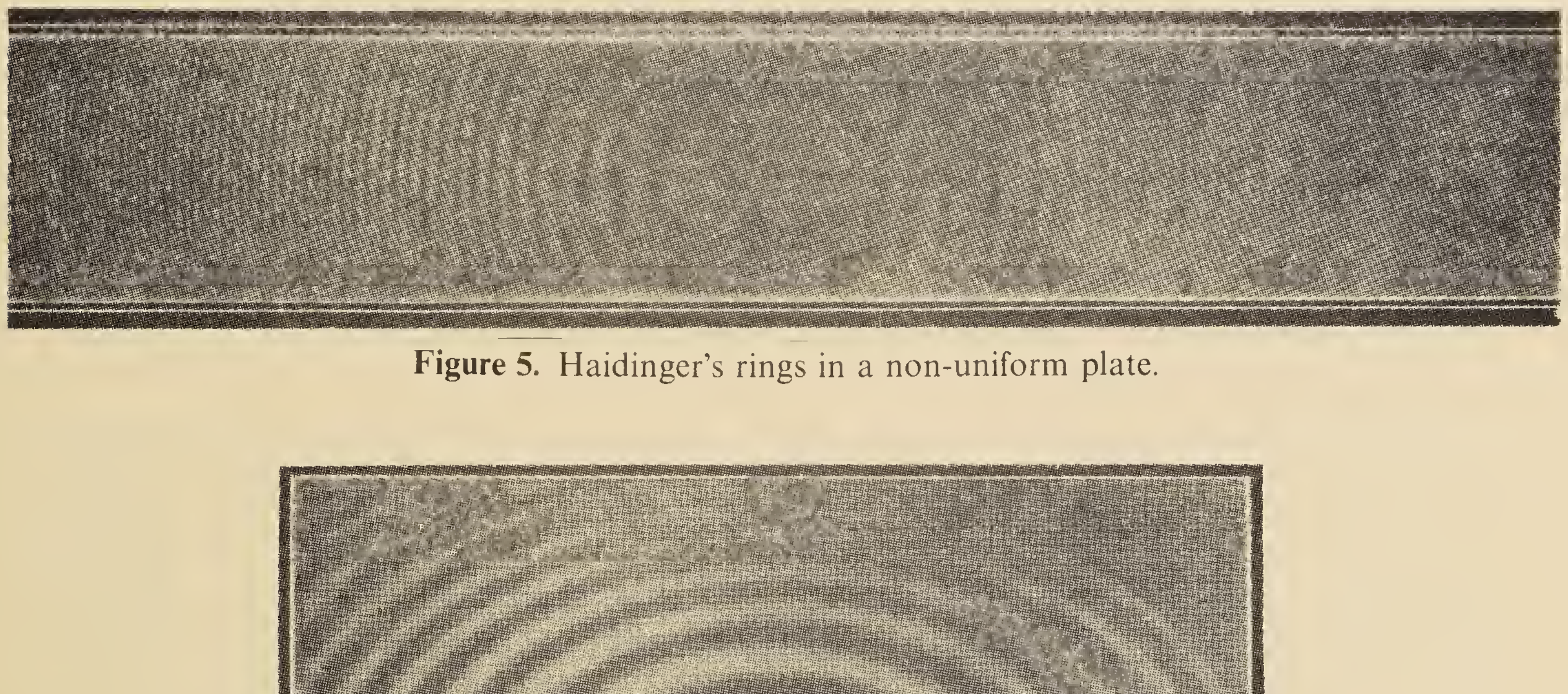

**Figure 5.** Haidinger's rings in a non-uniform plate.

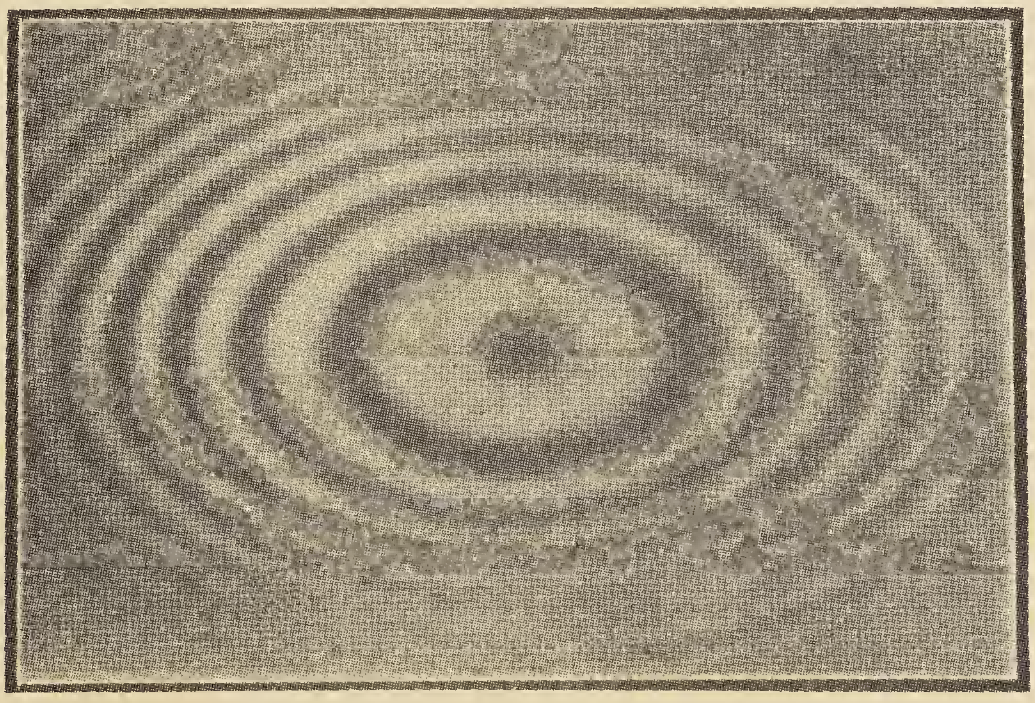

**Figure 6.** Haidinger's rings in a cylindrical plate.

plate of mica* with the viewing arrangement described above. The configuration of the rings then depends on the distance of the eye from the plate. If the plate be uniform in thickness, the pattern seen is determined solely by the variation of the obliquity with which the surface of the plate is viewed by the observer, and it is readily verified that it is equivalent to the normal Haidinger system modified in the same way as the image of a set of concentric circles would be, if seen by reflection at the curved surface of the plate (see figure 6).

*The Fabry–Perot etalon*: The Haidinger rings in a plate half-silvered on both sides and seen by transmitted light form the essential principle of the Fabry–Perot etalon which, as already mentioned, is a most useful and powerful appliance for the analysis of light. A very great improvement in the appearance of the fringes seen in transmission is effected by the half-silvering which makes the intensities of the interfering beams much more comparable, and as the result of multiple reflections, also largely increases their number. In practice, it is found more convenient to use a plate of air enclosed between two optically worked plane surfaces of glass; these surfaces are half-silvered and kept strictly parallel at a suitable distance apart by a separating ring of invar metal. The Fabry–Perot rings, as they are called, are observed when an extended source of monochromatic light is viewed in transmission through the plate. If the light used be highly monochromatic, they are seen as sharp bright circles on a dark background. As the result of the silvering and of the multiple reflections resulting therefrom, the transmitted light is much enfeebled and its intensity is negligible except in the precise directions for which all the emerging beams differ in path by the same integral number of wavelengths and can, therefore, totally reinforce each other. For a thick plate, these directions vary very rapidly with the wavelength, and the rings corresponding to closely spaced spectral components in the radiation are, therefore, clearly separated.

It is naturally desirable to use etalon plates of a fairly large area, as more illumination is thereby secured, which is a matter of great importance when working with very faint sources of light. The utility of the etalon is greatly enhanced when the separation of the plates can be varied to suit the problem under investigation. This is conveniently effected by having a selection of metal rings of different thicknesses, the aggregate of which, together with the plates themselves, makes up the total space within which the etalon is mounted. An invar ring of the thickness desired is placed between the plates, while other metal rings fill the gap outside. The etalon plates are adjusted to perfect parallelism by a delicate mechanism which exerts the minimum pressure necessary to tilt and hold them in position. It is convenient to place the etalon between the source of light and the slit of the spectrograph. An image of the interference pattern is focussed

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*C V Raman and V S Rajagopalan, *J. Opt. Soc. Am.*, 1939, **29**, 413.

carefully on the slit, and the adjustment of the etalon is made by trial till the best definition of the rings is obtained. The slit of the spectrograph is kept wide open so that a section of the ring pattern is recorded on the photographic plate, the different spectral lines, however, being separated by the dispersion of the spectrograph. Different etalon separations may be used to check the order of interference corresponding to any particular ring seen in the pattern. The absolute wavelength of the radiations can also be determined exactly from the positions of the rings, if the etalon separations are known.

*The Jamin interferometer*: This very useful instrument which has been extensively employed in refractometry is an application of the phenomenon known as Brewster's bands. Sir David Brewster observed coloured interference bands crossing the image of a source of white light seen by reflection successively at the surface of two plates of glass of equal thickness. The width of the fringes decreases with increasing inclination of the plates to each other.

The course of the interfering beams in the Jamin instrument is shown in figure 7. Part of the light incident on the first plate is reflected at its front surface, and then at the rear surface of the second plate; another part is reflected at the rear surface of the first plate and then at the front surface of the second. The paths of these two beams are equal, irrespective of the angle of incidence, provided the plates are of the same thickness and parallel to each other. If, however, the plates are inclined to each other, the paths are equal only when the incident beams make equal angles with the two plates; for other directions, the path difference would progressively increase. The appearance of fringes with an achromatic centre, and with a width diminishing with increasing inclination of the plates is, therefore, readily understood.

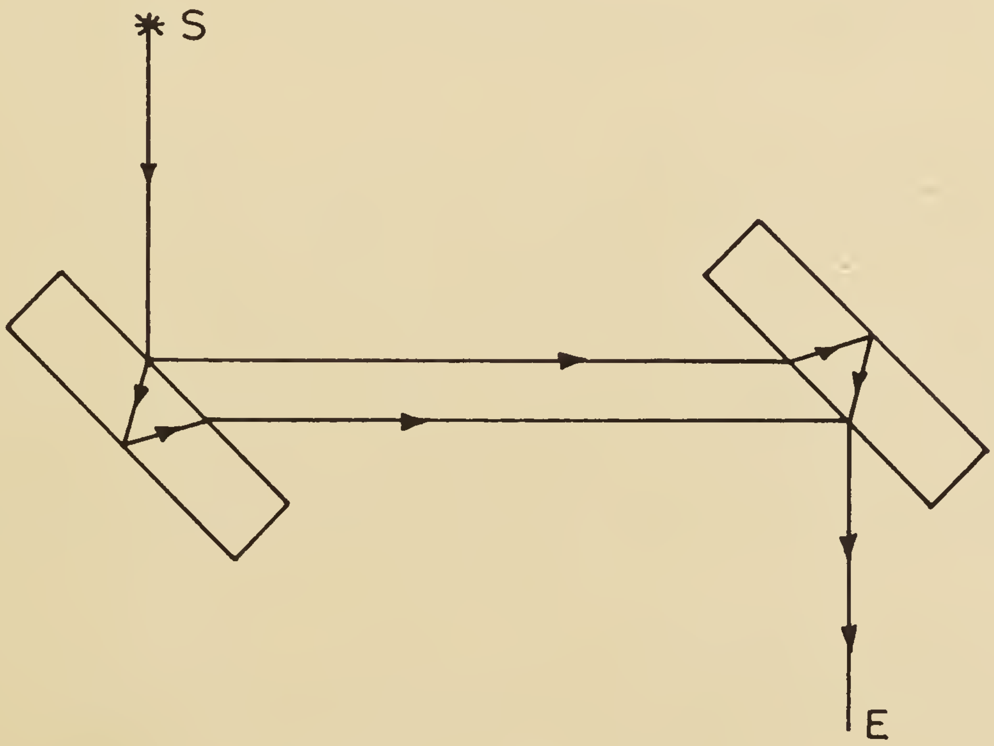


**Figure 7.** Principle of Brewster's bands.

Brewster's bands can, of course, be seen also with monochromatic light, and indeed the observations can be pushed much further with it. Ketteler, Lummer and others have studied the form of the interference figures over a wide range of incidences and of inclinations of the two plates relative to each other and also for plates of unequal thickness. The figures observed fall into two classes. These correspond to the cases in which the interference occurs under a path difference equal to the sum and the difference respectively of $2\mu_1 t_1 \sin\theta_1$ and $2\mu_2 t_2 \sin\theta_2$, these quantities being the relative retardation of the light beams reflected at the front and rear surfaces of each of the plates separately. It will be noticed that these quantities determine the Haidinger patterns of the two plates, and this suggests an alternative and very instructive way of regarding the theory of Brewster's bands.

It is evident from the diagram (figure 7) that the pattern seen by the eye placed at $E$ and viewing an extended source at $S$ is really the Haidinger system of reflected rings formed by the first plate and then again by the second, in other words, a multiplication of the intensities of the two systems.* The angular position of the dark rings in the two systems are given by the usual expressions

$$2\mu_1 t_1 \sin\theta_1 = n_1\lambda \quad \text{and} \quad 2\mu_2 t_2 \sin\theta_2 = n_2\lambda.$$

The effect of the multiplications of intensities is to give a series of superposition figures,† which may be classified as differentials and summationals of the first, second and higher orders, and the form of which may be derived graphically or analytically. As the Haidinger rings are widely separated at normal incidence, and after first closing up, open out again at more oblique incidence, the complete first order differential pattern consists of two sets of closed curves, the configurations of which relative to the Haidinger's rings are indicated in figure 8 for a particular inclination of two plates of equal thickness. It will be noticed that in the centre of the field, which corresponds to the symmetric direction, the fringes are straight. These are the Brewster bands observed in white light.

*Optical study of percussion figures*: We shall now consider as an example of interferences of the Newtonian type, a case arising in the study of the permanent deformation of plane surfaces by impact or static pressure. Hertz's well known theory of impact was found to be a correct description of the facts in the case of spheres impinging on each other, only if their surfaces are smooth‡ and highly polished and the velocity of impact is sufficiently small. Accordingly, for an experimental test of this theory as extended to the impact of spheres upon flat plates,§ it was decided to study the collision of polished hard steel balls on smooth

*A Schuster, *Philos. Mag.*, 1924, **48**, 609.
†C V Raman and S K Datta, *Trans. Opt. Soc.*, 1925, **27**, 52 and 1927, **28**, 214.
‡C V Raman, *Phys. Rev.*, 1918, **12**, 442.
§*ibid.*, 1920, **15**, 277.

**Figure 8.** Superposition of Haidinger patterns.

glass plates. It was then discovered that if the size of the balls or their velocity exceeded certain limits, the impact resulted in the production of percussion figures of beautiful geometric form in the glass plates.* A circular crack starts from the surface of the plate and spreads obliquely inwards in the form of a surface of revolution, revealing itself by the light which it reflects. The deformation of the external surface of the glass plate resulting from the collision is very conveniently exhibited by laying another flat glass plate on it, thus forming a wedge-shaped film of air between the two surfaces. The light of a mercury lamp passed by a green ray filter and incident nearly normally upon this film and reflected by it results in interferences which can be readily photographed, the camera being focussed on the percussion figure itself.†

It is evident that the method for the study of percussion figures illustrated in figure 9 can be extended to all solids which are capable of being polished to have a smooth reflecting surface. An inspection of the photograph reproduced shows three distinct regions in the figure. Firstly, there is a central area which is circular and is apparently unaffected by the impact, as is shown by the fringes passing through it being straight and parallel. Secondly, there is a narrow annular region of fracture full of a network of irregular fringes, showing severe injury to the surface. Thirdly and just beyond this, there is a sudden elevation of the surface which slopes down, first quickly and then more slowly, to the original level of the

*C V Raman, *Nature* (*London*), 1919, **104**, 113.
†C V Raman, *J. Opt. Soc. Am.*, 1926, **12**, 387.

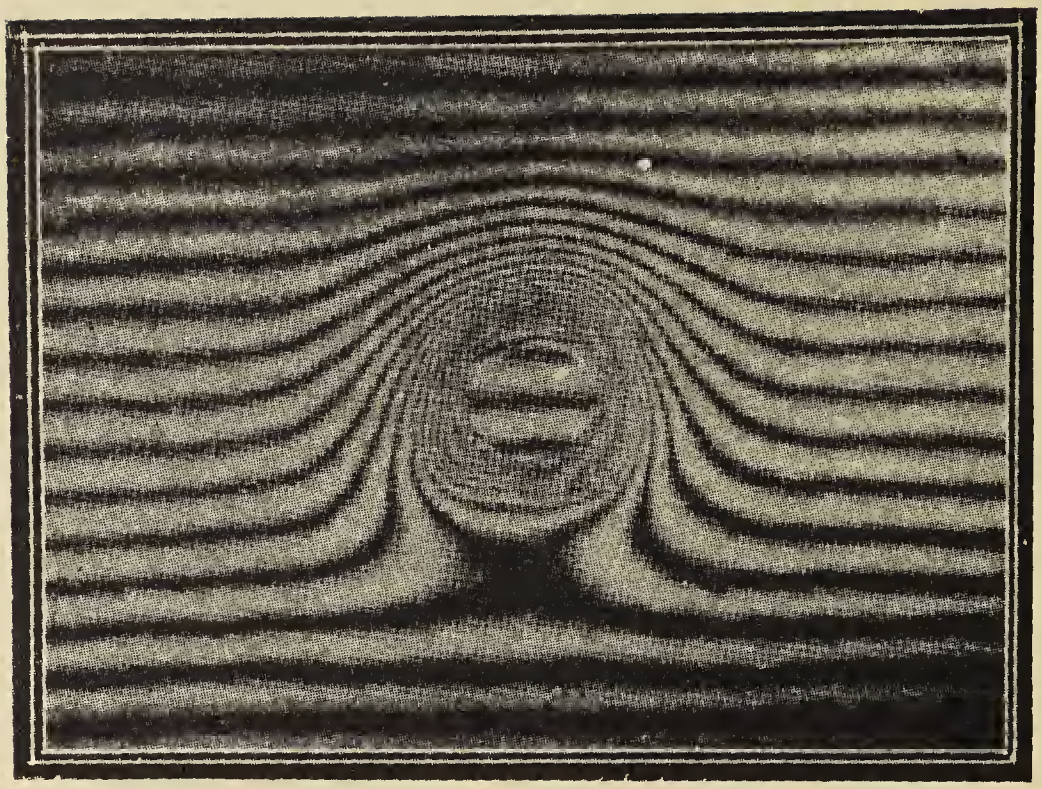

**Figure 9.** Percussion figure in a glass plate.

surface at the edge of an area which sets the limit to the percussion figure. Closer examination reveals another remarkable feature, namely, that the central area of the percussion figure, though it remains plane and apparently undisturbed, has, in reality, been *depressed below* the original level of the surface by an appreciable fraction of a wavelength, as shown by the fact that the course of the fringes outside the percussion area and within the central circle are distinctly out of register. This feature is observed in the percussion figures even with very thick plates of glass. Additonal details are given in a paper by K Banerji,* where further work is reported on with glass and metal plates. A brief note has also appeared on the percussion figures of crystalline rock-salt photographed by the same method.†

*Visibility of interferences in white light*: The colours exhibited by thin films illustrate the general principle that when the path-differences are sufficiently small, the effects of interference are perceptible to the eye even with white light. In all interference experiments, if the path-difference is zero and the maxima or minima of illumination for all wavelengths are therefore coincident at some point in the field, an achromatic fringe is there seen, appearing white or black as the case may be. Further out, the fringes are appreciated by the eye principally as alternations of colour, some six or seven of these being seen of gradually diminishing clearness. The explanation of these effects becomes clear when the interferences are examined with a spectroscope with the slit set parallel to the fringes, so that the interferences appear as dark and bright bands crossing the

*K Banerji, *Indian J. Phys.*, 1926, **1**, 59.
†S Smith, *Nature (London)*, 1931, **127**, 856.

spectrum. The number of such bands is small when the slit is near the achromatic fringe, but increases rapidly as the slit is moved away from it, till ultimately the bands are distributed over the spectrum with some approach to uniformity. The failure of the eye to appreciate any differences of colour or intensity in such circumstances is not surprising.

The number of orders of interference visible in white light may be greatly increased by causing the interfering beams to traverse approximately equal optical paths in two media of different dispersive powers. We may, for instance, using white light, interpose a glass plate in one of the arms of a Michelson interferometer and adjust the air path in the other arm to approximate equality. Several hundreds or even some thousands of interferences may then be seen and enumerated, the number depending on the thickness of the glass plate and its dispersive power, but the fringes are visible only as very small alternations of colour in the field.* The explanation of this effect is as follows: The addition of an optical path $D$ in glass of refractive index $\mu$ (relative to air) in one arm of the interferometer and of an air path $t$ in the other arm, changes the order of the interference at any point in the field by the number $(\mu D - t)/\lambda$, $\lambda$ being the wavelength in air of a particular region in the spectrum. The position of the interferences in the field is therefore stationary for small changes of $\lambda$, if the variation of this number is zero, that is if

$$D(\mu - \lambda \mathrm{d}\mu/\mathrm{d}\lambda) = t. \tag{1}$$

Equation (1) is equivalent to the statement that the retardation of a *wave group* produced by the extra path $D$ in glass is exactly balanced by the additional air-path $t$. The corresponding change in the order of interference, in other words, the shift of the "achromatic" band measured by the number of fringes is

$$D \cdot \mathrm{d}\mu/\mathrm{d}\lambda. \tag{2}$$

If the glass plate be thick or if its dispersive power be great, this shift may be quite large. But since $\mathrm{d}\mu/\mathrm{d}\lambda$ alters with the wavelength, neither the thickness $t$ of the compensating air-path, nor the order of interference $D \cdot \mathrm{d}\mu/\mathrm{d}\lambda$ for which equation (1) is satisfied, is even approximately independent of $\lambda$. The interferences are therefore "achromatic" only for a limited region of the spectrum, and the position of the "achromatic" band in the field shifts with the part of the spectrum under consideration. In other words, the achromatic band is "dispersed" by the introduction of the glass plate and the number of interferences visible in white light is thereby increased enormously, but only at the expense of diminishing the visibility of the individual fringes almost to the limit. The extraordinary sensitivity of the eye to small differences in colour in adjacent areas, however, enables such fringes to be observed and enumerated.

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*N K Sethi, *Phys. Rev.*, 1924, **23**, 69.

Sethi* to whom the foregoing explanation of the facts observed by himself is due, has shown that it covers also the increase in the number of interferences visible with white light in a plate of non-uniform thickness produced by viewing them through a dispersing prism—an observation originally made by Newton. Its correctness is proved by spectroscopic examination of the interference fringes at various parts of the field. It is then noticed that the interference bands are widely separated in a particular part of the spectrum and crowd together on either side of it. The region of the spectrum at which this effect is observed is found to vary with the part of the interference field under observation. Why the interferences are perceptible to the eye in spite of the great number of bands crossing the spectrum is thereby made intelligible.

*Interferences in polarised light*: The light reflected at both the surfaces of a transparent plate bounded by the same medium is completely polarised at a particular angle of incidence on the external surface, and if viewed through a polariser set so as to transmit only vibrations in the plane of incidence, is completely quenched. It follows that the plate would, at this incidence, exhibit no interferences by transmitted light for the vibration parallel to the plane of incidence. When the light falls more obliquely, the interferences reappear in the parallel component, but would continue to be weaker than those in the perpendicular component until grazing incidence is reached. Holding up a plate of glass obliquely against a mercury lamp and viewing the interferences through a polariser, it is readily verified that in the fringes seen by *reflection, the maxima* are broader and more intense, while the minima are narrower and more sharply defined for the perpendicular than for the parallel component of vibration. In the fringes seen by *transmission* we have the complementary effect, the minima being broader and darker and the maxima narrower and more sharply defined in the same circumstances.

The cases where the plate is bounded by two different media present some special features of interest. Such a situation arises when a film has one free surface and a solid or liquid backing. The phenomena are then influenced by the optical properties of both the film and the backing material, as also by the nature of the transition between them. The reflections at the front and back surfaces may be of very unequal strength, and this would naturally affect the liveliness of the interferences. The incidence (if any) at which the light is polarised by reflection and beyond which a phase-reversal occurs for the parallel component of vibration would also, in general, not be the same for the two surfaces. Hence the circumstances in which interference occurs would, in general, differ widely for the two components of vibration. If such a plate be viewed through a polariser held in front of the eye and the latter is rotated, striking changes in the colour and

*N K Sethi, *Proc. Indian Assoc. Cultiv. Sci.*, 1921, 7, 37.

intensity of the reflected light may be noticed depending on the obliquity of incidence. As a noteworthy example of this kind, we may mention the oxidation colours exhibited by a polished plate of copper which has been heated up in contact with air. The colours as seen in the parallel component are, in general, more vivid than in the perpendicular component; the two are also observed to be of a complementary character at sufficiently oblique incidences.

*Soap bubbles between crossed nicols*: Some beautiful and interesting phenomena are noticed* when a soap-bubble is placed between two crossed nicols or polaroids and viewed by transmission against a bright source of light. When only one of the nicols is present, the usual colours by transmitted light are noticed, except that they are slightly more vivid towards the end of the one diameter of the sphere and slightly less vivid at the ends of the perpendicular diameter. With both nicols present, a black cross appears on the surface of the bubble with its arms parallel to the vibration planes of the two nicols. Elsewhere, the surface of the bubble exhibits striking colours which recall those seen by reflected light in their vividness and are, indeed, complementary to the usual transmission tints. With a monochromatic light source, the interferences are as striking as those ordinarily seen by reflected light. Near the margins of the bubble, the minima are sharp and much finer than the maxima, thus reversing the effects usually observed in transmitted light. As the bubble thins down, the interferences successively disappear so that in the penultimate stage its entire surface is bright except for the dark cross. When, finally, the soap bubble goes "black", it retains a faint luminosity, while its spherical margin shines brightly as a crescent of light interrupted by the intersections with the black cross. These effects are illustrated in the series of six photographs reproduced in figure 10. These are arranged in order of increasing thickness of the soap film, the first being that of a bubble which has gone black near the top.†

When the nicols are set with their vibration planes not exactly at right angles, the black cross breaks up into two curved arcs or isogyres. These shorten and approach the margins of the sphere rapidly as one of the nicols is further turned round. With a thick film, the isogyres are themselves the most vividly coloured parts of the bubble. With thinner films it is noticed that when the nicol is turned so that the isogyre moves across an area of the bubble, the colour of the same alters to the complementary tint. With monochromatic light, the isogyres show notable alternations of intensity and appear distorted where they cut the interference curves, while the latter exhibit dislocations at these points which may amount to as much as half a fringe. A "black spot" on the bubble usually appears as a dark

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*Unpublished observations by the author.

†The so-called "liquid soap" which is commercially available can very conveniently be used for such experiments. Stable and uniform bubbles are obtained with a highly diluted solution of the same. This should be freshly prepared.

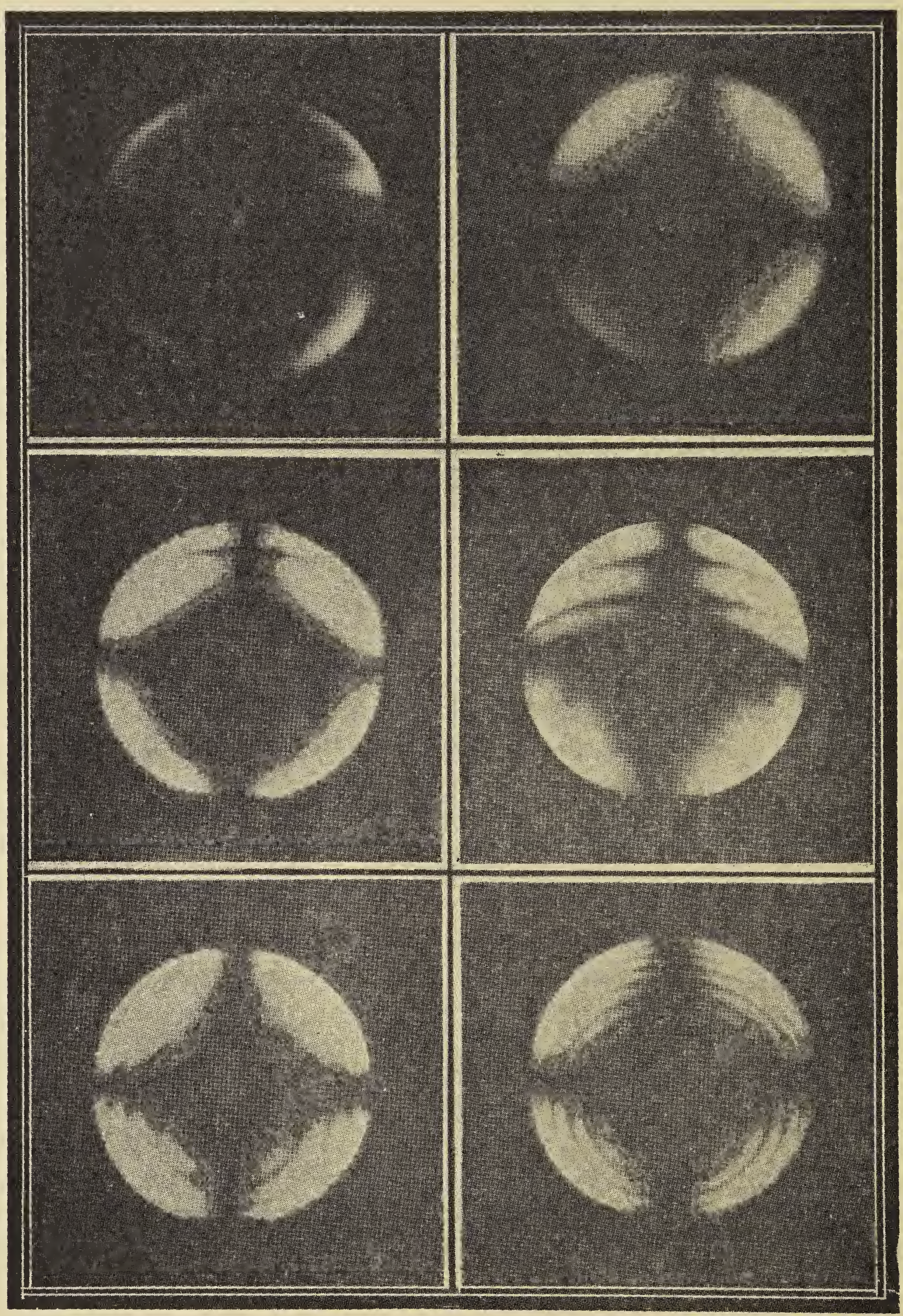

**Figure 10.** Soap bubbles between crossed nicols.

area on a bright background. But when it passes over one of the isogyres, its optical character reverses and it is then seen as a *bright spot on a dark background.*

The explanation of these effects has been discussed in a paper by K S Krishnan* on the assumption that the films are optically isotropic.† When plane-polarised light is incident on the bubble, the coefficients of transmission and internal reflection are different for the components of the vibration parallel and perpendicular to the plane of incidence. Hence, except when one or other of these components vanishes, the plane of polarisation is rotated by transmission or internal reflection at the surfaces; the light vector turns away from the plane of incidence in transmission and turns towards that plane further at each successive reflection. These rotations are, of course, superposed when a beam undergoes transmissions and reflections successively. In general, therefore, the second polariser fails to extinguish the transmitted and reflected beams. Their components emerging from it accordingly interfere and give the observed luminosity. The smallness of the rotation of the light vector in the transmitted beam taken together with its much greater magnitude and its opposite sense for the light vector in the reflected beam makes the relative amplitudes of these beams after passage through the second polariser comparable, and also involves a reversal of their relative phases. The vividness of the interferences and their similarity to those ordinarily exhibited by the bubble in reflected light are thus readily understood. At nearly grazing incidences, the transmission coefficients become small and the reflection coefficients increase considerably, and the resultant rotations of the light vectors for the transmitted and multiple reflected beams are also no longer in opposite directions. Hence, the considerations stated above require some modification at nearly grazing incidences. Taking these circumstances into account, Krishnan has given an explanation of the luminous crescent exhibited by the "black" bubbles.

*Haidinger's rings in crystals*: The natural cleavages of various crystals, e.g., mica or gypsum, enable us readily to obtain transparent plates with good optical surfaces which are suitable for the study of the Haidinger interferences. The technique of observation described in an earlier page—namely, that of holding a smooth white illuminated screen containing a viewing aperture very close to the specimen—is particularly well suited for use with crystals. For, with the close approach of the observer's eye to the plate made possible in this way, an extended area of surface coupled with perfect uniformity of thickness becomes unnecessary, and it is possible to see and photograph a large number of rings satisfactorily even with small and not quite perfect specimens *by reflected light.* A photograph of Haidinger's rings obtained in this way with a sheet of mica and the 4358 A.U. radiations of the mercury arc is reproduced as figure 11. The rings as seen by

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*K S Krishnan, *Indian J. Phys.*, 1929, 4, 385.

†It may be remarked that stratified films should, theoretically, be birefringent.

*normal transmission* are, of course, weak. But they may be greatly improved in this respect by half-silvering the surfaces of the plate, in which case the bright rings also become much sharper. For viewing the rings as formed by *reflection*, however, such silvering is unnecessary. An alternative way of observing the interferences of crystalline plates is to hold the specimen obliquely against a monochromatic source and to view the light reflected by or transmitted through it. In this case, of course, only a small part of the interference field is seen at a given time. But we have the advantage that the fringes are then widely separated and the minima (or maxima as the case may be) are also much sharper than in the rings formed by reflection or transmission at normal incidence.

On an examination of figure 11, it will be noticed that the rings do not appear with the same clearness everywhere, their visibility being a minimum along four arcs of roughly hyperbolic form. The interferences exhibit dislocations where they cut these arcs, the dark rings on one side running into the bright rings on the other and *vice versa*, while a distinct doubling of the interferences can be noticed where they run nearly parallel to the arcs of minimum visibility. The form of these arcs of minimum bears an obvious resemblance to the isochromatic curves in the birefringence colours exhibited by a sheet of mica in the polariscope. Indeed, the resemblance is seen to be perfect if the isochromatic curves are viewed with a plate *twice as thick* as that used for the observation of the Haidinger's rings and the observations include a sufficiently wide range of angles. The curves of minimum visibility have then exactly the same shape as the interference figures in polarised light.*

From the facts stated, it is evident that we have, in fact, two sets of Haidinger's rings which overlap and which are seen most clearly where they are in coincidence, and least clearly where the bright rings of one system falls on the dark rings of the other and *vice versa.* That these two sets correspond to light polarised with its vibrations respectively along two perpendicular directions can be directly verified by looking at the ring system through a nicol; as this is rotated, the lines of minimum visibility disappear absolutely in four positions of the nicol at right angles to each other. The two systems therefore also correspond to the two different velocities of propagation of light through the crystal, and the directions in space in which they appear are hence given by the two formulae

$$\delta_1 = 2dv_0/v_1 \cdot \cos r_1, \quad \delta_2 = 2dv_0/v_2 \cdot \cos r_2$$

where $v_0$ is the wave-velocity in air, and $v_1$, $v_2$ are the wave-velocities in the crystal, while $r_1$ and $r_2$ are the corresponding angles of refraction of the light into the plate. The lines of minimum visibility of the interferences accordingly correspond to

$$\delta_1 - \delta_2 = 2d \cdot v_0(\cos r_1/v_1 - \cos r_2/v_2) = (2p + 1)\lambda/2.$$

*T K Chinmayanandam, *Proc. R. Soc. London*, 1918, **A95**, 176.

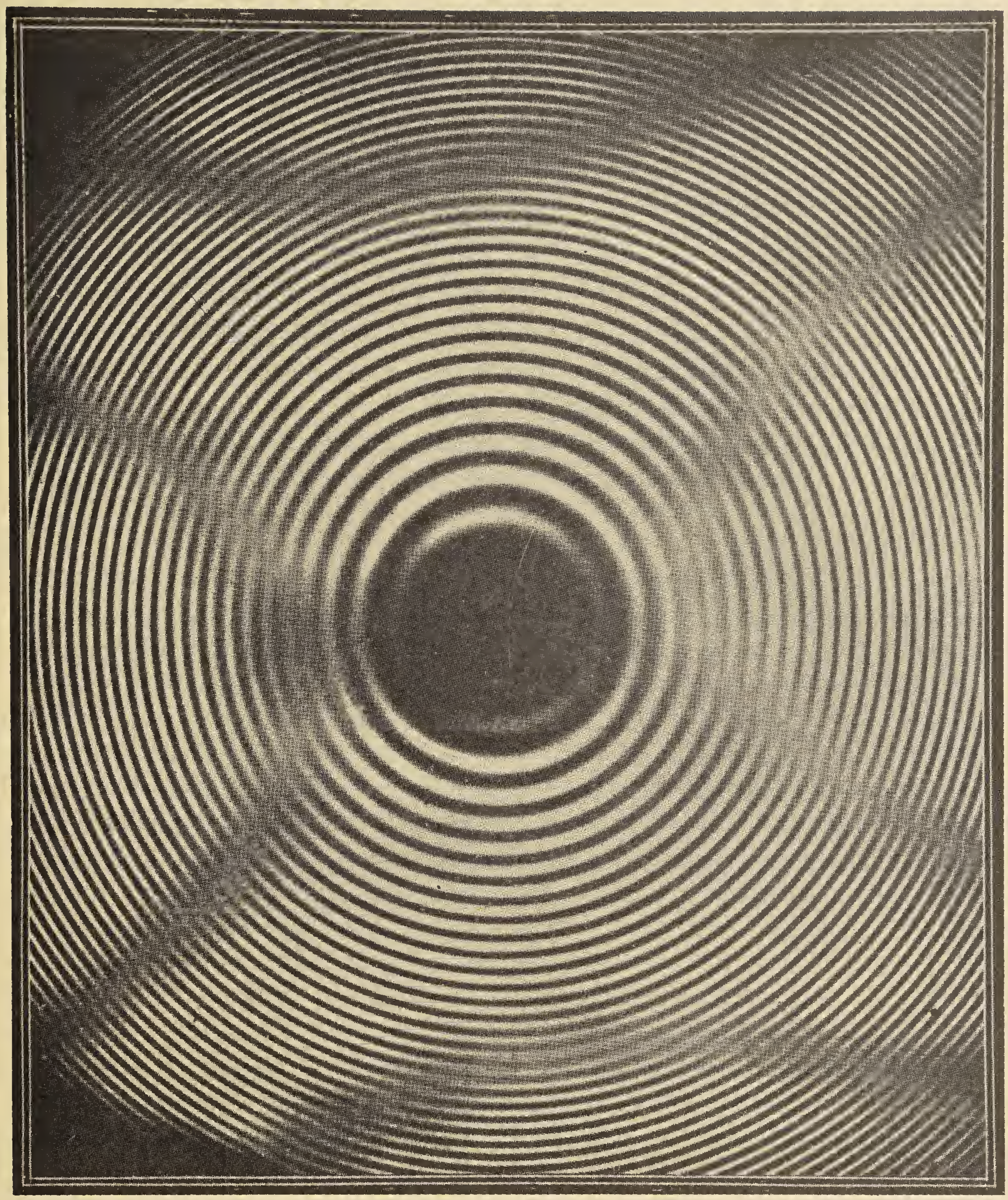

**Figure 11.** Haidinger's rings in mica.

This is also the well known formula giving the form of the isochromatic lines in convergent polarised light for a plate of thickness $2d$.

Since the wave-velocities $v_1$ and $v_2$ in a biaxial crystal depend on the direction of propagation, it is apparent that the rings cannot have a circular form as in the case of an isotropic plate. Their configurations can however be deduced from the

expressions given above and the known form of the wave-velocity surface in the crystal. They are, in general, curves of the fourth degree whose form depends on the crystal and on the direction in which the plate is cut. In the particular case when the plate is normal to the plane containing the binormals, and provided the angular separation of the latter is sufficiently great (as in the case of muscovite mica), the form of the curves in the vicinity of the normal to the plate can be shown to be approximately ellipses, their equations being

$$a^2y^2 + b^2x^2 = \text{constant}, \quad \text{and} \quad b^2x^2 + c^2y^2 = \text{constant},$$

where $a$, $b$, $c$ are the three principal wave-velocities in the crystal. It is readily shown that the overlapping of two sets of rings of this form would give a set of hyperbolae as the lines of minimum visibility.

The case of uniaxial crystals is also of interest. We may consider three typical examples, viz., a plate normal to the optic axis, a plate parallel to it and a plate cut at an angle of 45°. The configuration of the "isochromatic" curves in these cases is well known. They are in the first case, a set of widely spaced circles, in the second case a family of rectangular hyperbolae which are wide apart near the centre of the field and crowd together towards the margin, and in the third case a series of curved arcs running parallel and close to each other throughout the field. It follows that when unpolarised light is used, the Haidinger rings would be best seen with the first plate, less clearly with the second and should be scarcely visible in the third. Observations were made* with three plates of quartz five millimetres thick (figured by the firm of Hilger) and having the orientation stated, the rings being observed both by reflection and by transmission, in the latter case after half-silvering the plates. Polarisation of the incident light made a great improvement in the visibility of the rings as seen by reflection with the second plate; when it was rotated in its own plane, the rings were clearly seen in four positions and were very confused in four intermediate positions. The positions of the rings were also different for settings of the crystal at right angles to each other. The bifurcation of the rings could be clearly observed in the interference pattern as seen by transmitted light with the half-silvered plates, towards the margin of the field with the first plate and even at the centre with the second. It was evident from these observations that while both sets of Haidinger rings in the first plate were circular, in the second plate one set was circular and the other was elliptic.

*Colours of stratified media*: Several examples are known of substances exhibiting interesting optical effects ascribable to their possession of a periodic laminated structure. The optical behaviour of such substances includes a variety of phenomena, some of which lie outside the scope of the present lecture. We shall here consider only such of them as come under the category of the interferences of thin plates. Their essential features may be understood by considering the case of

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*P N Ghosh, *Proc. Indian Assoc. Cultiv. Sci.*, 1921, 7, 57.

a medium made up of a succession of alternate strata of two different substances having a thickness $d_1$ and $d_2$ and refractive index $\mu_1$ and $\mu_2$ respectively. A beam of parallel light enters and tranverses such a medium, its angles of refraction into the alternate strata being $r_1$ and $r_2$ respectively. The reflections to which it gives rise at the boundaries of each layer would extinguish each other by interference

$$\text{if} \quad 2\mu_1 d_1 \cos r_1 = n_1\lambda$$

$$\text{or if} \quad 2\mu_2 d_2 \cos r_2 = n_2\lambda$$

as the case may be. On the other hand, if the condition

$$2\mu_1 d_1 \cos r_1 + 2\mu_2 d_2 \cos r_2 = n\lambda$$

is satisfied, the reflections at all the boundaries form two sequences in each of which there is complete agreement of phase. Accordingly, if this condition be satisfied, the advance of the incident wave through the medium results in the successive reflections reinforcing each other. We have then a strong wave reflected *backwards* which ultimately becomes as strong as the primary wave itself. *Per contra*, when the wave thus reflected travels backwards, it meets the successive boundaries in the reverse order, and gives rise to a second series of reflections which join up and build a strong wave travelling *forward*. This being in the same direction as the incident radiation, they interfere with the result that the amplitude of the primary wave progressively diminishes until it is finally extinguished. The net result is thus a *total reflection* in the backward direction of the incident light when it has traversed a sufficient depth of the medium.

We may ask at this stage, how many laminae are required to give a sensibly perfect reflection? The answer depends on the strength of the resultant reflection from a single pair of strata in the medium. The greater the strength of the individual reflections, the more quickly would they add up so as to approach totality. It follows that the number of pairs of strata necessary to secure this result would be of the same order of magnitude as the reciprocal of the amplitude of the reflection from a single pair of strata. The incident wave would then penetrate into the medium to a depth not much greater than this number of strata. Accordingly, the rest of the medium is superfluous and may be removed without the totality of the reflection of this particular wavelength being sensibly affected.

The nature of the result to be expected if the relation

$$2\mu_1 d_1 \cos r_1 + 2\mu_2 d_2 \cos r_2 = n\lambda$$

is not exactly satisfied, clearly depends on the extent to which $\lambda$ diverges from the value given by the relation. It is *prima facie* evident that the reflection would be sensibly total also for values of $\lambda$ which differ from it, provided that the resulting disagreement of phase between the reflections from the first and last layers which sensibly contribute to its intensity is a sufficiently small fraction of the wavelength, say one-fourth or less. Accordingly, we infer that *the reflection would*

*be sensibly total over a finite range of wavelengths which is proportional to the reflecting power of a single pair of strata and is independent of the total number of strata present*, if this is sufficiently great. Outside the selected range of wavelengths, the intensity of reflection must fall off with extreme rapidity. For, any failure of totality of reflection would involve a greater depth of penetration and thus introduce more reflections which would conspire to extinguish the effects of the earlier layers by interfering with them.

It is evident from the foregoing discussion that when white light is incident on a regularly stratified medium, we obtain a reflection which is more or less perfectly monochromatic, the range of wavelengths included diminishing with the reflecting power of the individual laminations, a sufficiently large number of these being assumed to be present. The reflecting power of an individual pair of strata would be small and the reflection would therefore be highly monochromatic if the alternate strata are nearly equal refractive index or if one of them is of vanishingly small thickness. The number of laminations present influences the intensity and spectral character of the principal band of reflection only when the individual layers reflect so feebly that the radiation of the selected wavelengths penetrates the entire depth of the medium.

It is evident also that weaker subsidiary maxima of reflection would accompany the principal band of reflection in the spectrum. For, the wavelengths outside the range of total reflection would penetrate freely into the medium and the successive reflections would be all of comparable amplitude and of progressively altering phase. Hence, their resultant would not generally vanish but would remain finite, exhibiting oscillations of intensity which progressively diminish as we may move away from the principal band of reflection. The intensity and spread in the spectrum of these subsidiary maxima of reflection would be determined by the number of laminations present. They would be most noticeable when this number is small.

It should be remarked also that the *distribution of intensity of the subsidiary maxima of reflection may be unsymmetric with respect to the principal band of reflection.* For, this distribution would depend on the strength of the reflection by a single pair of strata, and the principal band may well fall in a region where the strength of such reflection alters rapidly with the wavelength. If, for instance, one of the layers is of very small thickness in comparison with the other, the principal *maximum* of reflection would fall in a region of the spectrum in the near vicinity of the *minimum* of intensity in the reflection by a single pair of strata. In such a case, it is evident that the subsidiary maxima would be weak on one side and strong on the other side of the principal maximum.

It is evident also that, in general, several orders of monochromatic reflection would be possible in the spectrum, depending upon the thickness of the stratifications and their refractive index. The relative intensities with which these orders appear would depend on the ratio of the thickness of the alternate strata and for particular values of this ratio, some of the orders of reflection would be

missing. Why this should happen becomes clear if we express the periodic variation of refractive index along the normal to the stratifications as a Fourier series of harmonic components. Provided that the variations of refractive index are sufficiently small, each Fourier component thus derived can give rise only to one order of reflection which is observable when the wavelength of the incident light and the angle of its incidence satisfy the relation $2\mu\Delta \cos r = \lambda$, $\Delta$ being the spacing of the Fourier component. The absence of a particular Fourier component would thus involve the non-appearance of the corresponding order of reflection. This way of approaching the subject is of value as it enables us more generally to appreciate how the character of the stratifications determines the spectral distribution of intensity amongst the various orders of reflection. It is evident, in particular, why stratifications with a discontinuous distribution of refractive index favour the appearance of a large number of orders of reflection. It should be remarked, however, that the non-appearance of particular orders of reflection with certain types of discontinuous variation of refractive index can also be explained without the aid of the Fourier analysis, by a direct calculation of intensities.

The mathematical theory of the phenomena described above in general terms has been developed in a simple and elegant manner by G N Ramachandran,* and we proceed to give a sketch of it. As already indicated, we have, within the medium, two streams of energy, one travelling forwards and the other backwards. Each layer contributes to the backward stream of energy by reflecting part of the forward stream, and *vice versa*, while it reduces the strengths of these streams directly by transmission through it. Accordingly, if we denote the reflection and transmission coefficients of each layer by the complex numbers $r$ and $t$ respectively, then we have

$$R_s = rT_s + tR_{s+1}$$

$$T_s = rR_s + tT_{s-1}$$

where $R_s$ and $T_s$ represent the amplitudes of the backward and forward streams of energy at a point midway between the $(s-1)$th and the $s$th layers. Combining successive equations of the above type, we obtain

$$T_{s-1} = yT_s - T_{s+1} \quad \text{and} \quad R_{s-1} = yR_s - R_{s+1},$$

where $y$ stands for $(1 - r^2 + t^2)/t$. Assuming that there are only $n$ laminations we note that $R_{n+1} = 0$, from which it is readily deduced that

$$R_1 = f_n(y)R_n$$

$$T_1 = \left[\frac{1}{t}f_n(y) - f_{n-1}(y)\right]T_{n+1}$$

*G N Ramachandran, *Proc. Indian Acad. Sci.*, 1942, **16**, 336.

and

$$R_1 = \frac{r}{t} f_n(y) T_{n+1}$$

where $f_n(y)$ is the finite series $y^{n-1} - [(n-2)/\lfloor 1 \rfloor] y^{n-3} + [(n-4)(n-3)/\lfloor 2 \rfloor] y^{n-5} - \ldots$ to $(n-1)/2$ terms if $n$ is odd, and to $(n-2)/2$ terms if $n$ is even. Putting $y = 2\cosh\beta$, and $r/\sinh\beta = \sinh\alpha$, the series $f_n(y)$ sums up to the expression $\sinh n\beta/\sinh\beta$, and we get

$$\frac{R_1}{\sinh n\beta} = \frac{T_{n+1}}{\sinh\alpha} = \frac{T_1}{\sinh(\alpha + n\beta)}$$

where

$$\sinh^2\alpha = \frac{\{(r+t)^2 - 1\}\{(r-t)^2 - 1\}}{4r^2}$$

and

$$\sinh^2\beta = \frac{\{(r+t)^2 - 1\}\{(r-t)^2 - 1\}}{4t^2}.$$

These expressions are quite general, and involve no assumptions as to the nature of the reflecting layers. If now, we take the stratifications to be of the type considered in the preceding discussion, then by a simple application of the electromagnetic theory of light, it can be shown that $r$ and $t$ have the values

$$r = \frac{-\eta(\exp(ik\delta_1) - 1)}{(\exp(ik\delta_1 - \eta^2)\exp(\frac{1}{2}ik\delta_2)}; \quad t = \frac{(1-\eta^2)\exp(\frac{1}{2}ik\delta_1)}{(\exp(ik\delta_1) - \eta^2)\exp(\frac{1}{2}ik\delta_2)}$$

where $\eta$ is the reflecting power of the boundaries of separation between the two media, $\delta_1 = 2\mu_1 d_1 \cos r_1, \delta_2 = 2\mu_2 d_2 \cos r_2$, and $k = 2\pi/\lambda$. From these, putting $\frac{1}{4}k(\delta_1 + \delta_2) = \phi$, and $(\delta_1 - \delta_2)/(\delta_1 + \delta_2) = c$, we get

$$\sinh^2\alpha = (\cos^2\phi - \eta^2\cos^2 c\phi)(\sin^2\phi - \eta^2\sin^2 c\phi)/\eta^2\sin^2(1+c)\phi.$$

$$\sinh^2\beta = -4(\cos^2\phi - \eta^2\cos^2 c\phi)(\sin^2\phi - \eta^2\sin^2 c\phi)/(1-\eta^2)^2.$$

The expressions for the reflection and transmission intensities can be discussed using the foregoing values of $\sinh\alpha$ and $\sinh\beta$, and the results so obtained confirm those indicated by the general discussion. It is found that the spectrum of the light reflected at any angle can be divided into two regions, viz., those of the primary and the secondary maxima. These correspond to the regions where $\sinh\alpha$ is imaginary and $\sinh\beta$ is real, and *vice-versa*, respectively. The width of the former region, which determines the sharpness of the reflection, decreases as the number of laminations $n$ is increased, but soon reaches a limiting value. The secondary maxima, however, steadily increase in number as $n$ is increased.

To illustrate these points, the spectral distribution of intensity in the neighbourhood of the first order principal maximum has been drawn in figure 12 for 2, 10, 20, 50 and 100 plates. The reflecting power $\eta$ is supposed to be 0·1, and the value of

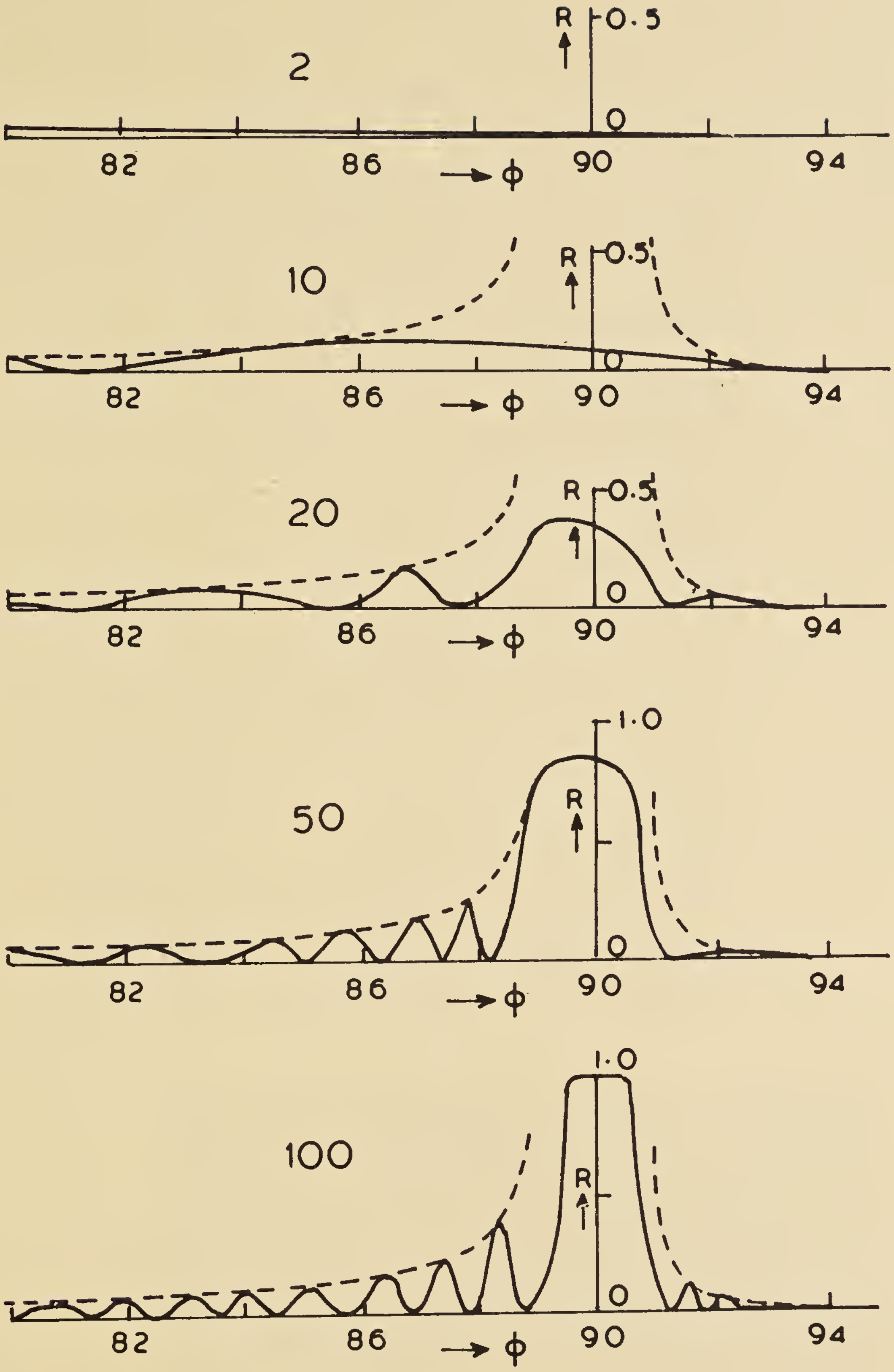


**Figure 12.** Spectral character of reflection by a regularly stratified medium.

$c = 0{\cdot}9$. The dotted line marks the curve on which the secondary maxima lie, and it shows how the intensity of these is widely different on either side of the principal maximum.

*Iridescence of potassium chlorate crystals*: Potassium chlorate crystallises in the prismatic class of the monoclinic system, commonly taking the form of flat plates whose faces are parallel to two of the crystallographic axes and are inclined to the third. Twins showing the characteristic re-entrant angles at the edges are not infrequent. The precise circumstances which result in the formation of crystals

exhibiting vivid colours are rather obscure. The seat of colour is usually a thin layer within the crystal parallel to its external faces and its power to reflect light appears to be the consequence of a repeated twinning within this layer. It is noteworthy that the coloured reflection vanishes when the plane of incidence of the light coincides with a plane of symmetry of the twinned crystal, while it is of maximum intensity when these two planes are perpendicular. Thus, when a crystal plate is held so as to reflect light obliquely and is turned round in its own plane, the colours alternately appear and disappear twice in each complete rotation. The intensity and colour of iridescence show wide variations. There are some crystals in which the reflections are so weak that they can only be observed with the flake mounted in Canada balsam between two glass prisms so as to eliminate all disturbing reflections. On the other hand, there are cases in which the coloured reflection is so intense that the crystal shines with an almost metallic lustre.

The spectral character of the reflected light is also very variable. In some cases, bands of varying widths are observed covering the whole of the visible spectrum. This is the case when the reflections are feeble and could, therefore, be ascribed to a few twinning planes only being present, possibly at considerable distances from each other and irregularly spaced (see figure 13). The character of the reflections

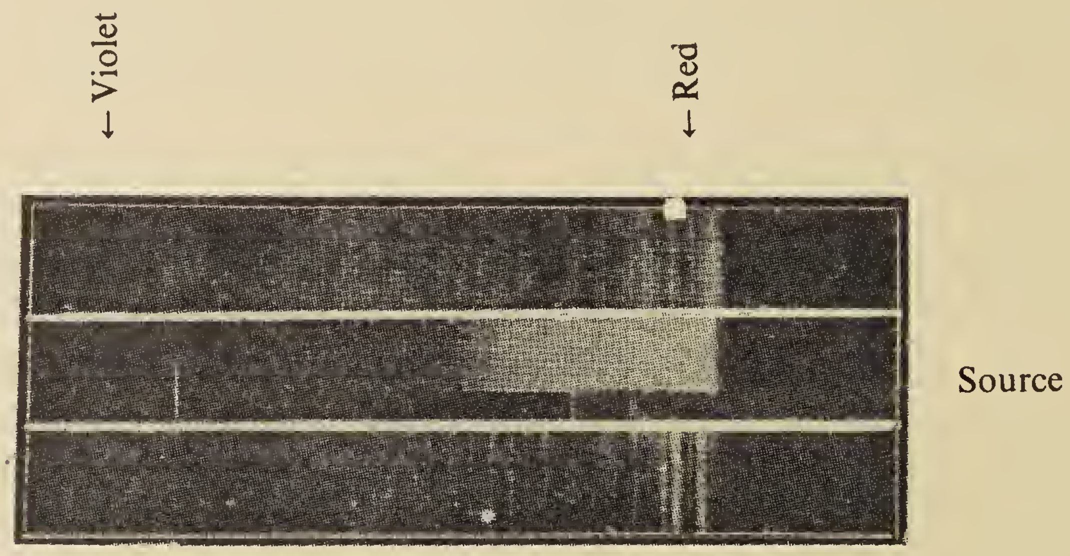


**Figure 13.** Reflection spectra of potassium chlorate crystals (feeble iridescence).

shown by strongly iridescent crystals is more remarkable, being frequently found to consist entirely of a narrow region in the spectrum (figure 14). It is evident that the reflecting planes are then numerous and are arranged with remarkable regularity. In one such case, L A Ramdas* observed no fewer than eight orders of reflection distributed in a regular sequence in the infra-red, visible and ultraviolet regions of the spectrum, each order of reflection being accompanied by a few

*L A Ramdas, *Proc. Indian Assoc. Cultiv. Sci.*, 1923, **8**, 231.

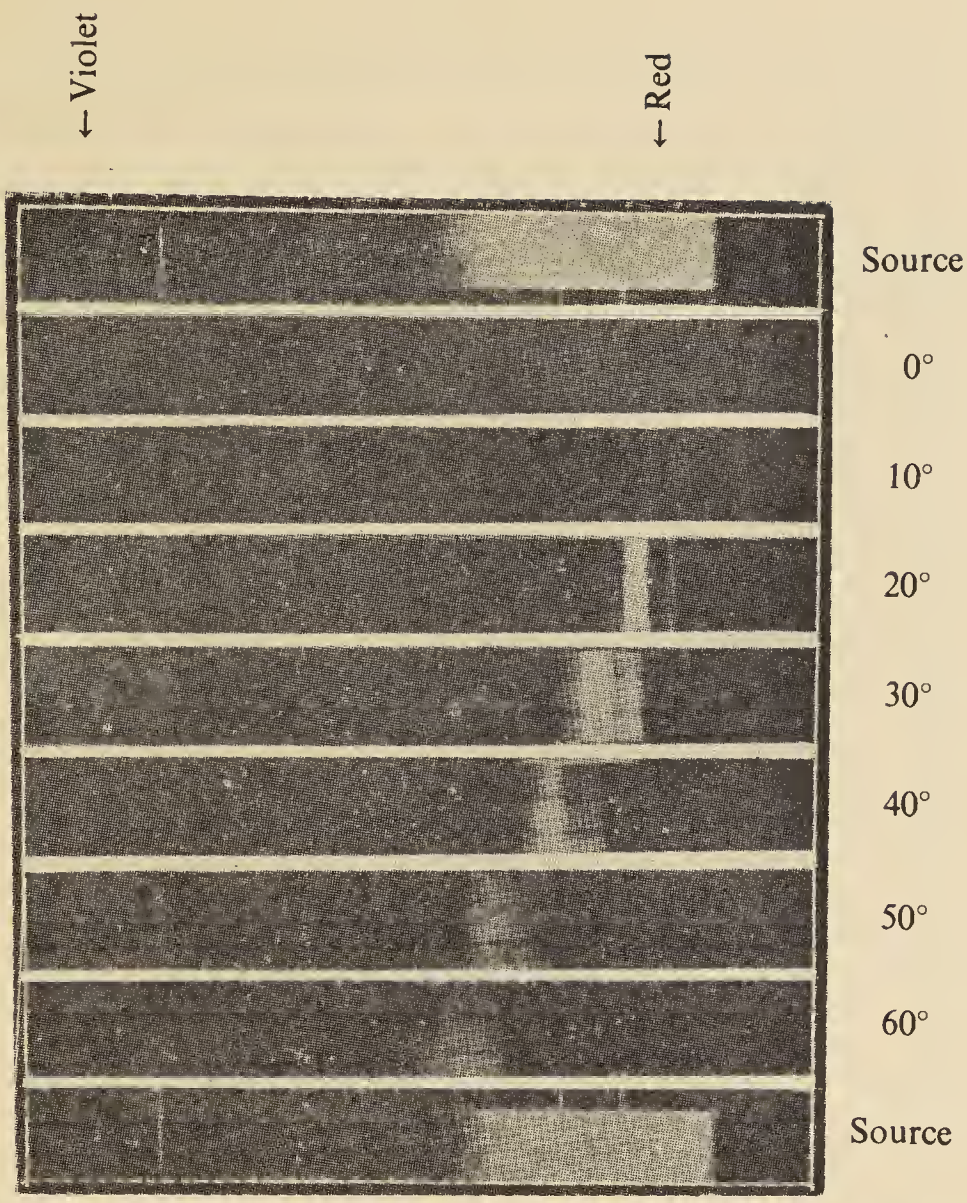


**Figure 14.** Reflection spectra of potassium chlorate crystals (strong iridescence and varying obliquity).

subsidiary maxima on either side. The general similarity in the appearance of the different orders indicated that these subsidiary bands were of similar origin in each case.

At strictly normal incidence, the twinning planes produce no optical effect, and there is then no hint of colour either by reflection or by transmission. On tilting the crystal even by a few degrees, the reflections appear, and their intensity increases rapidly as the incidence is made more oblique. The transmission colours are scarcely noticeable when the incidence is nearly normal, but rapidly becomes richer at more oblique incidences. The spectra of the reflected and transmitted light are, of course, complementary, a sharp bright band in reflection corresponding to an equally sharp dark band in transmission. (Compare figures 14 and 15) which refer to the same crystal). In the case of the strongly iridescent crystals, the selective reflections are practically total, and the corresponding extinctions in the transmitted light are therefore complete. The effect of varying the obliquity of incidence on the character of the spectra is very striking. The bands shift towards the violet end of the spectrum and at the same time rapidly widen out. The

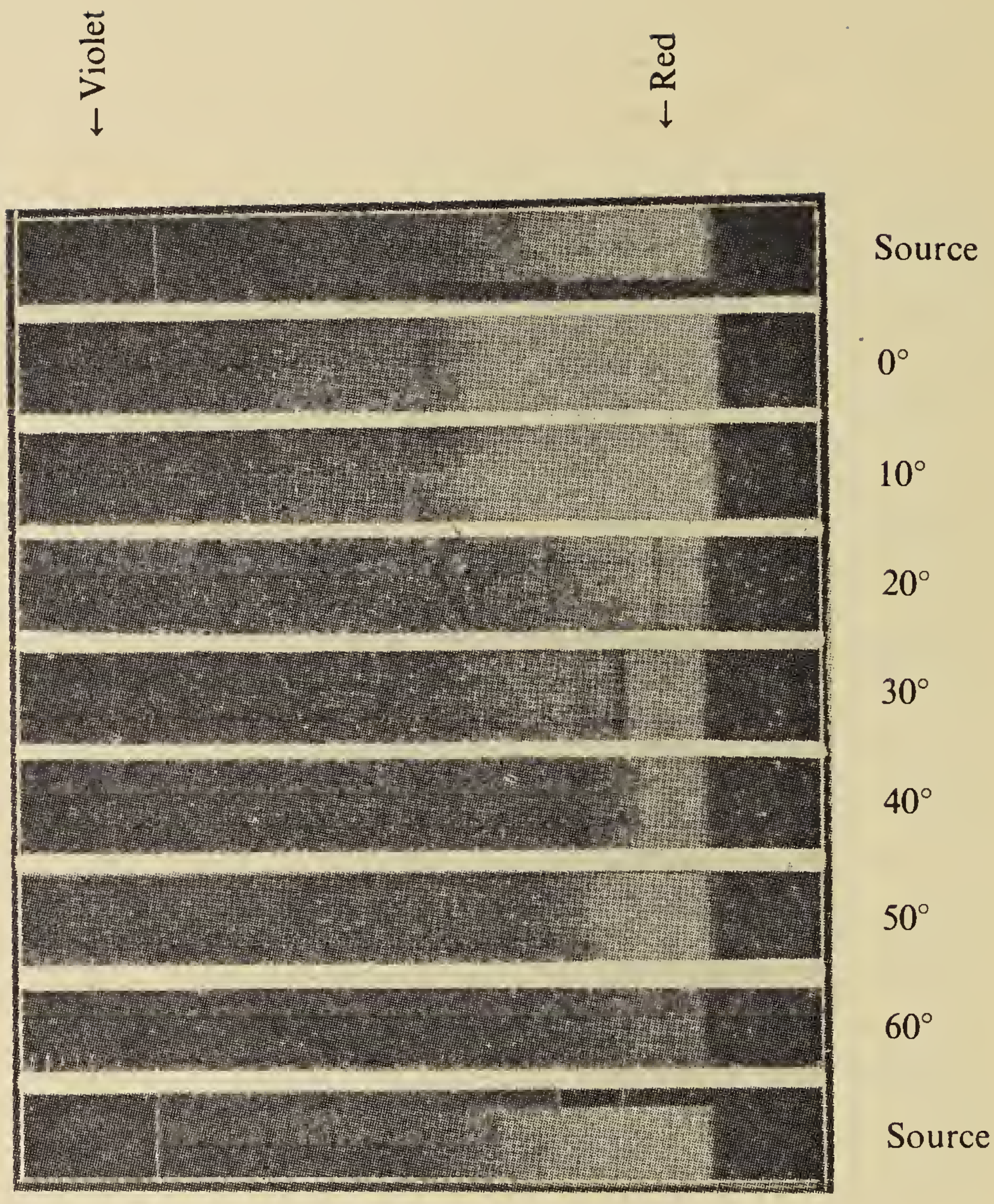


**Figure 15.** Transmission spectra of potassium chlorate crystals (strong iridescence and varying obliquity).

increased intensity of reflection and the enriched colour of the transmitted light are thus satisfactorily explained. With the particular crystal already mentioned, Ramdas found that the width $\Delta\lambda$ of the principal bands of reflection measured in Angstrom units decreased very markedly with increasing order of the reflection at any particular incidence. $\lambda/\Delta\lambda$ was not far from being the same for the different orders of reflection, but diminished rapidly with increasing obliquity of incidence, being about 150 near normal incidence, about 75 at 23° and 40 at 50° away from the normal.

The increasing width of the spectral bands of selective reflection at oblique incidences is evidently the result of the increased reflecting power of an individual lamina at such incidences, and is thus an illustration of the general theory of reflection by a stratified medium. A similar result may be brought about by keeping the angle of incidence constant, but rotating the crystal in its own plane. We have already remarked that when this is done, the reflection vanishes twice in each complete rotation. Accordingly, we should expect that the width of the spectral bands of selective reflection should oscillate as the crystal is rotated,

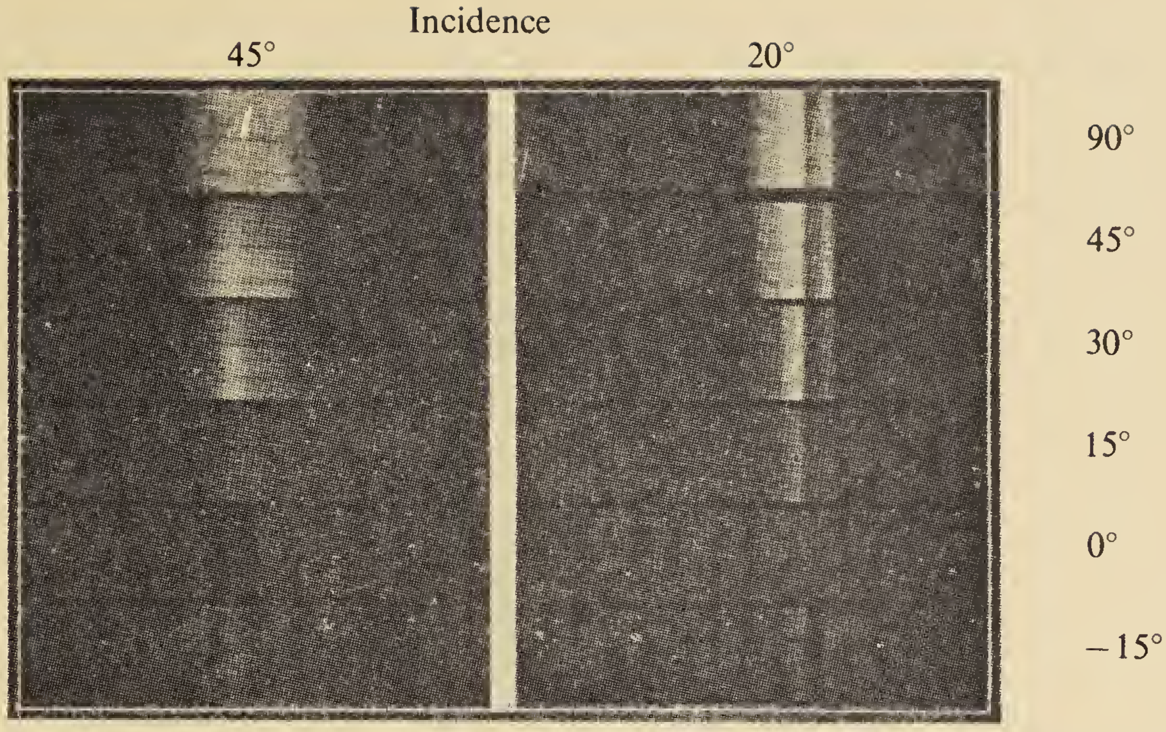


**Figure 16.** Reflection spectra of potassium chlorate crystals (effect of varying azimuth of reflection).

being least when the reflection is nearly extinguished and greatest when it is most intense. Further, these changes should be the more pronounced the greater the angle of incidence on the crystal. These anticipations from theory are completely confirmed in experiment.* (See figure 16).

A remarkable prediction from theory is that the direction of vibration of the light vector is turned through a right angle in the act of reflection from a twinning plane.† In other words, if the incident light is polarised in the plane of incidence, the coloured reflections should be polarised in the perpendicular plane, and *vice versa.* The late Lord Rayleigh to whom the explanation of the iridescence of potassium chlorate crystals is due, endeavoured to verify this prediction but found he could do so only in the particular case when the incidence of light on the crystal is nearly normal. As pointed out by him, the difficulty in observing the effect at more oblique incidences is the depolarisation of the light in its passage through the crystal before and after reflection at the twinning planes. This difficulty, however, disappears if the coloured layer be sufficiently close to the external surface of the crystal. Observations can then be made even at oblique incidences.‡ One polaroid may be used to polarise the incident light while the reflected light is viewed through a second polaroid and a direct vision spectroscope, the latter serving to eliminate the disturbing reflection at the external surface of the crystal. It is observed, in agreement with the theoretical prediction, that *the spectral bands of selective reflection are strong when the two polaroids are crossed and very weak when they are parallel.* A rotation of one

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*Unpublished observations by V S Rajagopalan and the present writer.

†Rayleigh I, *Scientific Papers*, **3**, 201.

‡Unpublished observation by the present writer.

polaroid alone when the other is absent makes no difference in the intensity of the coloured reflection. This is also in accordance with the theory.

It may be remarked that the existence of the twinned strata in potassium chlorate is no mere hypothesis, as they can be made visible by suitable sectioning, coupled with the use of polarised light for microscopic observation.* Viewing a crystal plate directly under the polarising microscope also discloses the difference between a twinned and an untwinned specimen. X-ray examination of twinned and iridescent crystals shows a doubling of certain groups of Laue spots which appear single in the untwinned crystals.† A similar behaviour is shown by the crystals in which twinning has been developed by heating up to nearly the fusion temperature and subsequent cooling. The question why twinning occurs with such facility and with such remarkable regularity in many cases in potassium chlorate is of considerable interest and is worthy of fuller elucidation. It is presumably connected with some special feature in the crystal structure of the substance.

*Structure and colours of opal*: Precious opal exhibits a striking play of colour. The finest specimens give brilliant monochromatic reflections over large areas, the colours ranging over the whole spectrum and altering with the angle of incidence of the light. Some specimens exhibit numerous small glittering spangles of colour, and others again an almost continuous sheen of iridescence. Some very beautiful and valuable opals are grey, blue or black in colour, the iridescence showing up by reflection against the dark background thus provided. Opals of a lighter tint are fairly transparent and in transmitted light exhibit hues approximately complementary to the colour of the reflected light. Opals also usually show a bluish-white opalescence overlying the reflected colours, and if such opalescence is strong, the colour seen by transmitted light tends to a honey-yellow, the complementary tints then being less conspicuous.

In examining the spectra of the reflected and transmitted light, it is important to fix attention upon a limited area of the opal showing a definite colour. The reflected light then appears as a narrow region in the spectrum, the width of which at normal incidence may be as little as 50 Å units or less. Corresponding to the bright band in the reflection, a black band appears in the transmitted light, indicating that the reflection is total (figure 17). On tilting the reflecting surface away from normal incidence, the bands shift towards the violet, broadening as they move. While these features are analogous to those observed with potassium chlorate, in almost every other respect the opal colours behave differently. They do not vanish when the incidence is normal, nor do they vary in intensity or spectral character with the azimuth of incidence. They are also polarised in the

*Rayleigh II, *Proc. R. Soc. London*, 1923, **A102**, 668.
†S C Sirkar, *Indian J. Phys*., 1930, **5**, 337.

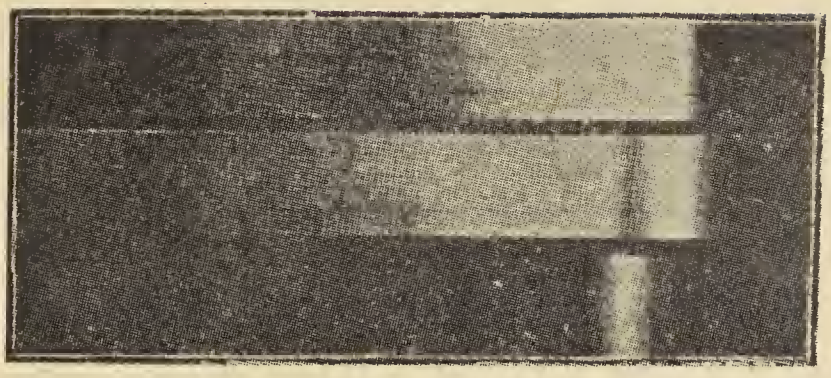



**Figure 17.** Reflection and transmission spectra of opal.

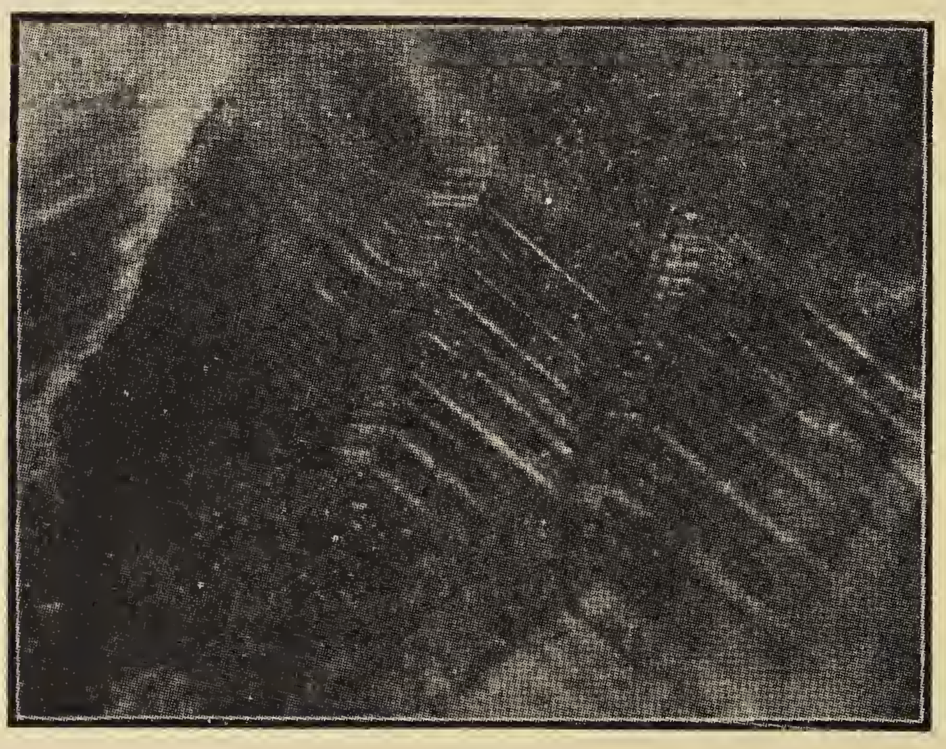
normal way, as may be shown by immersing the opal in carbon disulphide, thereby enabling the angle of incidence to be increased up to and beyond the angle required for complete polarisation. If polarised light is incident on the opal and the iridescent reflections are observed through a second polariser, they may be extinguished by holding the two polarisers in the crossed position, while in the parallel position the iridescence is seen with maximum intensity.

Examination of opal under the microscope reveals that the material possesses a lamellar structure of a remarkable character, there being actually three sets of interpenetrating planes of lamination geometrically related to each other in a manner recalling the rhombohedral cleavages of a crystal of calcite. Each set of laminations is capable of giving a monochromatic reflection of which the wavelength varies with the angle of incidence according to the usual formula. If, in a particular area, all three sets of laminations are exposed, they should be capable of giving reflections of a wavelength determined by the respective angles of incidence of the light upon them. But it is obvious that if the incident light be parallel, it would not be geometrically possible to observe the three reflections simultaneously. Normally therefore, if one set of laminations appears bright by reflection, the two others would, in general, appear dark. If, however, light be allowed to fall on the specimen from different directions at the same time, it should be possible to see the reflections from two or even all the three sets of laminations simultaneously. They would then be visible as sharply divided areas exhibiting different colours in close juxtaposition. The effect is readily observed and is a striking demonstration of the existence of the three sets of laminations.

Figures 18 and 19 are photographs of the same area on a piece of Australian opal, nothing being varied except the direction of illumination. The surface of the specimen was roughly parallel to the exposure of one of the three sets of laminations exhibiting a bright green iridescence. This appears as an extended luminous area in figure 18. The second set of laminations presented exposed surfaces of lesser area appearing as transverse bands and lines cutting across the

(18)

(19)



**Figures 18 and 19.** Laminated structure of opal.

main luminous area. As seen visually under the microscope, these exhibited a violet reflection. The third set of laminations appears as dark lines crossing the second set obliquely in figure 18. In figure 19 the illumination was so arranged that this third set of laminations appeared as bright yellow lines running through the whole field, the other two sets of laminations being perfectly dark. Effects of this kind may be conveniently studied by placing the specimen on the stage of a microscope, illuminating it from above in any desired direction and rotating the stage.

As the spacing of the stratifications is of the order of the wavelength of light, we can scarcely hope to be able to observe them directly under the microscope except

in specially favourable circumstances. Their physical nature and origin is, as yet, an unsolved problem, though numerous studies by X-ray and other methods have been reported in the literature. The spectral character of the reflections indicates that these stratifications are numerous and regular, and also that the reflective power of an individual lamina is not great, thus definitely excluding such crude hypotheses as for instance, the presence of cracks containing air. It appears probable that the material of the stratifications differs little in refractive index from that of the opal substance, or, alternatively, that the reflecting layers are thin even in comparison with the spacing of the laminations. Precise measurements of the polarising angle of reflection may assist in reaching a decision as to the nature of the material present.

*Colours of mother-of-pearl*: A nacreous layer with a characteristic lustre and iridescence is present in the shells of a great many mollusca. The nature of the material is, however, far from being identical in the different classes of mollusca, e.g., the Bivalves, the Gastropods and the Cephalopods. This is indeed evident from the striking variations in the general appearance of the nacre as well as of its density and other physical properties. Closer examination reveals remarkable variations in the structure and optical properties of nacre, not only of these great classes of mollusca but also of individual genera and species. These facts as well as the ready availability of the material, and the ease with which it can be worked and polished, make mother-of-pearl a substance of considerable interest to the student of optics. Large-sized shells of *M. margaratifera, Turbo, Trochus, Haliotis* and *Nautilus* are easily obtained, and when their nacreous layer is exposed and polished, they make striking exhibits. The iridescence of a large shell of *Turbo* thus prepared may be effectively displayed by placing an electric bulb inside it. The soft and glowing colours of the light which then diffuses out of the shell make an impressive demonstration of its optical properties.

A convenient way of examining the colours of mother-of-pearl is to cut out a piece of the shell parallel to its surface and after grinding it down to a suitable small thickness to polish its surfaces and mount it in Canada balsam between two cover slips of glass. To illustrate the influence of the thickness of the piece on the spectral character of the effects, the material may be worked into the shape of a wedge, not very thick at one end and tapering off to extreme thinness at the other. The transmitted light may then be examined by placing the mounted specimen right up against the slit of a pocket spectroscope. The transmission colour as visually observed is very weak at the thin end and becomes more vivid with increasing thickness. The spectroscope indicates that this effect is due to the spectral band of extinction being very narrow at the thin end and widening out with increasing thickness. This widening is evidently the result of small variations in the spacing of the laminations and illustrates the optical principle that a lack of perfect regularity in the stratifications may actually improve the intensity of iridescence.

Mother-of-pearl consists, in the main, of calcium carbonate which, remarkably enough, is present in the optically biaxial form of aragonite together with varying amounts of an organic substance known as conchin which serves to hold the substance together. The laminations characteristic of nacre are, in fact, made up of successive layers of aragonite in the form of very thin crystalline plates which are held together by the cementing material, there being an immense number of such layers, running roughly parallel to the natural surface of the shell. Examination of thin sections under the polarising microscope shows clearly that the aragonite crystallites all lie with their *c*-axes more or less exactly normal to the general direction of the laminations. The *c*-axis is the direction of vibration for which the refractive index of aragonite is least (1·530), while the refractive indices corresponding to the other axes are larger and nearly equal (1·680 and 1·685). For directions of incidence of the light not far from the normal, the effective refractive index of the crystalline plates is, therefore, in the neighbourhood of 1·68. The refractive index of conchin is probably about the same as that of solid gelatin (1·53), and the difference between this and the index of aragonite (1·68) is fairly large. Hence, if the thickness of the conchin films were at all comparable with those of the aragonite layers, the reflecting power of the individual laminae would be so large that the reflections by a large number of them would not exhibit any very marked spectral selectivity. A consideration of the relative densities of aragonite, mother-of-pearl and conchin, however, indicates that the conchin layers should be very thin in comparison with the aragonite crystals, and this conclusion is independently supported by a microscopic examination of the material. It follows that the reflecting power of an individual lamination should be small, and the fact that mother-of-pearl gives monochromatic reflections and extinctions in much the same way as potassium chlorate and opal thereby becomes intelligible. It is also evident that the relative thickness of the conchin and aragonite layers in different varieties of nacre should influence its optical properties quite as much or even more than the variation in the spacing of the laminations. This view is supported by spectroscopic studies which reveal that the greater vividness of colour exhibited by the mother-of-pearl from certain mollusca, e.g., *Haliotidae*, is largely a matter of the greater width of the spectral bands of extinction and transmission.

The angle at which the laminations meet the surface of the shell varies but is usually small; it may be adjusted to any desired value by suitably cutting and polishing the material. Unless the polishing has been unduly prolonged, the structure of nacre becomes evident under the microscope when its surface is illuminated in such manner that the direct reflection does not enter the field of view. Numerous sharp bright lines on a dark background are then seen (figure 20); these are the intersections of the conchin layers with the surface of the shell, and their configuration depends upon the curvature of the intersecting surfaces and the angle at which they meet. The sharpness of the lines and the character of the optical effects observed makes it evident that the conchin layers

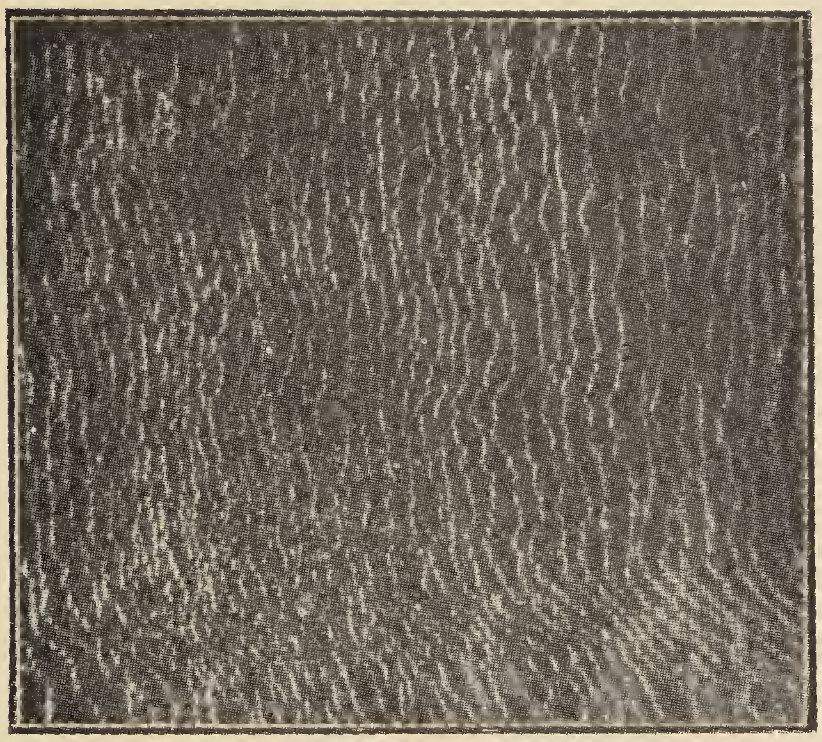


**Figure 20.** Surface of mother-of-pearl under dark-ground illumination.

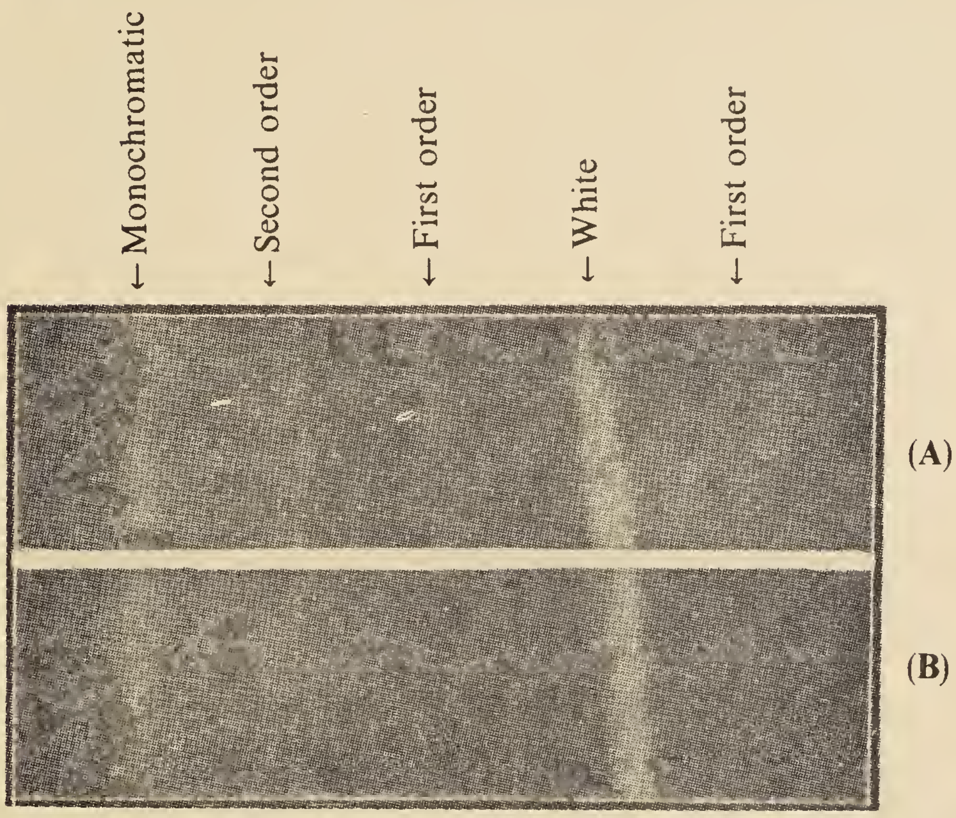



**Figure 21.** Reflection and diffraction by the surface of mother-of-pearl.

are excessively thin compared with the aragonite crystallites which they separate.

A surface structure such as that illustrated in figure 20 necessarily gives rise to diffraction effects when a pencil of light falls upon it and reflected by it. Figure 21 (A) exhibits the group of diffraction spectra given by a moderately well-polished surface and received on a screen, while figure 21 (B) is the effect observed when the surface is covered by a little Canada balsam and a glass cover-slip. It will be noticed that the diffraction spectra seen in figure 21 (A) have all vanished in figure 21 (B), except the third order on the left which persists with undiminished intensity and in an unaltered position. This is evidently the monochromatic

reflection by the internal laminations.* As we shall see in a later lecture, it is a simple consequence of diffraction theory that the directions in which the spectrum of a particular order and wavelength and the selective internal reflection of the same order and wavelength appear, coincide. This fact enables us readily to evaluate the spacing of the laminations optically and to connect their separation as seen on the surface with that directly observed (though with difficulty) in transverse sections under the microscope.

Mother-of-pearl is not only a stratified medium but also a heterogeneous one. This results in a diffusion of light which increases with the thickness of the material and exhibits itself in several ways. While a part of the incident light is selectively reflected, the rest is diffused backwards, forwards and laterally, while the regularly transmitted light progressively suffers extinction. The diffusion backwards, i.e., towards the source of light, results in mother-of-pearl exhibiting a body-colour which is *complementary* to the iridescence. The effect is conspicuously seen when the material is viewed in directions in which the iridescence is not visible. Indeed, the body-colour is also present superposed on the iridescence, thereby diluting its spectral purity. How great such dilution is can best be realised by viewing the shell under the light of the open sky. The iridescence and body-colour then completely overlap, and the shell appears dead-white, even though under directed illumination it may be strongly iridescent. In certain cases, however, as for instance, some of the Californian Haliotidae, the iridescence is very striking even in diffuse light. It would appear that in such shells the body-colour is suppressed by the presence of a strongly absorbing material in the conchin layers.

The diffusion of light in nacre is, at least in part, due to the aragonite crystallites which give the substance a granular structure. This may be demonstrated by viewing a distant source of light through a thin piece of mother-of-pearl polished and mounted in Canada balsam between glass cover-slips and held normally in front of the eye. A diffusion halo is then observed surrounding the source of light, its form being radically different with the mother-of-pearl from the three classes of mollusca, namely the Bivalves, the Gastropods and the Cephalopods.† The haloes differ in detail while retaining the general features of the class for the different individual genera and species. Their configuration is determined by the size, shape, orientation and spacing of the aragonite crystallites and is illustrated for three typical cases in figure 23. The conclusions regarding the structure of mother-of-pearl in the three classes of mollusca reached from a study of the diffusion haloes are independently confirmed by various other methods, viz., by a microscopic study of thin sections,‡ by their X-ray diffraction patterns§ and by

*C V Raman, *Proc. Indian Acad. Sci.*, 1935, **A1**, 574.
†C V Raman, *Proc. Indian Acad. Sci.*, 1935, **A1**, 859.
‡V S Rajagopalan, *Proc. Indian Acad. Sci.*, 1936, **A3**, 572.
§S Ramaswamy, *Proc. Indian Acad. Sci.*, 1935, **A1**, 871; and 1935, **A2**, 345.

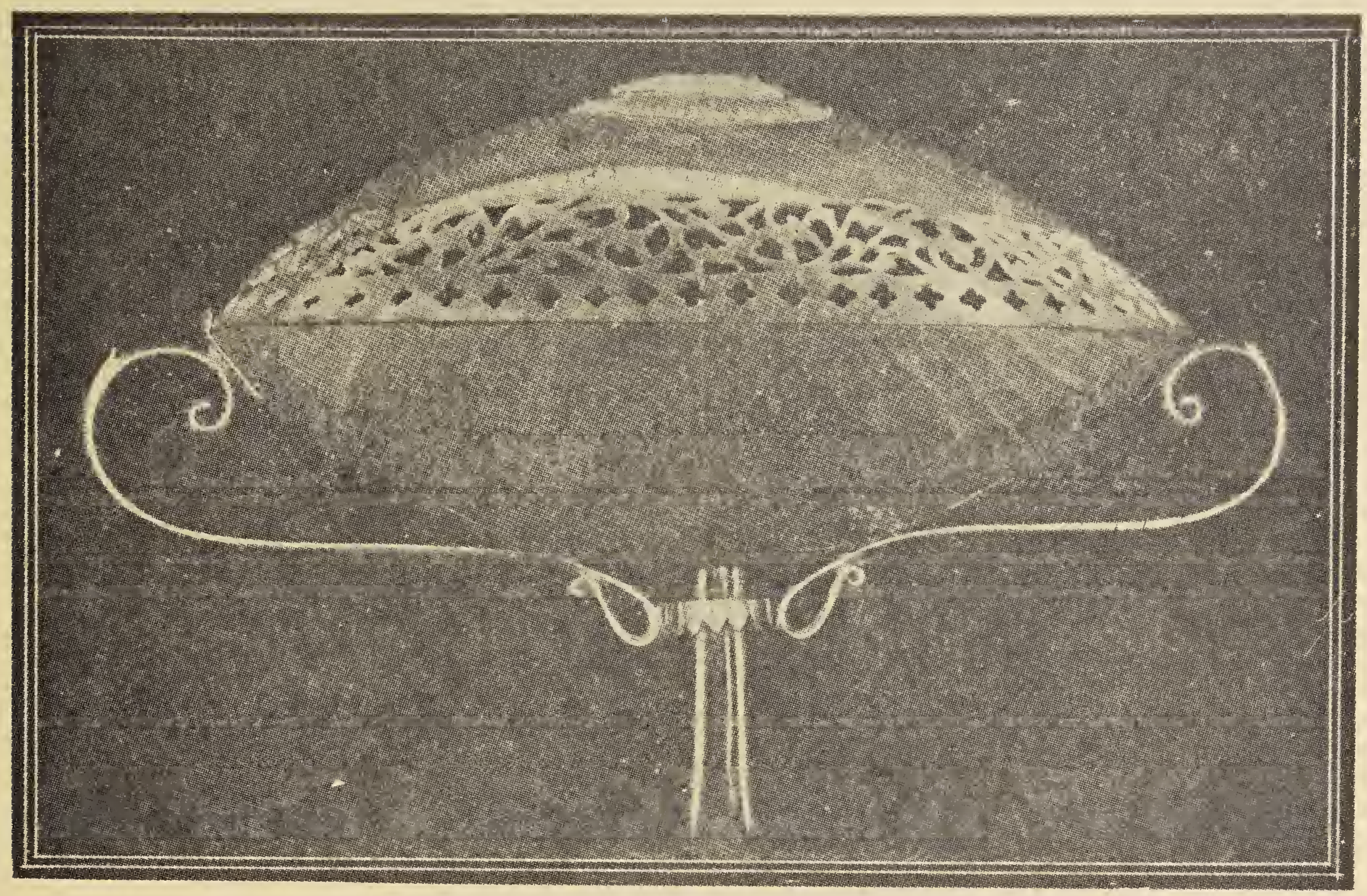


**Figure 22.** Bands of body-colour in a polished shell of *Turbo*.

observations of the birefringence, the magnetic anisotropy* and the elastic behaviour of nacre† from the different sources. We shall not enter into further details here, as this would take us beyond the limits of our present subject.

A lamination spacing of about 0·5 $\mu$ may be taken as roughly representative of the strongly iridescent varieties of mother-of-pearl; this would give a third-order reflection in the brightest part of the visible spectrum, the second order being in the near infra-red, while the fourth order would be at the violet end of the spectrum. A tenth of a millimeter would accordingly be sufficient to include about 200 laminations, and if these were all uniform and equally effective, the bands of reflection and extinction would have a width of only 10 Å, while at the same time, the effect of diffusion would be minimised. The advantage of using a relatively thin piece of the material for spectroscopic observations is thus apparent. With selected specimens and holding the slit of the spectroscope parallel to the contour lines of colour, reflection and extinction bands fully as sharp and straight as those observed with potassium chlorate crystals and with opal may be seen with mother-of-pearl.

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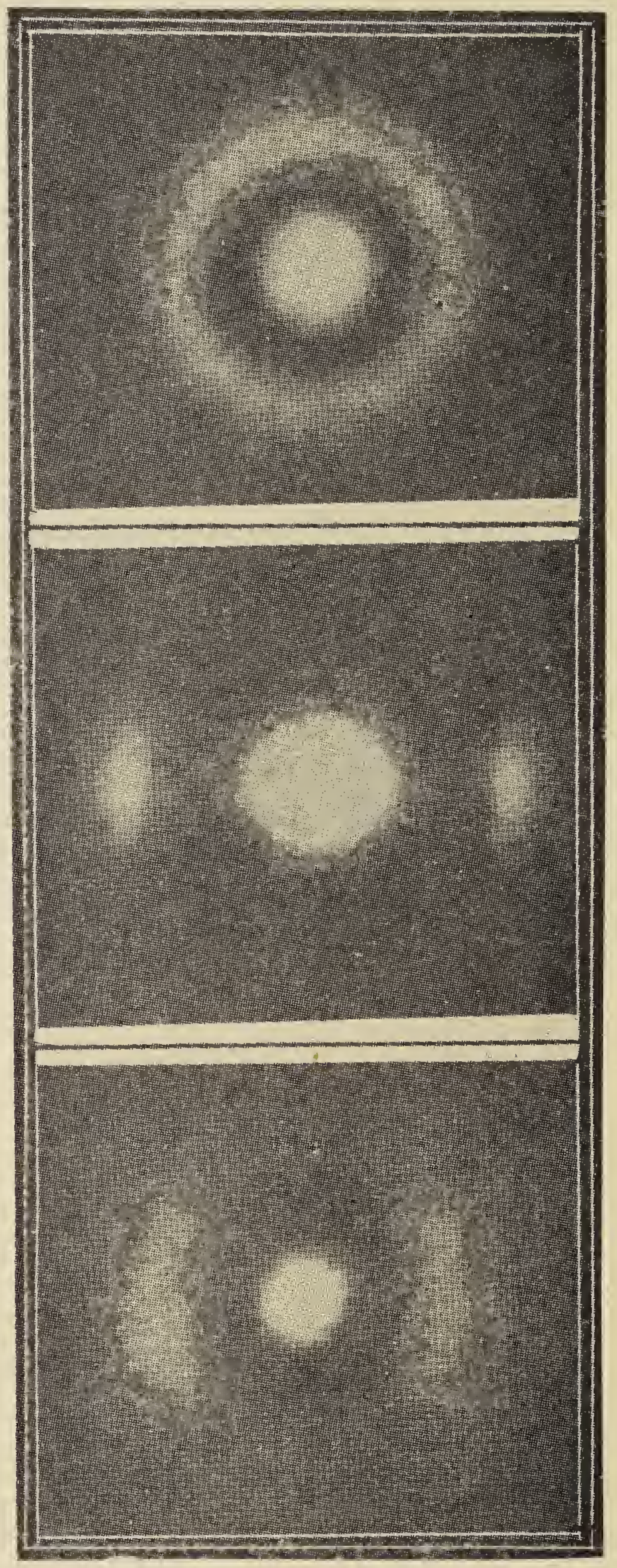
*P Nilakantan, *Proc. Indian Acad. Sci.*, 1936, **A2**, 621.
†P S Srinivasan, *Proc. Indian Acad. Sci.*, 1937, **A5**, 463.



Gastropods (*Turbo*)

Bivalves (*M. margaratifera*)

Cephalopods (*Nautilus*)

**Figure 23.** Diffusion haloes of mother-of-pearl.

The extinction of the transmitted light due to diffusion in traversing the nacre varies greatly with the class of mollusc. The mother-of-pearl from *Trochus*, for instance, is remarkably opaque even in thin layers, while the shells of *Turbo* and *M. margaratifera* are much more transparent than those of *Haliotis* or *Nautilus*. As is generally the case with turbid media, the extinction coefficient increases rapidly as the wavelength is diminished, becoming very great in the violet and ultra-violet regions of the spectrum. The colour of the transmitted light which at

**Figure 24.** Shell of *Nautilus* photographed as a translucency.

first is complementary to the selective reflection, assumes a reddish tinge and ultimately becomes a deep red with increasing thickness of the plate. One curious result of this is that specimens which selectively reflect at the red end of the spectrum are more opaque than those which give green or blue reflections. With thicknesses greater than a few tenths of a millimetre, light does not penetrate but only diffuses through; the colour of the diffusing light at first is complementary to the selective reflection only for moderate thicknesses and assumes a reddish tinge with increasing thickness. One remarkable property characteristic of nacre is that light can diffuse *in a direction parallel to the laminations to a distance of some centimetres*, while the diffusion normal to the laminations is limited to a few millimetres at the utmost. This effect is illustrated in figure 25 in which an extended area of the nacre appears luminous though only a small region at its centre was illuminated.*

This effect illustrated in figure 25 is no doubt a consequence of the special structure of nacre. The shape and size of the aragonite crystallites vary with the class of mollusc, the smallest dimension being from 2 to 8 $\mu$ as shown by the angular diameter of the diffusion haloes, while their thickness is of the order 0·5 $\mu$. The number of reflecting boundaries traversed per unit path of the light is thus many times greater transverse to the laminations than parallel to them. The difference in refractive index between the aragonite and the conchin also

*C V Raman, *Proc. Indian Acad. Sci.*, 1935, **A1**, 859.

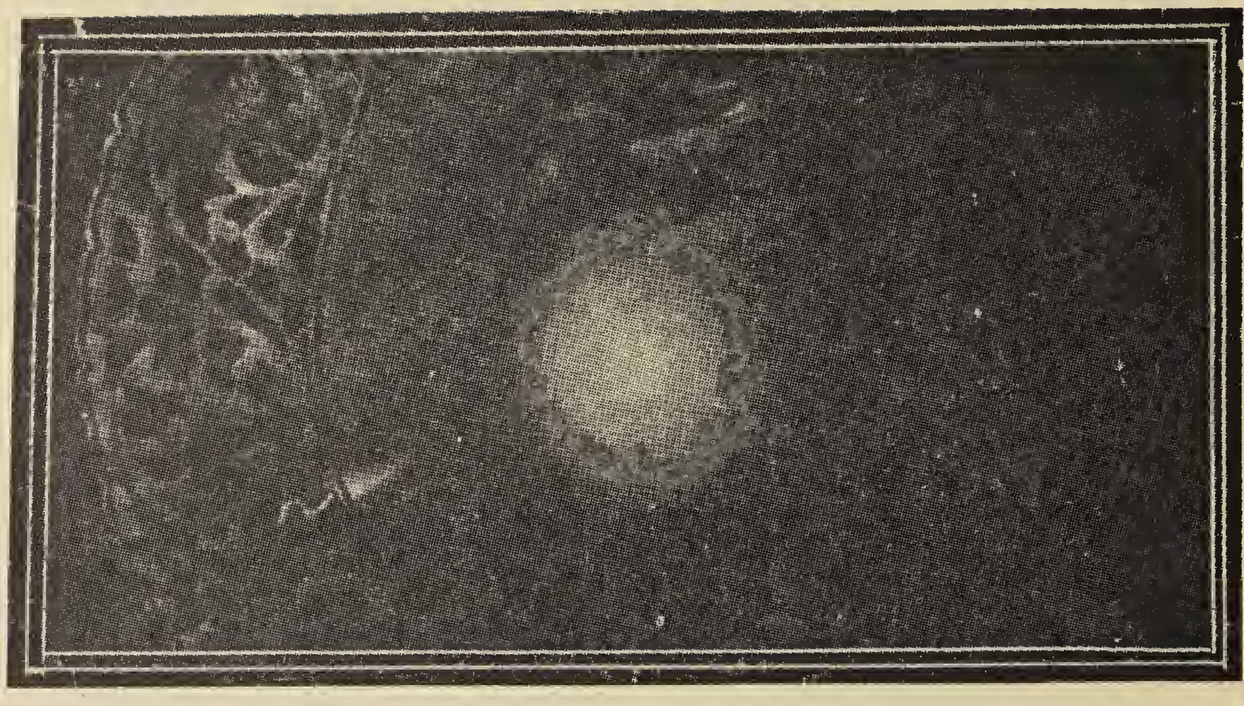


**Figure 25.** Diffusion of light in *Margaratifera* parallel to laminations.

effectively vanishes for light travelling parallel to the laminations with its electric vector normal to them. These features conspire to enable the light to penetrate to great depths along the aragonite layers.

*Iridescence of decomposed glass*: Ancient glassware excavated from archaeological sites is often distinguished by a beautiful iridescence. It is frequently possible to detach iridescent flakes from such glass, exhibiting rich colours by transmitted light complementary to those seen by reflection. The laminar structure of decomposed glass becomes evident on examining the edges of the flakes under the microscope. It is noticed also that the flakes are often not plane, but consist of shallow cups like watch-glasses fitting together perfectly and dividing the area into a larger number of polygons. These exhibit rich colour which is uniform over a large area but differs slightly at the margin and centre of each cup, owing to the difference in obliquity of observation. The beautiful regularity in curvature and the sharpness of the polygonal edges are evident in figure 26 which reproduces the interference rings in monochromatic light seen by transmission between the upper surface of the flake and a sheet of mica laid above it, and viewed under a microscope.*

Deeper cavities are sometimes seen which may be spherical or ellipsoidal in shape. Seen between crossed micols in the polarising microscope, such cavities exhibit a dark cross (figure 27) intersected by rings of colour due to the varying obliquity of the surface.

From the fact that the laminations in an iridescent flake adhere pretty firmly to each other, it is evident that they are *ordinarily in optical contact and are not*

*C V Raman and V S Rajagopalan, *Proc. Indian Acad. Sci.*, 1940, **A11**, 469.

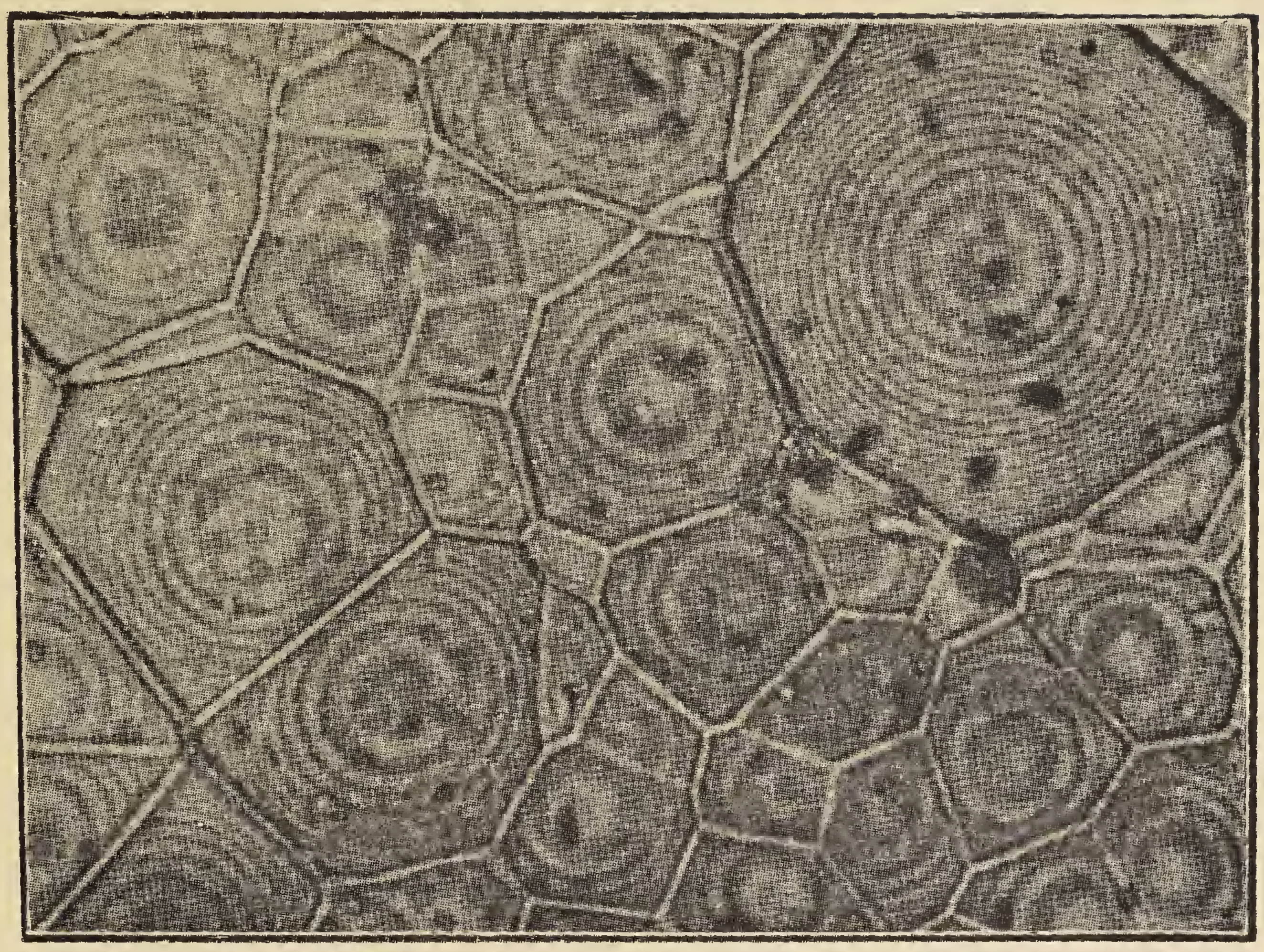


**Figure 26.** Polygonal sub-divisions in iridescent glass.

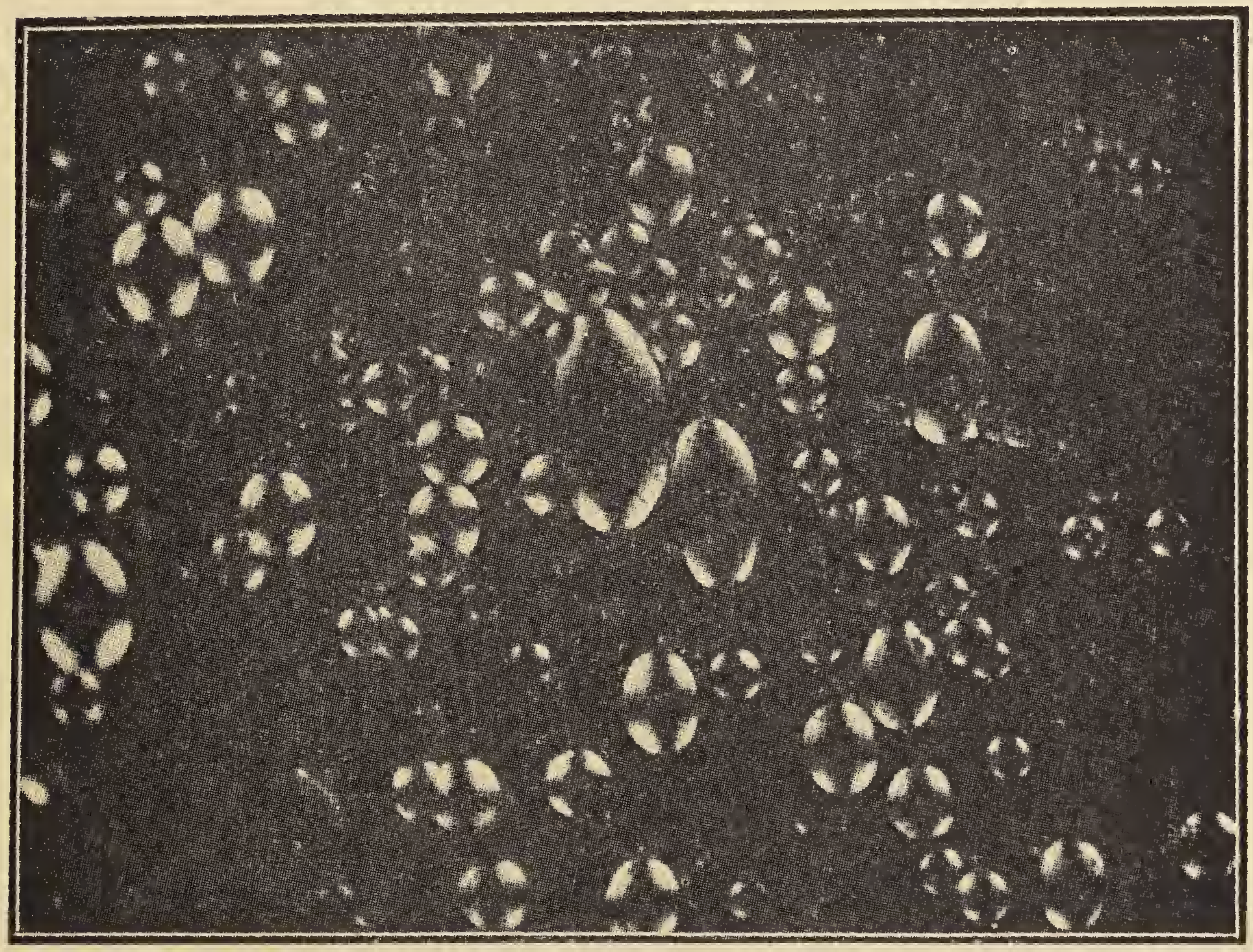


**Figure 27.** Deep cavities in iridescent glass under polarised light.

*separated by films of air*. When the flakes are viewed under the microscope in monochromatic light, the areas of colour appear uniform in intensity, but in those cases where a film of air has entered and broken optical continuity, it reveals itself immediately by the appearance of bright and sharply defined interference fringes

**Figure 28.** Fringes due to intruding air films.

on a dark ground (figure 28). Such fringes move about when the flake is lightly pressed and recover their shape when the pressure is removed. The presence of films of air also disturbs the uniformity of colour of the flakes as seen in white light under the microscope.

An insight into the nature of the laminations is obtained on immersing a flake in a cell of liquid and gradually varying the refractive index of the latter. The transmission colour disappears immediately, but the reflection colour continues to be visible though weakened, provided the refractive index of the liquid is either lower or higher than of the glass ($\mu = 1{\cdot}46$). It is noticed, however, that the colour of the flakes when immersed in carbon disulphide ($\mu = 1{\cdot}63$) is decidedly different from the colour when immersed in water ($\mu = 1{\cdot}33$). These observations indicate that the decomposed flake, though optically a continuous medium, has an open or porous texture into which liquid can penetrate, the distribution and size of the cavities varying in such manner as to give the material a laminated structure. A striking proof of this is given by the appearance of the flake when it is removed from the immersion liquid and allowed to dry. When still saturated with liquid, it

appears quite colourless. But immediately the drying commences, it becomes almost perfectly opaque and transmits no light, while it is silvery white as seen by reflection. The opacity then slowly clears up, the flake exhibiting the usual colours when completely dry. These observations indicate that in drying, the liquid withdraws first from the largest cavities, while it continues to fill the finer pores; as a consequence, the optical stratifications are actually intensified in the first stage of drying and their reflecting power is enhanced as compared either with the perfectly dry or with the completely saturated flake. The porous structure of ancient glass is also shown by placing a small flake covered with a piece of cellophane on the stage of the microscope, and then pressing or rolling a blunt steel point firmly on its surface. The colour is then observed practically to disappear from the area of pressure and does not recover on its removal.

It would seem that the laminations responsible for the colour are tolerably regular. This is indicated by a spectroscopic examination which shows that the flakes may be roughly classified in two divisions. Flakes of the first kind appear red or orange red by reflected light and blue or bluish-green by transmission; while flakes of the second kind show the complementary colours in each case. Typical spectra of the two groups are reproduced in figures 29(A) and 29(B). Spectra of the first kind are roughly analogous to those obtained with the

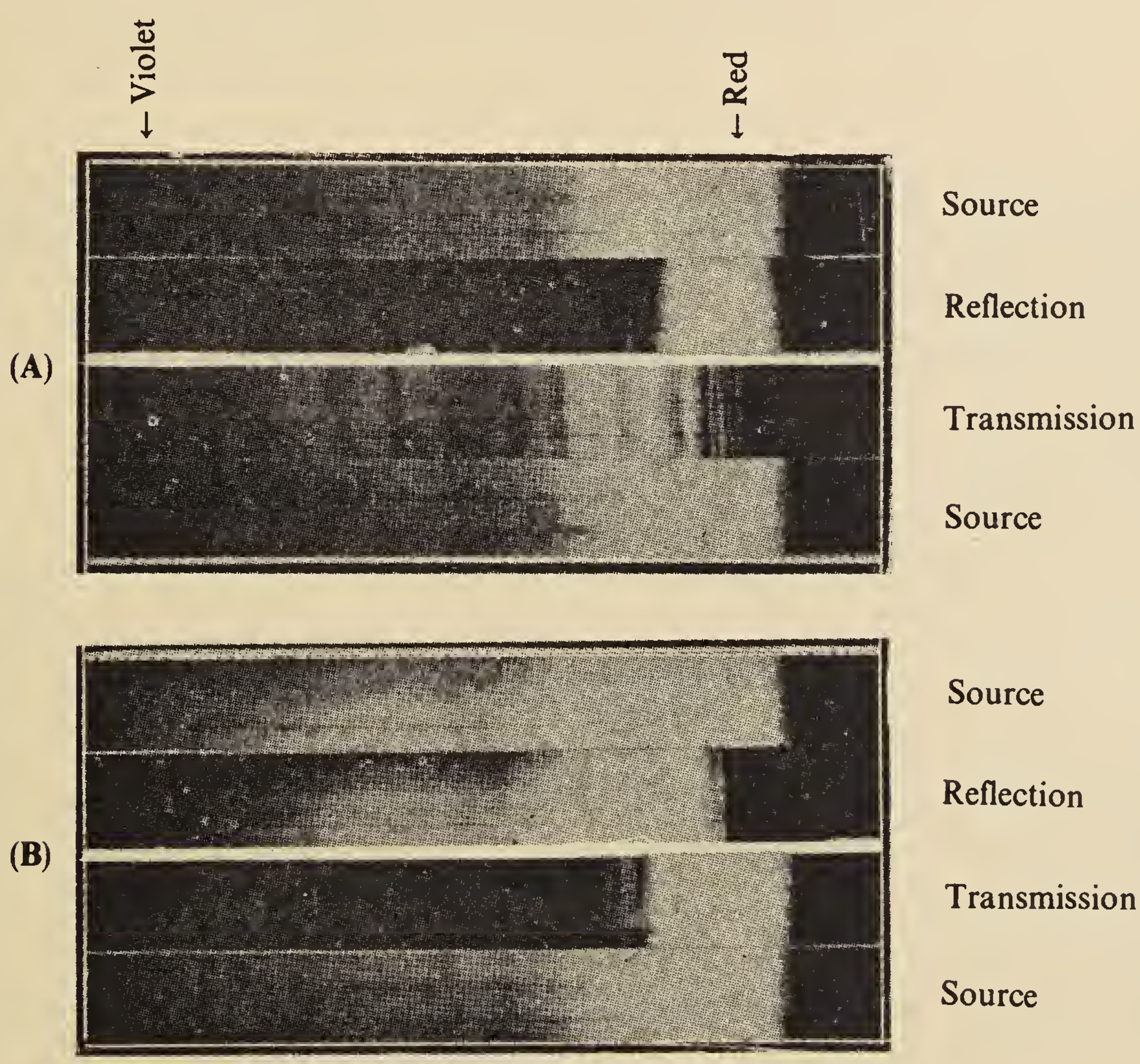



**Figure 29.** Spectrograms of ancient decomposed glass.

stratified media already discussed. The spectra of the second kind are evidently of a different nature. They are analogous to those encountered when we examine the light transmitted by a thin film enclosed between highly reflecting surfaces, in which case most of the light is reflected except certain narrow regions in the spectrum which are transmitted. It would appear that in such cases the laminations have a specially large reflecting power.

Modern glassware when buried underground or exposed to chemical action often develops iridescence. But this is superficial and the transmission colours are poor. Examined by reflected light under the microscope, however, some beautiful effects may be observed. A distinctive type of decomposition often seen is one in which the surface of the glass is divided up like a map into areas of uniform colour with sharply defined boundaries. In other cases, again, we have numerous small cavities pitting the surface and appearing by reflected light as circular rings of colour. These may be separate but may and indeed often also appear closely grouped together over extensive areas. Glass which has been attacked severely frequently exhibits numerous hollow cup-like cavities adjoining each other, the walls of such cavities being in many cases themselves pitted by still smaller depressions. Some particularly remarkable cases may be seen in which the decomposition has proceeded symmetrically outwards from a nucleus on the surface and has at the same time spread downwards into the glass, forming a succession of thin concentric laminae. The cavities formed by the removal of the decomposed material in such cases appear like circular craters which go down in

**Figure 30.** Crater-like forms in decomposed glass.

a series of terraces to the deepest area at the centre, these terraces exhibiting brilliant colours by reflected light. Numerous such craters may be seen adjoining each other in figure 30 in which a few craters are also visible in which the material resulting from decomposition is still *in situ.**

The porous texture of glass which has been chemically rendered iridescent may be prettily shown by placing a small drop of monobrom-naphthalene on the

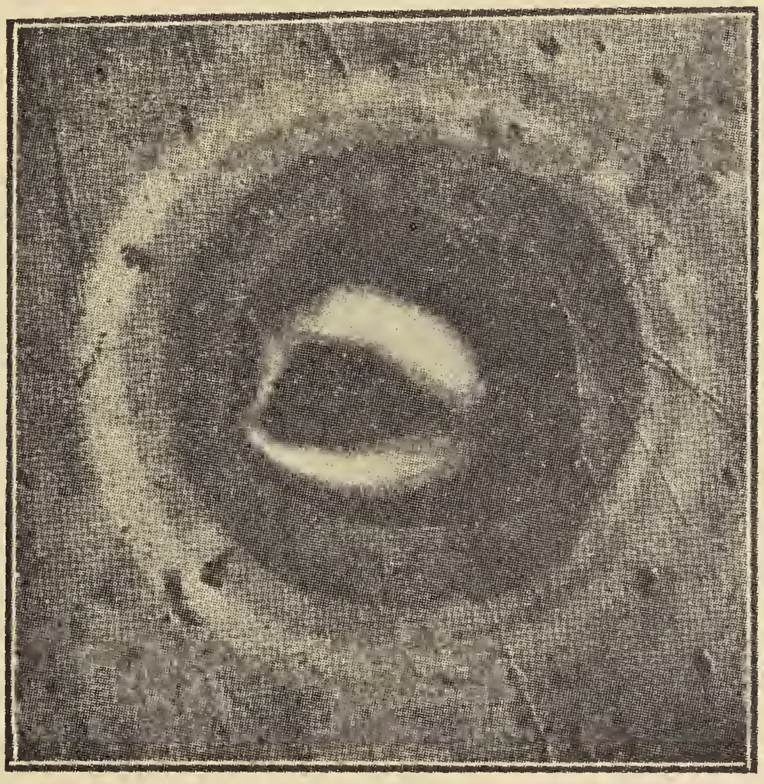


**Figure 31.** Adsorption of monobrom-naphthalene by iridescent surface.

surface and observing it by reflected light (figure 31). Surrounding the drop appears a circular area of film saturated with liquid, while beyond this is seen a succession of dark and bright rings which indicate a variation in the quantity and distribution of the liquid adsorbed by the film. These rings exhibit varied colours which are even more striking than the colour of the part of the film free from liquid. Monobrom-naphthalene being non-volatile, the pattern seen round the drop of it is static. The case is, however, different when a quickly-spreading and volatile liquid such as benzene is used; the colour patterns due to its adsorption and subsequent evaporation change with great rapidity.

*The Lippmann photographic films*: Besides the four examples of a laminated structure giving iridescence which we have dealt with in detail, there are several others which would have been well worthy of discussion in these pages, had space permitted. For instance, the metallic colouration of many beetles has been shown to be due to the presence of stratified layers in their wing cases.† The brilliant

*C V Raman and V S Rajagopalan, *Proc. Indian Acad. Sci.*, 1939, **A9**, 371.
†Rayleigh II, *Proc. R. Soc. London*, 1923, **A103**, 233.

colouration exhibited by many birds, insects and fishes may, at least in some instances, be of the same general nature. Amongst artificially produced periodic structures, pride of place must be given to the well known Lippmann films in natural colour obtained by the action of stationary light waves on photographic films. The theory set out in the preceding pages gives a satisfactory explanation of the experimental facts observed with such films. The appearance for instance, of subsidiary bands *on one side only* of the principal maximum in the spectrum with films intended to exhibit a monochromatic reflection is clearly in accord with the theoretical expectations* (see figure 12).

The frequency with which laminated structures are forthcoming in nature is rather remarkable. The mechanism giving rise to such structures appears to be of a varied nature, viz., rhythmic crystallisation, multiple twinning, periodic precipitates, colloidal aggregation, and the natural processes of biological growth. Perfect regularity in the laminations is not always to be expected, nor is it essential for the production of vivid colour. Indeed, a lack of regularity may enable the medium to give a sensibly total reflection and a corresponding extinction over a wider range of wavelengths than would otherwise be possible, and thereby to enhance the intensity of the colours observed without notably diluting their purity. This feature is of particular importance in those cases where the reflecting power of the individual laminations is small and a great many of them are present.

*R V Subramanian, *Proc. Indian Acad. Sci.*, 1941, **A13**, 467.

## Lecture II

# Diffraction of light


principles of geometrical optics indicate that the propagation of light is influenced by the presence of obstacles in its path in a manner determined by the form and properties of the obstacles. A polished metallic sphere, for instance, would reflect the light falling upon it and hence would cast a shadow. More complicated would be the effects due to a transparent obstacle, e.g., a drop of water, as both the reflections and refractions at its surface would have to be considered. The determination of the resulting light intensity everywhere in the field in a case of this kind is a problem in the theory of diffraction. It is evident that the indications of geometrical optics must be supplemented by those of the wave-theory, in view of the possibility of interference arising between rays of light which cross each other after traversing different paths. In the case of the metallic sphere, for instance, the entire field outside the region of shadow should exhibit interferences between the incident and reflected rays. Similarly in the case of the liquid drop, the interferences between the direct and refracted rays would, in addition, need to be considered. The question also arises whether the conclusions derived from ray optics remain valid in all circumstances. In the particular case of the metallic sphere, for instance, the geometric shadow is sensibly perfect in the vicinity of the sphere, but at a sufficient distance behind it, observation reveals the presence of a bright spot of light at the centre of the shadow. How is this fact to be reconciled with the concepts of ray optics?

It is evident that the study of diffraction raises a fundamental issue, namely, the relationship between the ray and wave concepts of light. An adequate discussion of diffraction phenomena necessarily includes a consideration of these two aspects of optical theory and a reconciliation between them. The geometric approach provided by ray optics has the merit of simplicity and enables us to obtain an intuitive grasp of the phenomena. Hence, instead of considering

diffraction phenomena as falling solely within the scope of wave-optics and as indicative of a failure of geometrical optics, it is a much more satisfactory procedure to bring diffraction within the scope of ray optics by appropriately framing our fundamental concepts. How this may be done is illustrated by the optical effects produced by a corrugated surface which we shall now consider in some detail.

*Refraction of light by a corrugated surface*: The phenomena which we shall now consider may be experimentally studied with a ripple tank, utilizing the fact that the surface of a liquid agitated by a linear system of ripples of definite frequency and wavelength is a corrugated surface. A narrow slit illuminated by a mercury lamp and followed by a collimating lens may be used as the source of light. The beam after passing through the liquid surface or after reflection by it is viewed through a telescope focussed for parallel rays. We may consider here a parallel beam of light which passes vertically upwards through a ripple tank containing

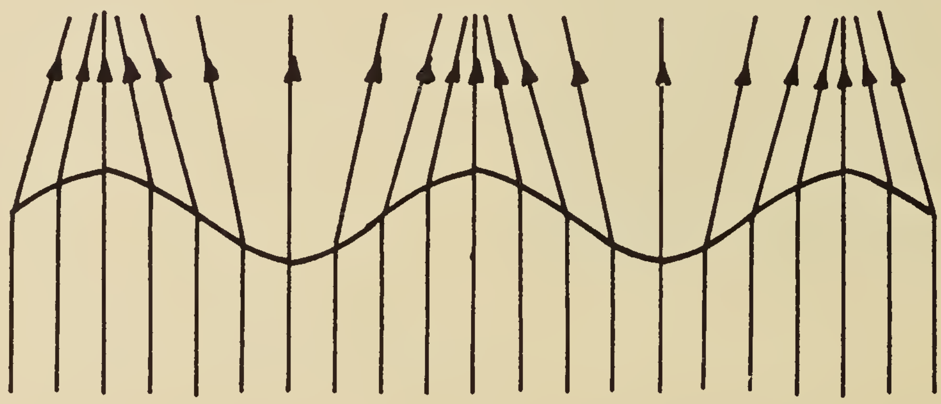


**Figure 32.** Rays emerging from ripple tank.

water. Figure 32 represents the geometric courses of the rays of light on emergence from the liquid. The rays would be divergent over the concave areas and convergent over the convex areas of the surface, but at a sufficiently great distance would be divergent throughout. These elementary considerations of ray optics describe correctly the effects observed in the immediate vicinity of the surface. But at points sufficiently removed from the surface, they no longer represent the facts correctly.

We may ask ourselves, why is the geometric theory valid near the surface of the liquid and why does it apparently fail at points sufficiently far removed from it? The answer to these questions must be found in the fact that the disturbance on emergence from the liquid surface is no longer a simple train of plane waves. As remarked earlier in our discussion on the interference of light, *the principle of*

*rectilinear propagation of light and the principle of interference do not in any way contradict each other.* But if apparent contradictions are to be avoided, it is necessary that we should recognise only waves of constant type on the one hand, and the rays normal to them on the other, as the proper basis for the description of the optical field. In our present problem, for instance, we should analyse the disturbance on emergence from the liquid surface into its component plane waves. The subsequent movement of such plane waves along their respective normals would furnish a description of the optical effects which is equally correct from the wave and the ray points of view and which is valid for every part of the field, both near and far from the surface of the liquid.

We have already seen (figure 2) that two plane wave-trains superposed on each other result in a stratification of the amplitude and phase of the disturbance in the field. The spacing $2D$ of such stratifications measured in a plane bisecting the angle between the wave-fronts is given by the formula $2D \sin \psi = \lambda$. This relation may be readily generalised. The superposition of sets of wave-trains travelling in various directions lying in a common plane and making angles $\psi_n$ with some fixed direction in it, the relation $2D \sin \psi_n = n\lambda$ being satisfied, ($n = 0, \pm 1, \pm 2$, etc.), would result in the most general type of disturbance in which the amplitude and phase vary periodically in the plane normal to the fixed direction. Hence, putting $2D = \lambda^*$, where $\lambda^*$ is the wavelength of the ripples, we may represent the light emerging from the liquid surface by such a set of plane waves travelling along the directions $\psi_n$ given by the formula $\lambda^* \sin \psi_n = n\lambda$, ($n = 0, \pm 1, \pm 2$, etc.). Accordingly, when the light is viewed through a telescope focussed for infinity, a set of monochromatic images of the original light source would be observed on either side of the original direction of the light beam. The directions and intensities of these images would correspond to the plane waves into which the disturbance emerging from the liquid surface has been analysed. It will be noticed that the formula is the well known one for the diffraction spectra due to a grating, but it has been obtained here directly from fundamental concepts, without introducing the principle of Huyghens.

Figure 2 gives us other useful indications regarding the geometric character of the optical effects to be expected in our problem. Consider the pair of wave-trains proceeding along the directions $\psi_{\pm n}$. As shown by the figure, the space periods of the resultant disturbance perpendicular to the fixed direction and parallel to it are respectively $\lambda/\sin \psi_n$ and $\lambda/\cos \psi_n$. Hence, the resultants of the wave-trains $\psi_0$, $\psi_{\pm 1}$, $\psi_{\pm 2}$, $\psi_{\pm 3}$, etc., have space periods $\infty$, $\lambda^*$, $\lambda^*/2$, $\lambda^*/3$, etc., along any plane normal to the fixed direction, thus being in strict harmonic relationship. Along the fixed direction, however, the "wavelengths" of the resultants are $\lambda$, $\lambda/\cos \psi_1$, $\lambda/\cos \psi_2$, $\lambda/\cos \psi_3$, etc. As these lengths are not identical the resultant disturbance would fluctuate as we proceed away from the liquid surface along the light beam. It can be readily shown that the disturbance would undergo a periodic cycle of changes, repeating itself completely when we advance a distance $2\lambda^{*2}/\lambda$ or any

multiple of it, away from the surface. This is verified on putting $p\lambda = (p - n^2)\lambda/\cos\psi_n$, where $p$ is an integer, and using the approximation

$$\cos\psi_n = (1 - \psi_n^2/2) = (1 - n^2\lambda^2/2\lambda^{*2}).$$

We obtain immediately $p\lambda = 2\lambda^{*2}/\lambda$. In other words, the result of superposing the waves $\psi_{\pm 1}$ on $\psi_0$ would repeat itself *once* when we advance a distance $2\lambda^{*2}/\lambda$, while that of superposing $\psi_{\pm n}$ on $\psi_0$ would repeat itself $n^2$ times within the same range. The appearance of the light field would, therefore, show fluctuations of a complex periodic character as we move away from the liquid surface, the spacing of the whole cycle being $2\lambda^{*2}/\lambda$.

An expression is readily found for the amplitudes of the plane wave-trains into which the disturbance emerging from the liquid surface is analysed. The light vector in the emergent wave when the surface of the liquid is plane may be taken proportional to

$$\sin(2\pi\nu t - 2\pi z/\lambda) = \sin Q.$$

The retardation of phase produced at a given epoch by ripples of amplitude '$a$' progressing along the $y$-axis is

$$2\pi a(\mu - 1)/\lambda \cdot \cos 2\pi y/\lambda^*.$$

We shall denote this for brevity by $v \cdot \cos\phi$. Accordingly, the expression for the light vector as modified by the presence of the ripples is proportional to sin $(Q - v\cos\phi)$. This may be expanded and written in the form

$$\begin{aligned} J_0(v)\sin Q - J_1(v)[\cos(Q+\phi) + \cos(Q-\phi)] \\ - J_2(v)[\sin(Q+2\phi) + \sin(Q-2\phi)] \\ + J_3(v)[\cos(Q+3\phi) + \cos(Q-3\phi)] \\ + J_4(v)[\sin(Q+4\phi) + \sin(Q-4\phi)], \quad \text{etc.} \end{aligned}$$

It is easily verified that the terms $\sin Q$, $\cos(Q \pm \phi)$, $\sin(Q \pm 2\phi)$, etc., represent plane waves travelling along the directions we have already indicated as $\psi_0$, $\psi_{\pm 1}$, $\psi_{\pm 2}$, etc. The intensities of these waves are accordingly proportional to $J_0^2(v)$, $J_1^2(v)$, $J_2^2(v)$, etc. As extensive tables of the Bessel functions are available, these quantities may be readily found for any assigned value of $v$. It may be remarked that, as is to be expected,

$$J_0^2(v) + 2J_1^2(v) + 2J_2^2(v) + 2J_3^2(v) + \ldots\ldots = 1,$$

so that the incident energy is merely redistributed amongst the different spectra.

*Experimental verification of theory*: The behaviour of the Bessel function when the order and the argument are varied is well known.† The changes in the

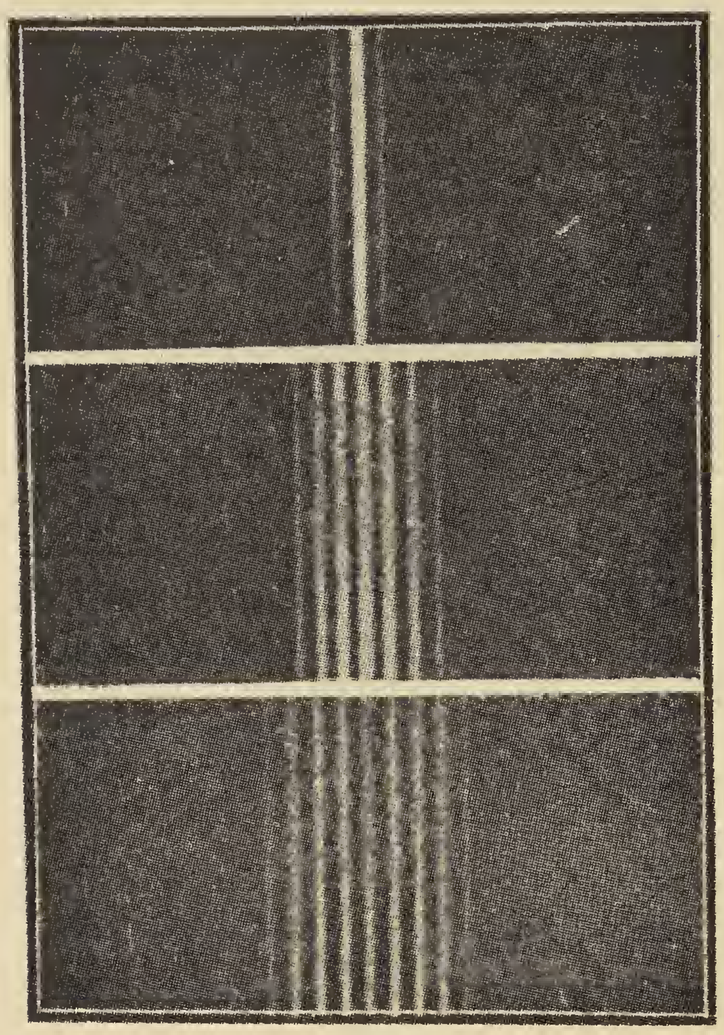
†See, for instance, Jahnke–Emde, *Tables of Functions*, Second Edition, 1933, Section XVIII.

configuration of the diffraction pattern with increasing values of $v$ can, therefore, be readily visualised. When $v$ is zero, we have only the central component. As $v$ increases, the first orders begin to appear and increase in intensity while the intensity of the central component steadily falls off. The second order spectra then begin to appear. With further increase of $v$, a stage is reached when the first order spectra are very conspicuous, the second order spectra fairly strong and the third orders begin to appear, while the central component has nearly vanished. For still larger values of $v$, the second orders are stronger than the first, while the central component has reappeared and the third orders are in fair strength. Further changes of the same general nature occur for larger values of $v$, the spectra fluctuating in intensity, and the higher orders gaining at the expense of the lower ones.

**Figure 33.** Relative intensities of spectra due to corrugated wave.

The relative intensities of the spectra for various values of $v$ from 0 to 8 are represented in figure 33 which is taken from a paper by Raman and Nath,* dealing with the somewhat analogous case of a beam of light which has passed through a liquid column carrying ultrasonic waves. As will be seen from the figure, spectra of increasingly higher orders continue to appear as $v$ becomes larger and take up the greater part of the energy, though some of the spectra of lower orders still have fair intensities. The fluctuations in the intensity of any particular order with varying $v$, and of the spectra of different orders with constant $v$ are both characteristic features of the case. The disappearance of particular spectra corresponds to the zeroes of the Bessel function of the orders concerned and furnish a sharp criterion for the value of the maximum phase-retardation which is operative. It is worthy of remark that the increase in the number of spectra and consequent enlargement of the angular width of the pattern with the increasing amplitude of the ripples, as shown by figure 33, roughly correspond with the increasing divergence of the rays on emergence from

*C V Raman and N S Nagendra Nath, *Proc. Indian Acad. Sci.*, 1935, **A2**, 406.



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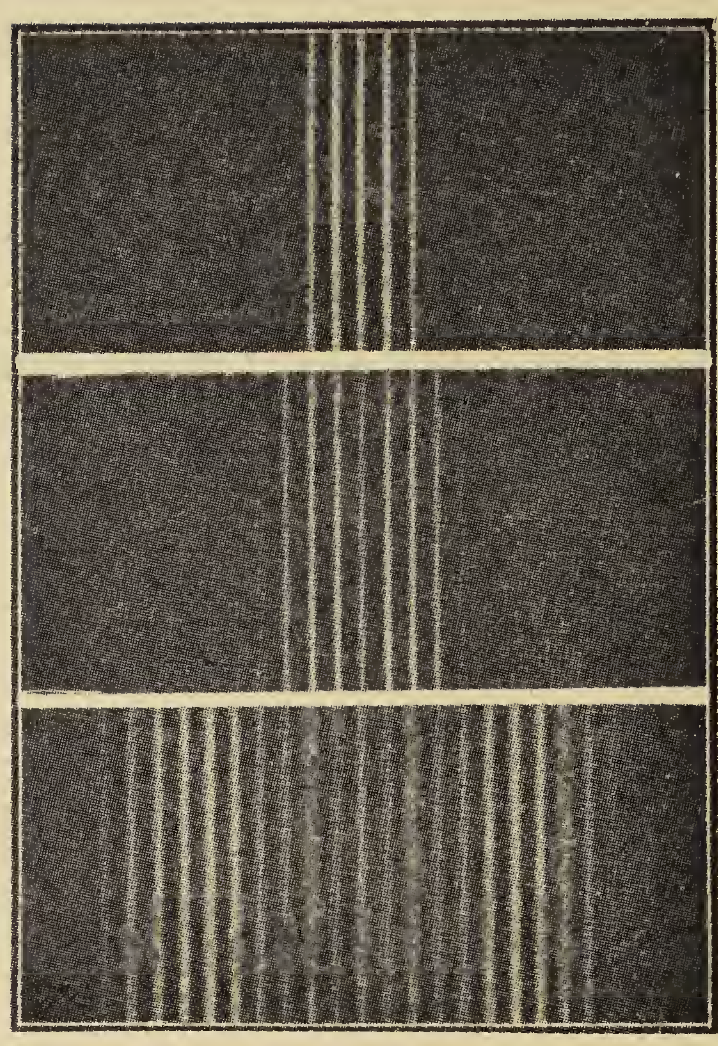

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**Figures 34 and 35.** Diffraction of light by ripples on water.

the liquid surface which is indicated in figure 32. Figures 34 and 35 reproduce a series of photographs* obtained with a ripple tank and exhibit a beautiful concordance with the theoretical results. It should be remarked that these pictures were obtained with *progressive* ripples, the boundaries of the tank being too far away to give a disturbing reflection. Theory indicates that the diffraction pattern by *stationary* ripples would be of a different character. This is readily understood, because the relative intensities of the different orders of spectra depend on the maximum retardation of phase produced by the ripples. This is a constant quantity for a progressive motion, but varies periodically with time for a stationary oscillation of the liquid surface. In the latter case, the diffraction pattern as recorded on the photographic plate would be a time average in which the special features depending on the particular value of $v$ would have been almost completely smoothed out. It is noteworthy that in either case, the diffraction patterns can be seen without stroboscopic aid. Indeed, stroboscopic illumination would make no difference in the diffraction pattern as observed with progressive ripples. With a stationary ripple pattern, however, the diffraction pattern seen would change with the phase at which the flashes of illumination are

*D S Subbaramiah, *Proc. Indian Acad. Sci.*, 1937, **A6**, 333.

given. By altering this phase, it should be possible to follow the changes in the structure of the pattern corresponding to all the values of $v$ from zero upwards to the maximum.

As the corrugation of the liquid surface produces changes of phase but no appreciable changes of amplitude in the light beam on its emergence, a microscope focussed on the surface would fail to give any indication of the existence of the ripples. When the plane of observation is moved away from the surface, however, alternations of light intensity would develop, which may be observed with stroboscopic aid for progressive ripples, and without such aid for a stationary ripple pattern. In the latter case, what we observe is an average intensity effect. The spacing of the pattern seen would therefore be $\lambda^*/2$, while for progressive ripples, the spacing would have the full value $\lambda^*$. As remarked earlier, the nature of the pattern would vary in a cyclic manner with the movement of the plane of observation. The spacing of the complete cycle is $2\lambda^{*2}/\lambda$ for a progressive wave, but if we ignore a lateral displacement of the pattern and consider only its general appearance, the period of the cycle would be one half of this, namely, $\lambda^{*2}/\lambda$. If, therefore, the plane of observation coincides with the liquid surface or is removed from it by a distance $\lambda^{*2}/\lambda$ or a multiple thereof, the ripples would be unobservable, while at intermediate positions, complex patterns would be seen the nature of which depends on the amplitude of the ripples. For stationary patterns seen without stroboscopic aid, the wavelength for the average intensity effect is effectively halved, so that the period of the cycle is only $\lambda^{*2}/2\lambda$. The general theory of visibility of periodic structures including the case of ultrasonic waves of optical gratings for which $\lambda^* \gg \lambda$ has been given by Nagendra Nath.†

*Fresnel and Fraunhofer patterns*: An application of the principle that the ray-optical and wave-optical descriptions of a light-field should be completely equivalent enables us to find the effect of restricting the aperture of a beam of light in any manner. To illustrate the essential features of the problem, we consider the comparatively simple case of the passage of light through a plane diffraction grating made up of parallel strips of equal width which are alternately transparent and opaque. Exactly as in the case of a corrugated refracting surface discussed earlier, we analyse the disturbance emerging from the grating into sets of plane waves travelling in various directions, starting from the disturbance at its surface as determined by its assumed properties.

The waves incident normally on the plane grating are represented by the expression $A \sin 2\pi(vt - z/\lambda)$. At the plane of the grating ($z = 0$), this reduces to $A \sin 2\pi vt$, and we assume that this is also the disturbance emerging from the transparent strips, while over the opaque strips the disturbance vanishes. The light-field thus described is periodic over the surface of the grating with

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†N S Nagendra Nath, *Proc. Indian Acad. Sci.*, 1936, **A4**, 262.

wavelength $\lambda^*$ and may be therefore represented by its Fourier expansion

$$\frac{1}{2}A\sin 2\pi\nu t + \sum_{s=1,3,5,\ldots} 2A\sin 2\pi\nu t\frac{\sin 2s\pi y/\lambda^*}{s\pi}.$$

We now reintroduce the co-ordinate $z$, and write the emerging disturbance in the form

$$\frac{1}{2}A\sin 2\pi(\nu t - z/\lambda) \pm \sum_{s=1,3,5,\ldots} \frac{A}{s\pi}\cdot\cos 2\pi(\nu t - z/\lambda \mp sy/\lambda^*).$$

The first term represents the undeviated plane waves of diminished strength emerging from the grating, while the others represent a series of diffraction spectra in which the even orders are missing. These spectra appear with equal amplitudes but with opposite phases in directions equally inclined to the primary beam on either side of it. The amplitudes of the diffracted plane waves are inversely proportional to the order of the spectrum, in other words, to the sine of the angle of diffraction. Considering the situation at the edges $b = 0, \pm\lambda^*, \pm 2\lambda^*$, etc., on the surface of the screen, we notice that the diffracted plane waves traverse each of these edges in various directions but in identical phases. The diffracted disturbances may therefore be regarded as made up of sets of *cylindrical* waves diverging normally from these edges with an amplitude inversely proportional to the sine of the angle of diffraction. A similar situation also presents itself as the intermediate edges $y = \pm\lambda^*/2, \pm 3\lambda^*/2$, etc., except that the phases are now reversed. It follows that the cylindrical diffracted waves diverging from an edge have *opposite* phases according as they appear on the illuminated or the dark side of it. The diffraction spectra emerging from the grating may be considered as the result of the interferences of these cylindrical waves. On this view, the non-appearance of the spectra of even order is a consequence of the cylindrical waves from the equidistant edges being alternately in opposite phases.

The result which thus emerges, namely, that the boundary between light and shadow at the edge of a screen is a source of diffracted radiation having opposite phases on its two sides, evidently does not depend on the particular disposition of the edges in the case considered nor on their being infinitely extended or straight. It is indeed valid generally, for edges of finite length as also for curved edges. Further, the assumption that the alternate strips are completely opaque is also not essential, since a sudden transition of any kind—in amplitude or phase or both—on the two sides of a boundary of arbitrary form in a light-field would give rise to similar effects. The recognition of these consequences of optical theory is the key to an understanding of the diffraction phenomena which arise from the passage of light through apertures or its obscuration by obstacles of arbitrary form and nature. They enable us to describe these phenomena in geometric terms related to the form of the apertures or obstacles. Further, they lead us naturally to an understanding of the more recondite aspects of diffraction theory, including especially the influence of the material and thickness of the screens, and the

configuration of the edges (viz., whether they are sharp, wedge-shaped or rounded-off) on the observed phenomena.

A procedure closely analogous to that described above may be adopted to find the effect of passage of light through a single slit. We start by assuming a series of parallel slits of width $a$ at regular intervals $\lambda^*$ and then pass to the limit when $\lambda^* \to \infty$. As in the case considered above, a Fourier expansion gives the result of the passage of the light through such a grating in the form of a series. It is readily shown that this may be written in the form

$$\frac{2Aa}{\lambda^*}\cdot\left[\frac{1}{2}+\sum_{s=1,2,3,\dots}(-1)^s\cdot\frac{\sin(\pi a\sin\theta_s/\lambda)}{(\pi a\sin\theta_s/\lambda)}\right],$$

where $\lambda^* \sin\theta_s = s\lambda$. As $\lambda^*$ tends to $\infty$, the diffraction spectra appear in more closely contiguous directions, and their number being great, we may neglect the first term in comparison with the rest. On squaring, we obtain an expression of the familiar type for the intensity in the diffraction pattern given by a rectilinear slit. A similar expression is also obtained if we consider the diffraction pattern as the result of the interference of the cylindrical waves having opposite phases emitted by the two edges.

The validity of the treatment of diffraction problems outlined above is naturally subject to certain restrictions and in particular, depends greatly on the exactness with which the configuration and properties of any actual screen reproduce those assumed for the purpose of the Fourier analysis. It is evident also that the results of the analysis would progressively tend to deviate from the facts as the size of the structures considered approaches the wavelength of light. For, we would then actually have a terminated sequence of spectra and not an infinite sequence as assumed. These difficulties become much more acute when we consider cases in which the light is incident very obliquely on the screens or apertures.

Instead of analysing the disturbance emerging from an aperture into sets of infinitely extended plane waves, we may follow a different procedure and express it as the summation of sets of spherical waves having their origins continuously distributed over the area of the aperture. This, in fact, is the familiar approach to diffraction theory based on the principle of Huygens due to Fresnel which is generally adopted in treatises on optics. In this treatment, the intensity at any point in the light-field is expressed as a surface integral taken over the illuminated area of the aperture and then evaluated. Apart from its historic interest and its usefulness in certain cases for purposes of computation, it is evident that this classic procedure has not much to recommend it from a physical point of view. It postulates secondary sources of radiation at points of space where there are no real sources and no material particles which can serve as secondary sources. Indeed, if we examine the matter closely, we find, on carrying out the summation of the effects of the postulated secondary sources, that they disappear from the picture, leaving only secondary radiations having their origins on the boundary

of the aperture. That this is the case will be shown a little later when we consider the problem of diffraction by apertures or obstacles with curvilinear boundaries.

*Diffraction by an equilateral aperture*: The fundamental role played by the boundaries of an aperture in determining the physical and geometric characters of its diffraction pattern may be suitably illustrated by a simple case, namely, that of an aperture bounded by three sharp straight edges in the form of an equilateral triangle.

The light from a point source passing normally through such an aperture placed at a distance from it and after diverging further falls on a photographic plate; the pattern thus recorded with short exposures is reproduced as figure 36 and with large exposure as figure 37. Figure 38 is a greatly enlarged reproduction of the pattern recorded when an image of the light source as seen through the aperture is brought to a focus on the photographic plate by a convex lens. On comparing figure 36 and figure 38, we notice that the former exhibits trigonal and the latter hexagonal symmetry. The complete dissimilarity of the Fresnel and Fraunhofer patterns which these pictures suggest is negatived by a comparative study of figures 37 and 38. We then realise that the fainter outlying parts of the Fresnel pattern which are recorded on a strongly-exposed plate present marked similarities with the features observed in the Fraunhofer patterns.

The Fresnel pattern illustrated in figure 36 and figure 37 may be completely described by the statement that in the strongly illuminated triangular area,

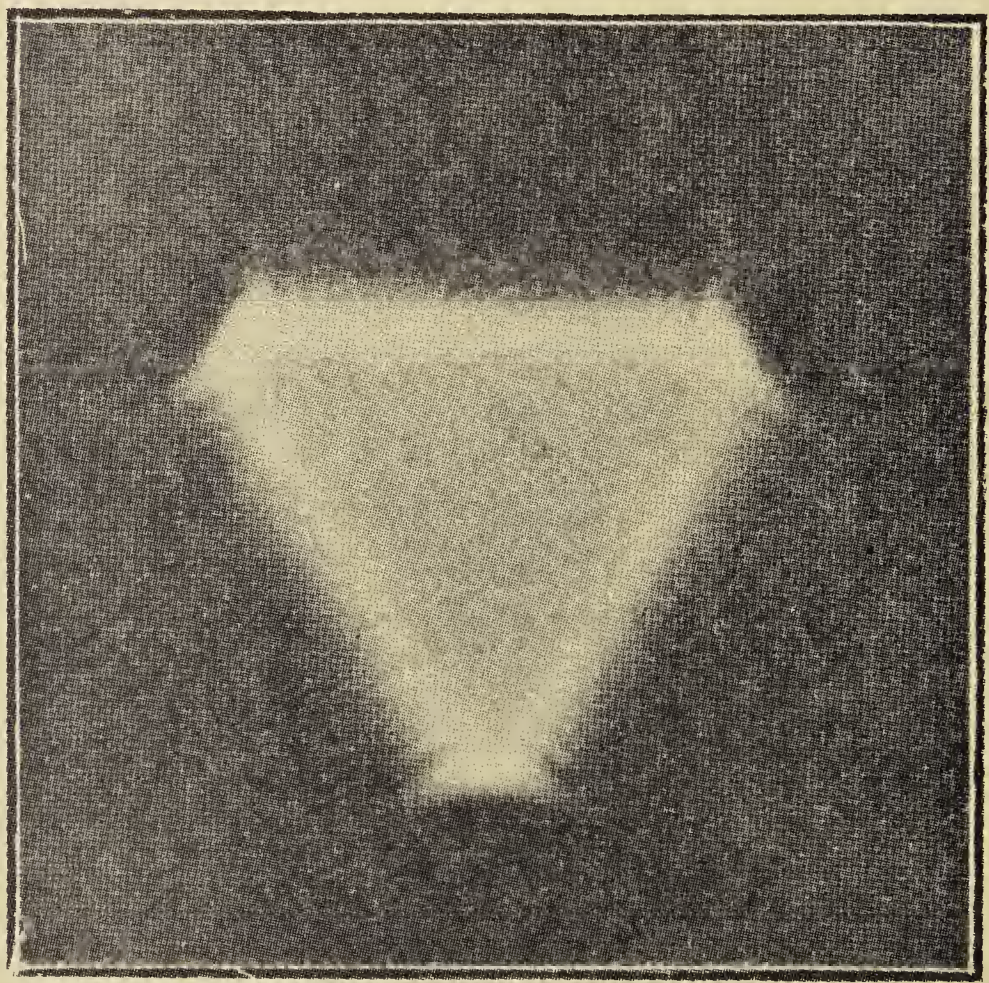

**Figure 36.** Fresnel pattern of equilateral aperture (weakly exposed photograph).

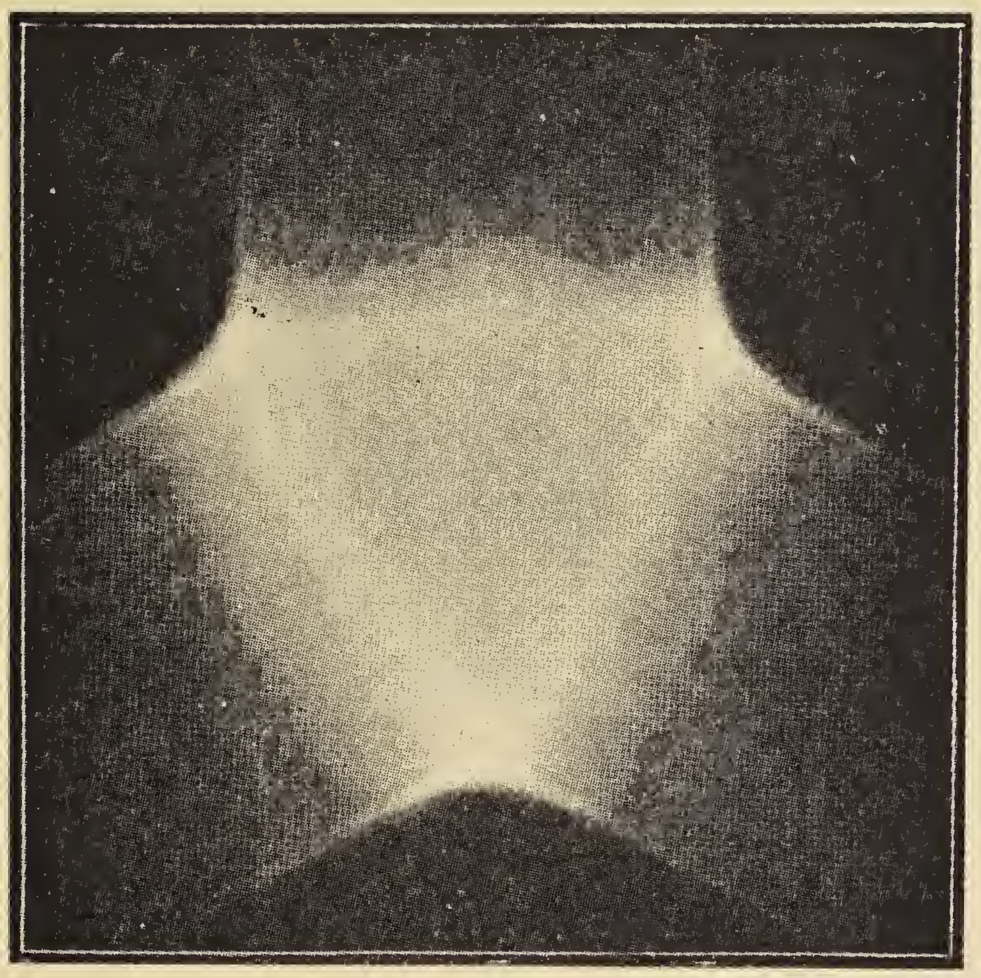

**Figure 37.** Fresnel pattern of equilateral aperture (strongly exposed photograph).

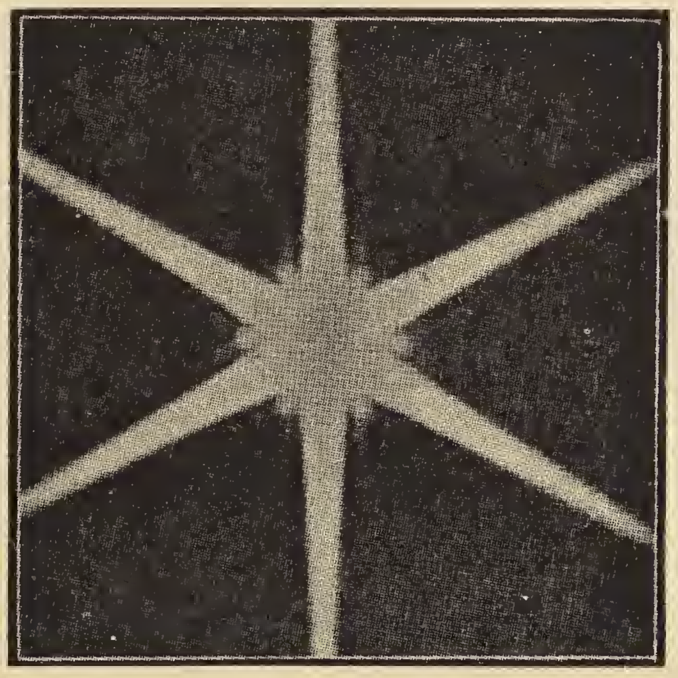

**Figure 38.** Fraunhofer pattern of equilateral aperture.

spherical waves of light passing directly through the aperture are superposed upon and interfere with the cylindrical waves diverging from each of its three edges, while in the fainter outlying regions, these cylindrical waves appear by themselves. The interference of the cylindrical waves from each edge with the primary spherical waves gives the bands of alternating intensity which are noticed running parallel to the same edge. Outside the central illuminated

area, an area of hexagonal shape is noticed where the cylindrical waves from all the three edges are seen superimposed. Further out still, we have a star-shaped hexagonal pattern where they appear superimposed in pairs; still further out, they appear individually as streamers. Running parallel to the edges of these faintly illuminated areas, we observe bands of alternating intensity which are most prominent in the vicinity of the vertices of the triangle; in this neighbourhood, they take the form of hyperbolic curves having their asymptotes normal to the sides of the triangle.

The Fraunhofer pattern illustrated in figure 38 may be described as the result of the collapse and disappearance of the central illuminated area in figure 37, consequent on the lens bringing the spherical waves which pass through the aperture to a geometric focus. In other words, the Fraunhofer pattern is due exclusively to the radiations from the edges of the aperture. Where the radiations from all the three edges are effectively superposed, we have the central hexagonal area of the pattern. Surrounding this, features are visible which are due to the cylindrical waves being superposed on each other in pairs and giving observable interferences. Further out still, the effect of each edge is observed by itself in the arms of the six-rayed star which forms the most conspicuous feature of the pattern. The rays of this star arise from the cylindrical waves diverging on either side of each edge; owing to the astigmatism of these waves, the broad streaks to which they give rise in figure 37 have contracted laterally (but not longitudinally) into the narrow streaks seen in figure 38. The fainter bands running parallel to the rays of the star are essentially similar in their origin to the bands running normal to the edges seen in the Fresnel pattern. *They arise from the interference with each other of the cylindrical waves from different parts of the same edge.* Along the central line of each ray of the star, the whole of the corresponding edge is effectively in the same phase. But when we move away from the central line, the radiations from different parts of the edge no longer agree in phase. We may then replace the line source parallel to the edge by point sources of diffracted radiation, one at each end; these give the interferences running parallel to the rays of the star. The Fraunhofer pattern may indeed with justice be described as arising from the interferences of radiations from three point sources placed respectively at the three vertices of the triangle. The amplitudes and phases of these radiations depend in a characteristic manner upon the angle of diffraction, thereby influencing the general appearance of the pattern to a notable extent.

*Diffraction caustics and foci*: We shall now proceed to consider the special phenomena exhibited by apertures and obstacles with *curvilinear* boundaries. It is useful in the first place to remark upon the relationship between the effects produced by an opaque obstacle and by an aperture in an opaque screen when they have the same form and situation relatively to the source of light. The illuminated areas being complementary, the sum of the effects in the two cases would everywhere be the same, being that due to the source itself. Accordingly, if

we subtract from the undisturbed effect of the light-source, that actually observed at any point in the shadow in one case, the difference would be the effect observed in the illuminated area in the other, and *vice versa*. The form of the boundary being the common feature, this reciprocal relationship indicates that we are dealing in both cases with the same physical phenomenon, namely, a diffracted radiation having its origin at the boundary. That this appears as a positive contribution in one case and as a subtractive effect in the other, or *vice versa*, is readily understood, since the boundary radiation has opposite phases on the shadowed and illuminated sides respectively.

We have now to define more precisely the character of the radiation from a diffracting edge of arbitrary form. Here again, the essential features of the case may be deduced from the simplest assumptions. Spherical waves are assumed to diverge from a point-source $Q$, and at a distance $D$ from it, meet an opaque screen covering the wave-front except over an aperture bounded by a curve of arbitrary form. We represent the disturbance emerging from the aperture as a summation of spherical waves having their origins distributed continuously over its area.* Accordingly, the disturbance reaching a point $P$ on a distant screen is written as the integral

$$\iint \frac{A}{\lambda R} \sin 2\pi(vt - R/\lambda) \mathrm{d}S,$$

where $\mathrm{d}S$ is an element of area on the aperture at the point $O$ (say), and $R$ is the length $OP$. It is readily shown that

$$\mathrm{d}S = Rd\, R\mathrm{d}\varepsilon \cdot D/PQ,$$

where $\varepsilon$ is the angle between the plane $OPQ$ and some fixed reference plane passing through $PQ$. Integrating with reference to $R$, we obtain

$$\int_0^{2\pi} \frac{A \cdot D}{2\pi \cdot PQ} \cos 2\pi(vt - R/\lambda) \mathrm{d}\varepsilon.$$

$R$ now signifies the distance from $P$ of points on the boundary of the aperture and is, therefore, to be regarded as a function of $\varepsilon$. The integration with respect to $\varepsilon$ from 0 to $2\pi$ may be written as an integration over a complete circuit of the boundary of the aperture; $\mathrm{d}s$ being an element of arc on the boundary, and $\phi$ the angle which it makes with the plane passing through $PQ$ and the element, we obtain

$$\int \frac{A \sin \phi}{2\pi R \sin \theta} \cdot \cos 2\pi(vt - R/\lambda) \mathrm{d}s,$$

*The variation of the amplitude of the assumed spherical waves with their direction of propagation is unimportant for our present purpose and is, therefore, ignored.

where $\theta$ is the angle between the incident ray reaching the element d$s$ and the diffracted ray starting from it and reaching the point of observation. Thus, it appears that *each line-element of the boundary is a source of diffracted radiation, its strength being inversely proportional to the sine of the angle of diffraction and directly proportional to the sine of its inclination to the plane of diffraction.*

The foregoing result is quite general and may be used to evaluate the intensity in both Fresnel and Fraunhofer patterns, it being always remembered that the integral represents the disturbance originating at the boundaries of the aperture or obstacle, and does not include the undisturbed effect of the light source. The latter, if present, must therefore be added to the expression. It will be noticed that the contribution from each element of the boundary becomes large and changes sign when $\theta$ passes through zero. It is this reversal of phase of the radiation from the edge that secures the observed continuity of the illumination when we pass from the region of shadow to the illuminated region in diffraction patterns of the Fresnel type. The *amplitudes* of the radiations received at any point in the field from different parts of the edges depend principally upon the angles $\theta$ and $\phi$, while their *phases* vary with $R$. It follows that the resultant effect would be contributed mainly by the parts of the edge for which $\theta$ is small and $R$ is a maximum or a minimum. The latter condition automatically ensures that $\phi$ is $\pi/2$ or $3\pi/2$ and $\sin\phi$ is therefore numerically a maximum. Ordinarily, therefore, the diffracted radiation has its origin principally at the point or points on the edge where this runs perpendicular to the rays reaching the points of observation. If, therefore, we draw a series of normals at various points on the geometric boundary between light and shadow on the receiving screen, these normals also define the directions in which the corresponding points on the edge are most effective in diffracting light.

In the language of geometrical optics, a focus is a point at which a set of reflected or refracted rays intersect, while a caustic is a curve to which a set of such rays is tangential. These concepts may evidently be applied *mutatis mutandis* to the rays diffracted by edges in the manner discussed above. Hence, provided the edges are sharp and smooth, besides being curved, we should be able to observe diffraction foci or diffraction caustics at the points where the normals to the geometric boundary of the shadow intersect or touch each other respectively.

Perhaps the best known example of a diffraction focus is the bright spot observed along the axis of the shadow of an opaque circular disc thrown by a point source of light. A smooth spherical obstacle also gives a similar bright spot, but with a notably different colour and intensity, for reasons which we shall consider more closely later.* It is worthy of remark that the edge of a circular *aperture* also gives a bright spot along the axis which may be rendered visible by blocking out the superposed illumination received directly through the aperture.

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*C V Raman and K S Krishnan, *Proc. Phys. Soc. London*, 1926, **38**, 350.

It should also be mentioned that an *incomplete* circular edge also gives a bright spot, though naturally of inferior sharpness and intensity.

The considerations set out above indicate that the evolute of the shadow of a curved boundary is the geometric figure along which its diffraction caustic is formed. That this is actually the case was demonstrated by observations with an elliptic boundary made by the present writer many years ago.* A circular opacity polished surface held so that it reflected light obliquely proved an excellent substitute for an elliptic aperture. Less satisfactory was a circular disc or a circular aperture with a sharp edge cut in a thin metal sheet and held obliquely. Qualitatively, the geometric evolute was found to be the locus of maximum intensity of the diffracted light. More exactly, the evolute represented the geometric limit within which the radiations from the concave part of the curved edge were concentrated. The curve of maximum intensity lay along and inside the evolute being accompanied by other and weaker interferences running parallel to it (see figure 39). These are due to the diffracted rays crossing each other at various small angles inside the evolute.

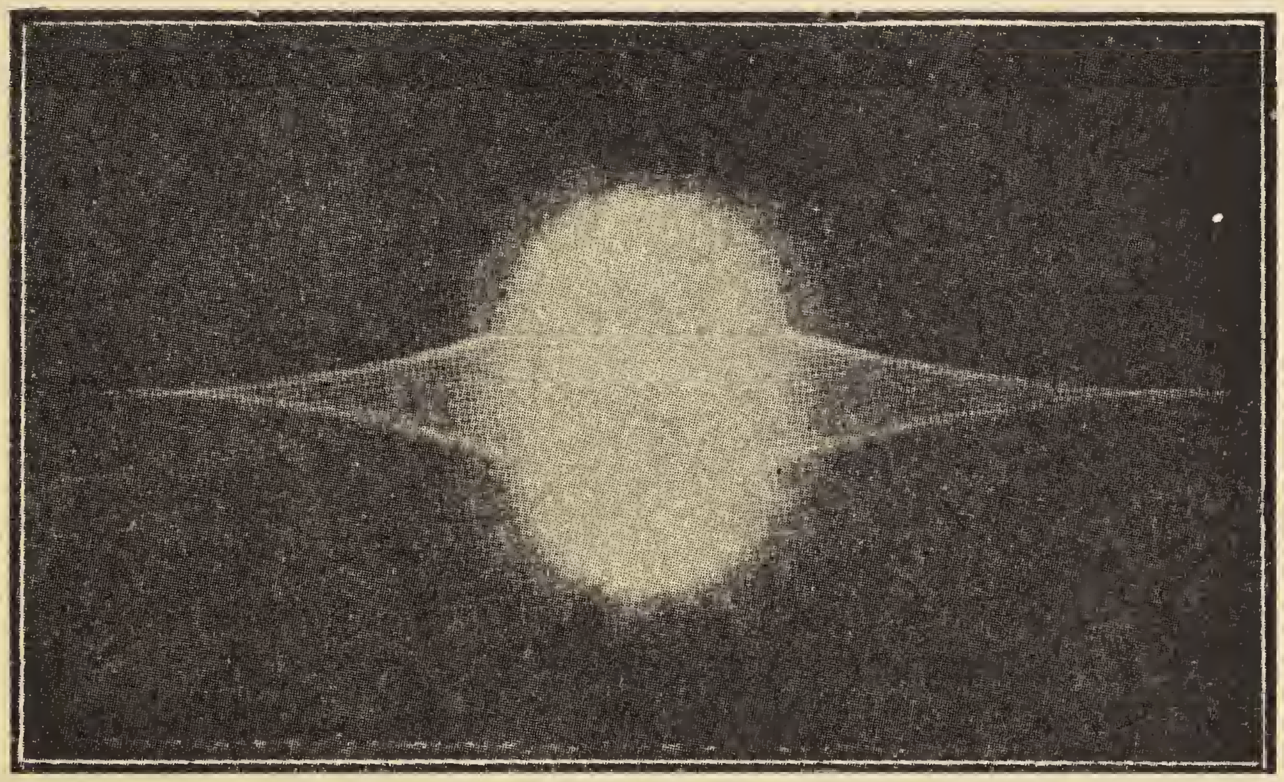

**Figure 39.** Diffraction caustic of elliptic aperture.

Figure 40 shows the Fresnel diffraction pattern given by an elliptic aperture in monochromatic light. The intensity in the diffraction pattern outside the elliptic area is mostly concentrated within its geometric evolute as indicated by theory. The pattern is crossed by two sets of interferences, one of them running parallel to the evolute and arising in the manner already explained, while the other set is

*C V Raman, *Phys. Rev.*, 1919, **13**, 259.

**Figure 40.** Fresnel pattern of elliptic aperture.

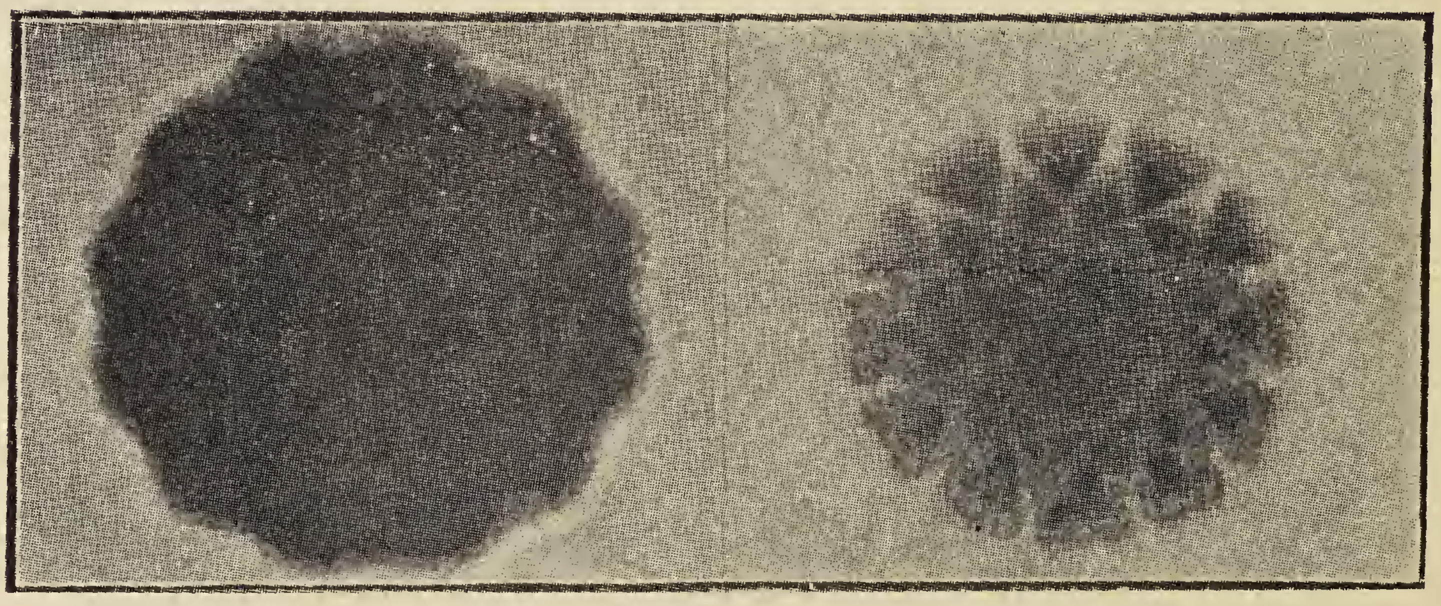

**Figure 41.** Diffraction by an undulating edge.

transverse to the evolute and arises from the interference of the effects due to the convex and concave parts of the edge.

Diffraction caustics are strikingly exhibited in the shadow of a disk with an undulating margin* (a nickel one-anna coin), two photographs of which taken with short and long exposures respectively are reproduced in figure 41. The diffraction pattern within the shadow which is recorded with the longer exposures follows the geometric form of the evolute of the undulating boundary. It exhibits a marked intensity at the cusps which may be regarded as foci, as well as a concentration of the luminosity along the branches of the evolute, with subsidiary interferences parallel to them. The effects in the vicinity of the cusps are very similar to those noticed when two branches of a caustic formed by refraction meet, e.g., in the case of an obliquely held transparent cylinder.†

*S K Mitra, *Philos. Mag.*, 1919, **38**, 289. Other cases of interest, notably that of a disk with milled edges are illustrated in this paper.

†T K Chinmayanandam, *Phys. Rev.*, 1918, **12**, 314.

*The heliometer diffraction figures*: The case of a semi-circular aperture has a special interest in its application to the form of the star-images seen in the heliometer; this instrument is a telescope with a divided objective, the two halves of which are capable of displacement relatively to each other along their common diameter. The case also offers an excellent illustration of the power of the geometrical method in discussing the configuration of Fresnel and Fraunhofer diffraction patterns and the relations between them. We shall here first consider the Fraunhofer pattern, and show how its geometric features may be deduced from the semi-circular form of the boundary.

We denote the angle between the incident and diffracted rays by $\theta$, and the complement of the angle which the plane containing these rays makes with the diameter of the semi-circle by $\psi$. When $\psi = 0$, the plane of diffraction is normal to the diameter; the radiations from the elements of the straight edge have then the maximum amplitude, while their phases are in agreement for all values of $\theta$. Accordingly, these radiations give a long bright streak in the pattern along a line perpendicular to the diameter. When $\psi$ is not zero, the phases disagree, and the interferences then arising would result in alternate minima and maxima of intensity running parallel to the streak along $\psi = 0$. As in the case of the equilateral aperture considered earlier, we may regard these interferences as due to point-sources of equal strength and opposite phases placed at the extremities of the edge. When $\psi = \pi/2$, the radiations from the straight edge vanish completely.

Considering now the curved part of the boundary, the amplitude of the radiations from a line element is a maximum at the point where the edge runs perpendicular to the plane of diffraction, and diminishes gradually to zero at the points where it runs parallel to that plane. The phases of the radiations from the line elements as received in the focal plane would, on the other hand, be stationary at the former point and alter at an increasing rate as we approach the two latter points. The resultant effect of all the line-elements may, therefore, be represented by that of a source placed at the point of stationary phase, supplemented by two equal sources located at the ends of the semi-circle. These latter vanish when $\psi = 0$, but become increasingly important as $\psi$ alters in either direction; in the limiting cases when $\psi = \pm\pi/2$, they represent the entire effect of the boundary.

Thus, in general the radiations from the straight and curved parts of the edge, taken together, may be replaced by the effect of three point-sources placed at the positions indicated in figure 42, namely, one on the curved edge and two at the corners of the aperture. The optical paths traversed by the radiations from these sources in reaching the focal plane would differ from each other by the three quantities $(\alpha + \alpha \sin\psi) \sin\theta$, $(\alpha - \alpha \sin\psi) \sin\theta$ and $2\alpha \sin\psi \sin\theta$, as will be seen from figure 42. If we denote $\alpha \sin\theta$ by $\zeta$, the quantities $\zeta$ and $\psi$ may be used to define the polar co-ordinates of a point in the focal plane. The interferences of the three sources, considered in pairs, would lie along the lines whose equations are

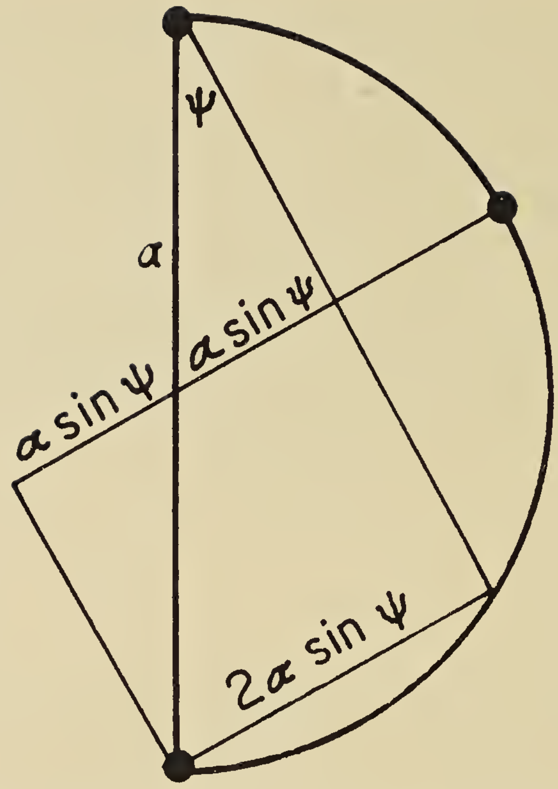


**Figure 42.** Diffraction by semi-circular boundary.

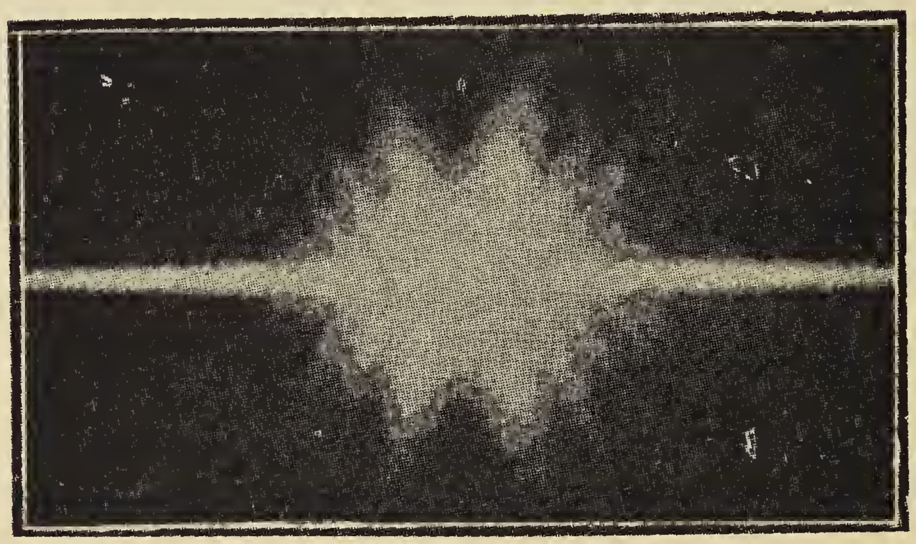

**Figure 43.** Fraunhofer pattern of semi-circular aperture.

$2\zeta \sin\psi = \text{constant}, (\zeta + \zeta \sin\psi) = \text{constant}$, and $(\zeta - \zeta \sin\psi) = \text{constant}$. The first equation represents the long streaks perpendicular to the diameter already mentioned, while the two latter represent sets of parabolae whose axes are *parallel* to the diameter of the semi-circle, but whose curvatures are oppositely directed with reference to the set of straight bands represented by the first equation. These features are beautifully exhibited by the photograph of the Fraunhofer pattern reproduced in figure 43. The three sets of interferences and their intersections completely determine the form of the pattern and the distribution of intensity in it.

As already remarked, the effect of the straight edge vanishes when $\psi = \pi/2$, while that of the semi-circle is equivalent to two sources placed at the ends of the diameter. The situation is then the same as for a complete circular boundary,

except that we have only one-half its effect. In other words, in the plane of the diameter, the pattern is identical with that of a complete circular aperture with the intensities reduced to one-fourth their values. When $\psi = 0$, the effect of the curved edge reduces to that of a single source placed at its midpoint; the interferences of this with the effect of the diameter would result in the straight bands due to the latter fluctuating in intensity along their length. These fluctuations diminish both relatively and absolutely, and ultimately disappear, as we move out along these bands. For, the elements of the straight edge remain in phase with each other, while those of the curved edge fall out of phase in an increasing measure as $\theta$ is increased. The effect of the latter, therefore, falls off much more rapidly than that of the former.

The line integrals over the straight and curved parts of the boundary are readily evaluated and by their summation, the intensity variations in the pattern may be found quantitatively. The former integral is

$$\int_{-a}^{a} A/2\pi f\cdot\sin\theta\times\cos(Z-2\pi s\cdot\sin\psi\cdot\sin\theta\cdot/\lambda)\cos\psi\,ds$$

$$=A\alpha/\pi f\cdot\cos Z\cdot\cos\psi/\sin\theta\cdot\sin\xi/\xi,$$

where $f$ is the focal length of the lens, $\xi$ stands for $2\pi\alpha\sin\psi\cdot\sin\theta/\lambda$ and $Z$ for $2\pi(vt-f/\lambda)$. The latter integral is

$$\int_{-\psi}^{\pi-\psi} A/2\pi f\cdot\sin\theta\times\cos(Z-2\pi a\sin\theta\cdot\sin\phi/\lambda)\sin\phi\cdot a d\phi.$$

When $\psi$ is equal to $\pm\pi/2$, this reduces to

$$A\cdot\frac{\pi a^2}{f\lambda}\cdot\frac{J_1(2\pi a\sin\theta/\lambda)}{(2\pi a\sin\theta/\lambda)}\cdot\sin Z,$$

which is half the value for a complete circle. The effect of the semi-circular arc is evaluated very simply for the case when $\psi = 0$. For, the point-source to which it is then equivalent must be such that together with an equal source at the opposite end of a diameter, it should give the effect of a complete circle. Using the well known semi-convergent expansion for the Bessel function $J_1(x)$, the amplitudes and phases of the sources may be so fixed that their interferences give everywhere the required intensity.*

Comparing the photographs reproduced as figures 43 and 44 respectively, we notice that the Fraunhofer pattern has both a horizontal and a vertical axis of symmetry, while the Fresnel pattern has only the former. This is an example of the general theorem that a Fraunhofer pattern always exhibits centro-symmetry, while the Fresnel pattern has no higher symmetry than the aperture itself. The

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*S K Mitra, *Proc. Indian Assoc. Cultiv. Sci.*, 1920, **6**, 1.

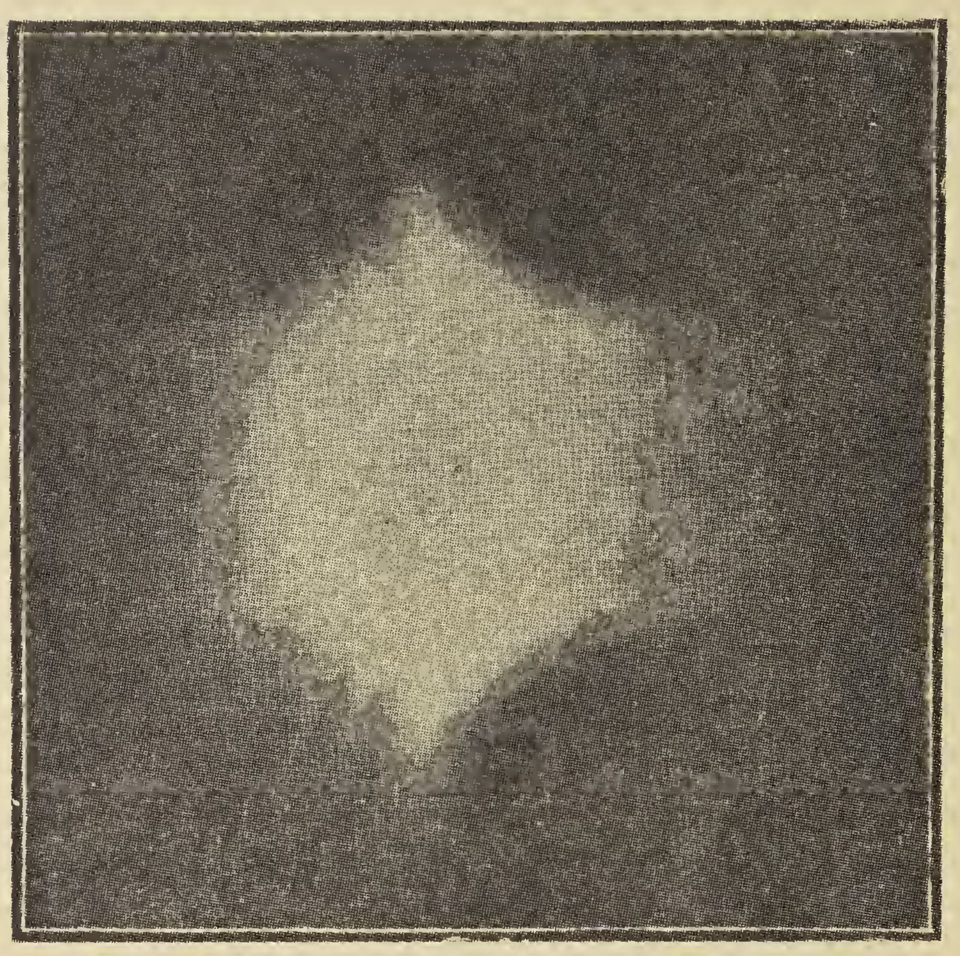

**Figure 44.** Fresnel pattern of semi-circular aperture.

centro-symmetry of Fraunhofer patterns is explained by the circumstance that they are due exclusively to the boundary radiations, the angle of diffraction vanishing at the centre of the pattern for all the elements of the edge; this is not the case for Fresnel patterns.

Various features are noticeable in figure 44 which may be explained by considering the radiations from the diffracting boundary and their mutual interferences. Those from the straight edge appear on both sides of the pattern as horizontal bands. Those from the curved edge are divergent on the left, but on the right converge to a focus and then diverge again. If longer exposures had been given, streamers would have been recorded diverging from the focus towards the right in various directions. The lack of symmetry of the Fresnel pattern about a vertical axis would then have been somewhat less conspicuous. The transverse bands seen in the fainter regions on both sides of the pattern arise from the interferences of the effects of the straight and curved parts of the boundary.

*Diffracting apertures in the Foucalt test*: The general view of diffraction phenomena outlined in the preceding pages is strikingly confirmed and illustrated by the effects noticed when an illuminated aperture is observed by the aid of the light diffracted by itself. To enable this, an achromatic lens of good quality is employed to form a focussed image of a point source of light. An aperture of the chosen form is placed immediately after the lens and restricts the area of the emerging beam. After reaching a focus, the light enters a second achromatic lens which forms an image of the illuminated aperture on a distant screen. The character of this image

is found to be greatly influenced by restricting the aperture of the second lens in any manner, the size and position of the openings in the focal plane through which the light enters the second lens determining the nature of the effects observed. The phenomena noticed are readily interpreted on the view that the light forming the Fraunhofer pattern in the focal plane of the first lens has its origin on the edges of the diffracting aperture. By varying the disposition of the openings which admit the light into the second lens, effects may be exhibited which demonstrate the various characters of the edge radiation to which reference has already been made, viz., the sudden reversal of its phase in passing from interior to exterior diffraction, the dependence of its intensity on the angle of diffraction and the concentration of the intensity at special points on the edge, namely, the poles of the point of observation and sharp terminations on the edge.

When the opening of the second lens is not restricted in any manner, it gives an image of the illuminated aperture reproducing its form with full definition. A remarkable transformation of the image occurs when a small obstacle is placed at the region of maximum intensity in the Fraunhofer pattern, so as to block it out. The image then formed is a delineation of the boundaries of the aperture, the area within the boundaries appearing more or less perfectly dark. It should be remarked, however, that the edges of the aperture do not appear as bright lines in the image so formed. On the contrary, they appear as *perfectly dark lines* bordered by alternately bright and dark fringes on either side, provided the aperture of the second lens is symmetrically disposed with reference to the centre of the pattern. The reason for this fact is readily understood. As was remarked earlier, the Fraunhofer pattern is centro-symmetric by virtue of the interior and exterior diffraction by every element of the edge being of equal intensity but of opposite phases. Hence, when the light admitted into the second lens includes both interior and exterior diffraction to equal extents, the two radiations cancel each other by interference and given an image of zero intensity; the diffracted light appears in the interference fringes bordering the image of the edge on either side. If, on the contrary, the aperture of the second lens is not centro-symmetric but admits the light only on one side of the focus, this would no longer be the case. The image of the edge then appears as *a bright line* bordered by alternate dark and bright fringes.

When the second lens is completely covered except for a small opening which admits the light reaching some chosen point in the pattern, the luminosity of the edge is, in general, observed only at the poles of this chosen point. Taking, for instance, the case of a circular aperture, only two bright spots are seen, one at each end of a diameter. The intensity of these spots falls off with the increasing angle of diffraction as the opening is moved away from the focus. It is desirable that, in such observations, the opening used is not either too small or too large; in the former case the image becomes weak and diffuse, and in the latter case, the radiations from too great a length of the edge are admitted. Even when a small opening is used, the spots at the two ends of the diameter of a circular aperture

lengthen into bright arcs and finally appear as semi-circles when the opening is brought close to the focal point. In this limiting position, the edge appears brightest where it is perpendicular to the plane of diffraction and is of zero intensity where it is parallel to this plane, as required by theory. Observations of this kind serve to illustrate the geometric theory of diffraction discussed in the preceding pages. An aperture of polygonal form, for instance, exhibits a luminous point at each of its corners which brightens and merges with the luminosity of the edge observed when the opening in the focal plane lies on the corresponding streamer in the pattern. A semi-circular aperture exhibits, except in special cases, three bright spots on its boundary in the positions indicated by figure 42, the relative brightness of these spots varying greatly with the position of the opening. In the particular case when the chosen position is on the horizontal axis of symmetry (figure 43), the entire diameter appears luminous, while if it is on the vertical axis, only two bright spots are seen at the ends of the diameter, as in the case of a complete circular aperture.

Figures 45(*a*) and 45(*b*) are photographs of apertures as seen by diffracted light,

(*a*) (*b*) (*c*) (*d*)

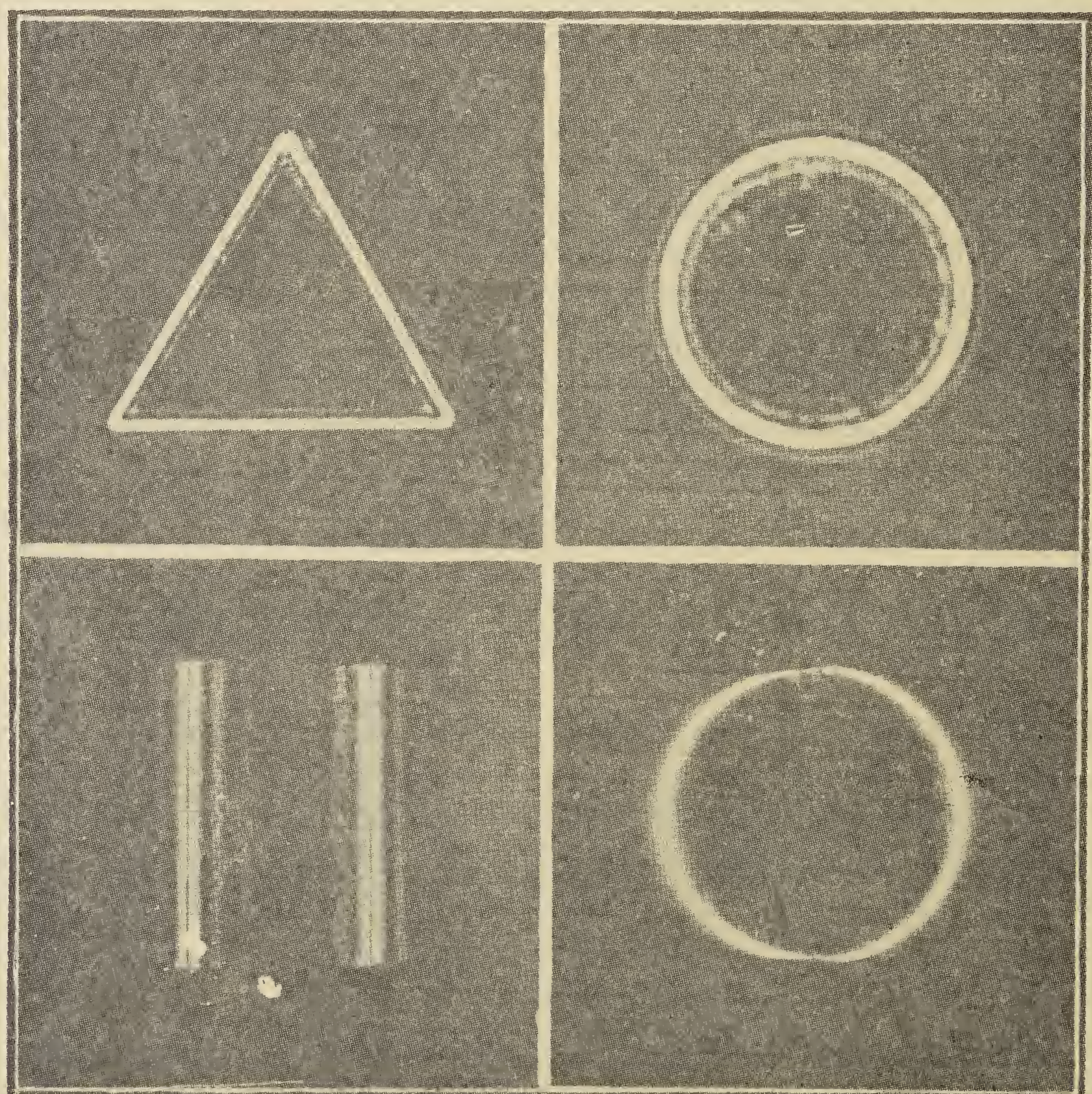

**Figure 45.** Diffracting apertures in the Foucalt test.

**Figure 46.** Circular aperture in the Foucault test.

the central rays being blocked out at the focus by a small sphere of wax; the edges appear in each case as dark lines bordered by bright lines. Figures 45(*c*) and 46 represent the appearance of the respective apertures when the direct rays are blocked out by a fine wire stretched across the Fraunhofer pattern through the focus. The broad fringes seen on either side of the edge in these two figures are in striking contrast with the extreme sharpness with which the geometric form of the edge itself is recorded as a dark line. These features become intelligible when it is recalled that the breadth of the diffraction fringes depends on the diameter of the wire stretched in the focal plane, while the whole effective aperture of the second lens determines the sharpness with which the edge is depicted. Figure 45(*d*) represents a circular aperture as seen in the usual form of the Foucault test, a knife-edge cutting off the light at the focus. It will be noticed that the edge now appears as a bright curve which is intense on the horizontal diameter and fades off towards the vertical.

The theory of the effects observed in the Foucault test is readily worked out in the relatively simple two-dimensional case* of a rectangular aperture. The light vector at any point in the focal plane of the first lens is, in this case, proportional to

$$2 \sin Z \cdot \sin (2\pi/\lambda \cdot \alpha \cdot x)/x,$$

where $\alpha$ is the semi-angular aperture of the opening which limits the area of the beam, and $x$ is the co-ordinate of any chosen point in the focal plane. We consider this disturbance as effective over the aperture of the second lens, and to find the effect due to it at the focal plane of the latter, we integrate it over the area, paying due regard to the phase differences which arise between the effects due to the elements of this area when the observation is in a direction making an angle $\beta$

*Rayleigh I, *Scientific Papers*, **6**, 455.

with the axis of the original light beam. The integral to be evaluated is

$$2\int dx\cdot\sin(Z + 2\pi/\lambda\cdot\beta\cdot x)\sin(2\pi/\lambda\cdot\alpha\cdot x)\cdot/x,$$

which may be written as

$$\sin Z\left\{\int dx\cdot\sin 2\pi/\lambda\cdot(\alpha+\beta)x/x + \int dx\cdot\sin 2\pi/\lambda\cdot(\alpha-\beta)x/x\right\}$$
$$+\cos Z\left\{\int dx\cdot\cos 2\pi/\lambda\cdot(\alpha-\beta)x/x - \int dx\cdot\cos 2\pi/\lambda\cdot(\alpha+\beta)x/x\right\}.$$

The integrals have to be taken between the limits $x_1$ and $x_2$ of the aperture covering the second lens. They reduce to the well known Si and Ci functions of which complete tables are available.* By assuming particular values for $x_1$ and $x_2$, the distribution of intensity in the focal plane of the second lens may be numerically worked out. In the case of the simple knife-edge test, for example, $x_2$ would be taken large and $x_1$ small, both having the same sign. The calculations show† that when the edge has covered only half the central band, the luminosity at the edges of the aperture is already six times greater, and when it has covered the whole of the central band, about 80 times greater than the illumination at the centre of the aperture. The computations also show that the luminosity of the areas between the edges alternately increases and diminishes, the successive maxima becoming smaller, as the knife-edge advances. The fluctuations of colour which are observed simultaneously over the whole area of the aperture are thus satisfactorily explained. When the knife-edge traverses an ultra-focal plane of the first lens, the two edges of the aperture appear of different intensity—as is to be expected, since we are then dealing with the Fresnel pattern of the aperture.‡ The great brightness and sharpness of the luminosity at the edges appear in these calculations as mathematical consequences of the behaviour of the Ci function which falls very steeply to an infinite negative value when its argument tends to zero. The intensity thus becomes very great in the directions $\alpha = \pm\beta$, provided $x_1$ is small and $x_2$ is large, both being of the same sign. On the other hand, when the apertures are symmetrically disposed, the Ci's being even functions of the argument cancel each other in the expressions for the intensity, while the Si's which are odd functions vanish for a zero value of the argument and reach a finite limiting value for large arguments. That the edges then appear as dark lines in the directions $\alpha = \pm\beta$ is thus readily explained, as also the sharpness of these lines when the full aperture of the second lens is operative. The case of a circular

*Jahnke–Emde, *Tables of Functions*, 1933, p.83.
†Rayleigh I, *loc. cit.*
‡S K Banerji, *Astrophys. J.* 1918, **48**, 50.

aperture can very similarly be dealt with in terms of the Si and Ci functions when the aperture of the second lens has a symmetric opening.*

*Diffraction by sharp metallic edges*: The edge of a razor-blade held in a beam of light is observed to diffract light through large angles, appearing as a luminous line, when viewed either from within the region of shadow or from the region of light. The light thus diffracted is found to be strongly polarised, but in perpendicular planes in the two regions. Gouy, who was the first to notice these effects, experimented with edges of various metals and discovered that both the colour of the light diffracted into the shadow and its state of polarisation depend in a remarkable manner on the material of the edge and on the extent to which it has been rounded off in the process of polishing. When viewed through a double-image prism from within the shadow, only that image of the edge appears coloured which is more intense and is polarised with the magnetic vector parallel to the edge. The second image which is fainter and is polarised with the electric vector parallel to the edge, appears perfectly white. When the incident light is polarised in any arbitrary azimuth, the diffracted light is, in general, elliptically polarised. The explanation of these phenomena will now be discussed.†

Earlier in this lecture, it was shown from theoretical arguments that the edge of an opaque screen functions as a source of cylindrical waves; these diverge both into the region of shadow and into the region of light, but in opposite phases in the two regions, their amplitude being inversely proportional to the sine of the angle of diffraction. In establishing this result, the edge was regarded as a line of discontinuity between the area in which the full effect of the incident primary waves is present and the area in which it is completely cut off. An aperture with sharp edges in a very thin sheet of metal makes a fair approach to the situation here contemplated. It should be remarked, however, that such a sheet would also reflect backwards such of the light falling on it as is not absorbed. The boundary of such an aperture is, therefore, a discontinuity in two distinct ways, firstly in respect of the light passing through it, and secondly in respect of the light reflected by the surrounding area. Diffraction effects have, therefore, to be considered arising out of these two distinct processes. The reflection of light was ignored in our earlier discussions, but if it is taken into account, the colour and polarisation effects discovered by Gouy find a natural explanation.

We shall, in the first instance, consider the case of a plane metallic screen bounded by a straight edge. Plane waves of light travelling towards the edge and incident on it in the direction $\phi = \phi_0$ (see figure 47) may be represented by the real part of the expression

$$\exp[ik\rho\cos(\phi - \phi_0) + ikct].$$

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*S K Banerji, *Philos. Mag.*, 1919, **37**, 112.

† C V Raman and K S Krishnan, *Proc. R. Soc. London*, 1927, **A116**, 254.

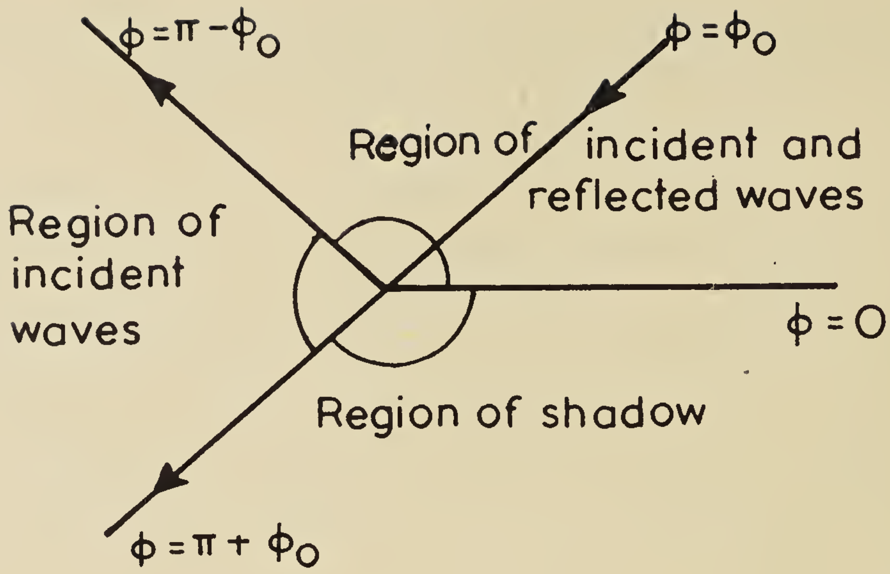


**Figure 47.** Reflection by half-plane.

The waves reflected by the upper surface of the screen and receding from it are given by the real part of

$$-(C_s + iD_s)\exp[ik\rho\cos(\phi + \phi_0) + ikct]$$

or of

$$+(C_p + iD_p)\exp[ik\rho\cos(\phi + \phi_0) + ikct].$$

The complex amplitudes for the reflected wave are those characteristic of the metal for the particular angle of incidence. The alternatives refer to the cases in which the electric and magnetic vectors respectively are parallel to the edge of the screen for both the incident and the reflected waves.

We have now to find expressions for a set of cylindrical waves radiated from the edge of the screen, the superposition of which on the incident and reflected wave-trains would give in the vicinity of the planes $\phi = \pi + \phi_0$ and $\phi = \pi - \phi_0$, effects which are the same as those indicated by elementary theory, but which would be valid also for large angles of diffraction. These requirements are met if we multiply the foregoing expressions by the factor $S$ given by the equation

$$S = \frac{\exp(i\pi/4)}{\sqrt{\pi}} \int_{-\infty}^{\sigma} \exp(-i\sigma^2)\mathrm{d}\sigma,$$

where

$$\sigma = \sqrt{2k\rho}\cos\tfrac{1}{2}(\phi - \phi_0), \quad \text{for the incident waves}$$

and

$$\sigma = \sqrt{2k\rho}\cos\tfrac{1}{2}(\phi + \phi_0), \quad \text{for the reflected waves.}$$

It is easily verified that the resulting products are solutions of the wave-equation in cylindrical co-ordinates. $S$ tends to different values, namely, unity and zero, when $\sigma$ is positive and negative respectively and is sufficiently large. The result of the multiplication is, therefore, to restrict the appearance of the incident and reflected wave-trains to the particular regions indicated in figure 47. The

multiplier $S$ is, however, not *exactly* unity or zero as the case may be; its *difference* from these values is given with all needful accuracy by the following expression (obtained by integration in parts):

$$-\frac{\exp(i\pi/4)}{\sqrt{\pi}}\cdot\frac{\exp(-i\sigma^2)}{2i\sigma}.$$

Substituting the values of $\sigma$ in this expression and multiplying it with the expressions for the incident and reflected wave-trains, we obtain finally, by addition, the result

$$\frac{1}{4\pi}\sqrt{\frac{\lambda}{\rho}}\cdot\exp[-i(k\rho-kct+\pi/4)]\cdot\left[\frac{1}{\cos\frac{1}{2}(\phi-\phi_0)}+\frac{C_s+iD_s}{\cos\frac{1}{2}(\phi+\phi_0)}\right].$$

or

$$\frac{1}{4\pi}\sqrt{\frac{\lambda}{\rho}}\cdot\exp[-i(k\rho-kct+\pi/4)]\cdot\left[\frac{1}{\cos\frac{1}{2}(\phi-\phi_0)}+\frac{C_p+iD_p}{\cos\frac{1}{2}(\phi+\phi_0)}\right].$$

These expressions represent cylindrical waves diverging from the edge and having an amplitude varying with the direction of the observation, the angle of incidence of the light and its state of polarisation, and also dependent on the optical constants of the metal. The latter appear in the complex reflection amplitudes which are given by the formulae

$$-(C_s+iD_s)=\frac{a-\cos\psi}{a+\cos\psi}$$

$$+(C_p+iD_p)=\frac{\omega^2\cos\psi-a}{\omega^2\cos\psi+a},$$

where $\psi$ is $(\pi/2-\phi_0)$, $\omega=n(1-ik)$ and $a^2=(\omega^2-\sin^2\phi)$; $n$ and $k$ are the index of refraction and the absorption coefficient, respectively, of the metal. It will be seen that the cylindrical waves have large amplitudes near the boundaries of the geometric shadow and of the geometric reflection, and that the phases of the components which become large are reversed in crossing these boundaries. Further, the components bear the same relation of amplitude and phase to their respective parent waves. Their interference with these waves would, therefore, correctly describe the diffraction fringes observed in the vicinity of these boundaries. The expressions being also valid for the large angles of diffraction, they contain within themselves the explanation of the experimental facts concerning the intensity, colour and state of polarisation of the light diffracted by sharp edges.

The foregoing treatment of the problem of diffraction by a straight edge is based upon Sommerfeld's solution for the physically unrealisable case of a perfectly reflecting half-plane, but differs from it in taking the actual properties of the screen into account. It might be remarked that a metallic knife-edge is more

appropriately regarded as a wedge than as a half-plane. The theoretical expressions (due to Poincare and others) for a perfectly reflecting wedge may, however, be modified in the same way by considering the actual reflecting power of the material of which it is made. Taking the wedge to be bounded by the surfaces $\phi = 0$ and $\phi = \chi$, the edge effect is given by formulae similar to those for a half-plane, $1/4\pi$ being replaced by $\sin(\pi^2/\chi)/2\chi$, and the expressions within the square brackets by

$$\left[\frac{1}{\cos\dfrac{\pi(\phi-\phi_0)}{\chi}-\cos\dfrac{\pi^2}{\chi}}-\frac{C_s+iD_s}{\cos\dfrac{\pi(\phi+\phi_0)}{\chi}-\cos\dfrac{\pi^2}{\chi}}\right]$$

or by

$$\left[\frac{1}{\cos\dfrac{\pi(\phi-\phi_0)}{\chi}-\cos\dfrac{\pi^2}{\chi}}+\frac{C_p+iD_p}{\cos\dfrac{\pi(\phi+\phi_0)}{\chi}-\cos\dfrac{\pi^2}{\chi}}\right]$$

as the case may be. Writing $\chi = 2\pi$, we immediately regain the preceding expressions for a half-plane.

The new feature introduced by considering the optical properties of the screen is that the edge radiation appears as a summation of two distinct effects. For normal incidence ($\psi = 0$), the components due to reflection are numerically identical but are different in sign for the two possible states of polarisation. If, therefore, the incident light be unpolarised, the light diffracted through large angles would be partially polarised, and this effect would be most marked in directions nearly parallel to the surfaces of the screen or wedge, since the quantities added or subtracted would then be nearly equal numerically. The electric vector parallel to the edge would be the stronger component in the region of light or exterior diffraction, and would be the weaker component in the region of shadow or interior diffraction.

The incident light being white, the colour of the radiation diffracted by the edge would be determined by two considerations. The wavelength appears as a factor in the expression for the intensity, and thus would tend to make the diffracted light reddish, irrespective of the state of polarisation of the incident radiation. The colour would also be influenced, and in quite a different way, by the appearance of the optical properties of the metal explicitly in the expressions. Since the amplitudes of reflection occur with opposite sign in the two cases, the wavelengths strengthened in one case will be weakened in the other, and *vice versa*. Thus, in the region of shadow, the colours for which the reflecting power of the metal is greater would appear strengthened in the electric vector perpendicular to the edge and weakened in the electric vector parallel to the edge. This effect would increase with the angle of diffraction, so much so that the favoured colours would become more and more marked in one component, and less and less marked in the other component, the further we proceed into the

region of shadow. Similar effects should occur in exterior diffraction, but with the parallel and perpendicular components exchanging places in respect of both colour and polarisation.

If the incident light be plane-polarised in an azimuth inclined to the edge, the diffracted light would exhibit a rotation of the plane of polarisation as well as ellipticity, the latter appearing as a consequence of the change of phase in metallic reflection. From what has already been remarked about the effects observed when the incident light is unpolarised, it follows that both the rotation and ellipticity would depend on the nature of the screen, and in the case of strongly coloured metals would be notably a function of the wavelength of the light. The determination of these two quantities gives us both the ratio of intensities of the parallel and perpendicular components and the difference of phase between them, and would, therefore, enable a more stringent test of the theory to be made than would be possible when incident unpolarised light is employed.

The results of the foregoing theory are in general agreement with the original observations of Gouy. Quite recently, the subject has been very carefully investigated by M Jean Savornin,* in whose memoir will be found also a complete bibliography of the subject and a detailed discussion of the earlier results of other workers. Savornin has quantitatively studied the phenomena and compared them with the results expected theoretically on the assumption that the screens are perfectly conducting half-planes or wedges, as also according to the modified formulae in which the optical properties of the material are taken into account. The evidence is decisively in favour of the latter procedure. Indeed, Savornin's data for the variation with the wavelength of the rotation of the plane of polarisation and the ellipticity of the light diffracted by a razor edge, and by the same when covered with gold by cathodic deposition, show such a striking resemblance with the curves deduced from the theory and the known optical properties of steel and gold respectively, as to leave no room for doubt of the essential correctness of our formulae. The ellipticity for a steel edge is found to be small and to diminish towards the red end of the spectrum, while the rotation increases at the same time, though only slowly. In the case of gold, the ellipticity shows a pronounced maximum at about 5,200 A.U., dropping off to smaller values at both smaller and longer wavelengths, while the rotation exhibits a minimum at about 4,900 A.U., and rises very steeply towards the red end of the spectrum. These observations are in full accord with the indications of our theory.

The formulae are equally successful in other respects. They completely explain the striking difference in colour of the two components of the diffracted light. They account for the rapid increase in the rotation of the plane of polarisation and in the ellipticity produced by tilting the plane of the diffracting edge away from the symmetric position in one direction or the other, the deviation of the

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*J Savornin, *Ann. Phys.*, 1939, **11**, 129.

diffracted rays remaining constant. They also explain in a quantitative manner the variation of these quantities and of the intensity of the diffracted light with the angle of diffraction. In making the comparison between theory and experiment, it is necessary to consider not only the optical constants of the material, but also the angle of the wedge, since the latter is found to influence the results very markedly, especially when this angle is at all considerable. It is also necessary to remember that the theory is strictly applicable only in the case of a perfectly sharp and straight edge—a state of affairs to which the observations show that a fresh razor blade may be a remarkably good approximation. It may be mentioned that the optical phenomena exhibited by such an edge are so striking and so easily observed that they should be personally familiar to every student of optics.

*Diffraction by semi-transport laminae*: As is well known, metals are ordinarily opaque to light, but in extremely thin layers transmit light. By cathodic deposition, or preferably by evaporation in vacuum, it is possible to obtain metal films of controlled thickness and consequently of any desired degree of transparency. By protecting part of the surface of a plate of glass or quartz by a razor edge held obliquely in contact with it, it is possible to coat the plate with a metallic film partially transmitting light and terminating in a straight edge,* leaving the rest of the plate clear. The same technique is also applicable for obtaining non-metallic films by evaporation in vacuum. The diffraction patterns of the Fresnel class due to semi-transparent laminae bounded by a straight edge exhibit interesting features not observed either with opaque screens or with transparent laminae. In the familiar diffraction pattern due to an opaque screen with a straight edge, we have a few fringes of rapidly diminishing visibility running parallel to the edge in the region of light, and a continuous illumination falling off steadily to zero intensity in the region of shadow. A semi-transparent film, on the other hand, exhibits a great number of interference fringes in the region of shadow, besides the fringes of moderate visibility in the strongly illuminated region. As the thickness of the film is increased and its transparency thereby diminished, the first few fringes in the region of shadow become less clear, while the subsequent ones gain visibility in spite of the diminished intensity of light in the field. The fringes having the maximum visibility move further into the region of shadow with increasing thickness of the film, until ultimately they disappear altogether. Another interesting feature is that the appearance of these patterns alters greatly with the colour of the light. This is a consequence of the marked variation of the transparency of the films with the wavelength.

Figure 48 reproduces the Fresnel fringes due to the edge of a semi-transparent silver film. To enable the effects observed on both sides of the edge to be seen simultaneously, a photometric wedge was set across the pattern when it was

*N Ananthanarayanan, *Proc. Indian Acad. Sci.*, 1939, **A10**, 477.

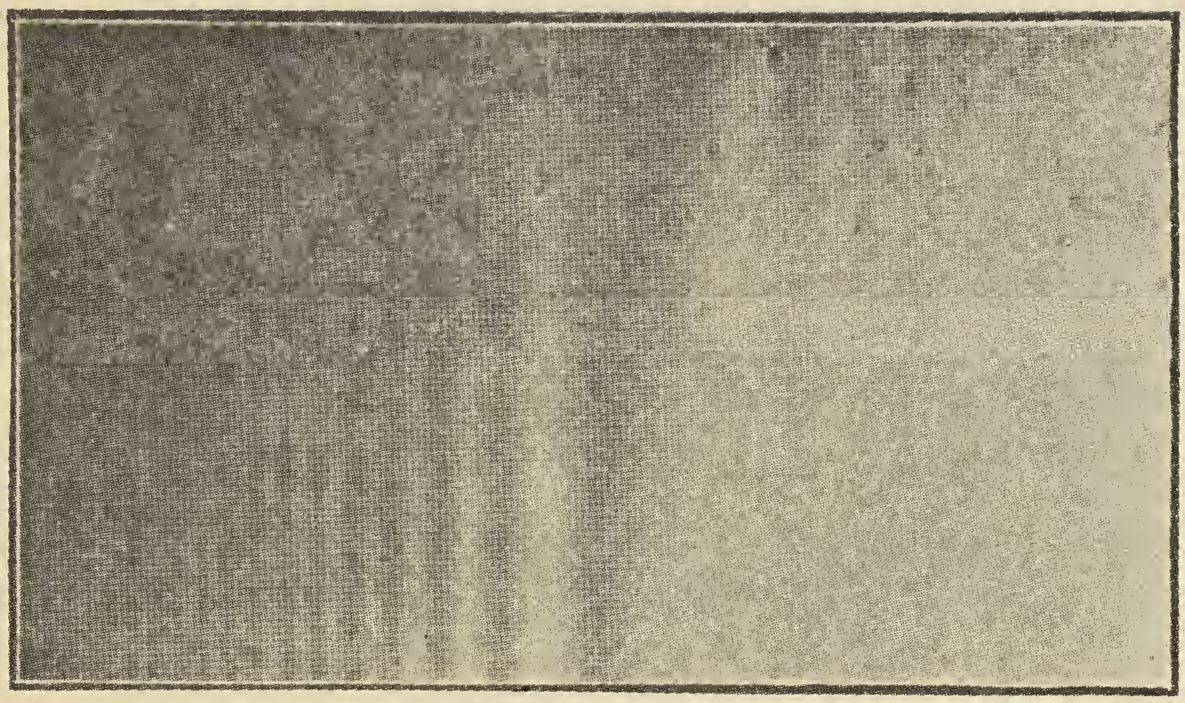


**Figure 48.** Diffraction by a thin metallic half-plane.


**Figure 49.** Diffraction by silver in the vicinity of 3250 A.U.

photographed. The fringes of low visibility on the strongly illuminated side, and the numerous fringes of high visibility in the shadow were thus recorded respectively in the upper and lower parts of the photograph. To exhibit the changes in the pattern with wavelength, the fringes may be set transversely on the slit of a spectrograph. The dispersion by the instrument then indicates how the position and intensity of the fringes alter with wavelength. Silver has a region of comparative transparency in the near ultra-violet at about 3,250 A.U. The measurements by Minor show that the optical constants $n$ and $k$ change rapidly in the vicinity of this band, $n$ falling steeply and $k$ rising equally rapidly with increasing wavelength. The influence of these changes on the configuration of the diffraction fringes is shown by the spectrogram reproduced in figure 49. A marked displacement of the fringes on the two sides of the band at 3,250 A.U. and

their practical disappearance inside it are the notabe features revealed by this record.*

The phenomena described above become intelligible when we regard the Fresnel pattern as arising from the interference of the cylindrical waves having their origin at the edge of the film with the plane waves regularly transmitted on either side of it. The high visibility of the interferences observed in the region of shadow is a consequence of the light transmitted by the film and that diffracted by the edge being of comparable intensity. The thicker the film, the further we have to move into the region of shadow for this situation to occur, while both nearer and still farther, the interfering waves differ greatly in amplitude, thereby diminishing the visibility of the resulting fringes. The positions occupied by the maxima and minima of intensity depend on the geometrical differences of path between the interfering plane and cylindrical waves, due correction being made for their initial phase-difference, and especially for the change in phase of the light transmitted through the film. These considerations also serve to explain the remarkable variations in the positions and visibility of the fringes noticed in figure 49 as we pass along the spectrum, and especially in the vicinity of the band of transparency at 3250 A.U.

By exposing a silver film having a sharp edge to the action of iodine,† it may be completely converted into a film of silver iodide which transmits the red end of the spectrum freely, but is nearly opaque at the violet end. Figure 50(a) reproduces the Fresnel pattern given by such a film in red light. Its approximately symmetrical character indicates that it arises, at least in part, from a difference in phase between the waves transmitted on either side of the edge. Figure 50(b) which reproduces the Fresnel pattern observed in violet light is a striking contrast. Here, the part covered by the film appears as in deep shadow; the photographic exposure necessary to record the fringes seen resulted in those of low visibility appearing in the strongly illuminated part of the field being effaced by over-exposure. Figure 50(c) is the Fresnel pattern of a somewhat thicker and therefore less transparent film in red light; this shows the fringes more clearly on one side than on the other. Figure 50(d) illustrates the importance of a sharp edge by exhibiting the effect of its absence on the Fresnel pattern due to a film of Canada balsam; it will be noticed that the fringes have completely disappeared from one side and show rapid changes in visibility on the other side. This is the result of the film sloping off at its termination instead of having a sharply-defined edge.

*Colours of the striae in mica*: As is well known, mica has a perfect cleavage, and the sheets obtained by splitting the mineral exhibit a remarkable uniformity of thickness, as is shown, for instance, by the perfection with which they exhibit

*N Ananthanarayanan, *Proc. Indian Acad. Sci.*, 1942, **A14**, 85. The photographs reproduced in figure 50 are also from his work.

†A tiny crystal of iodine placed on a silver film gives beautiful rings of colour.

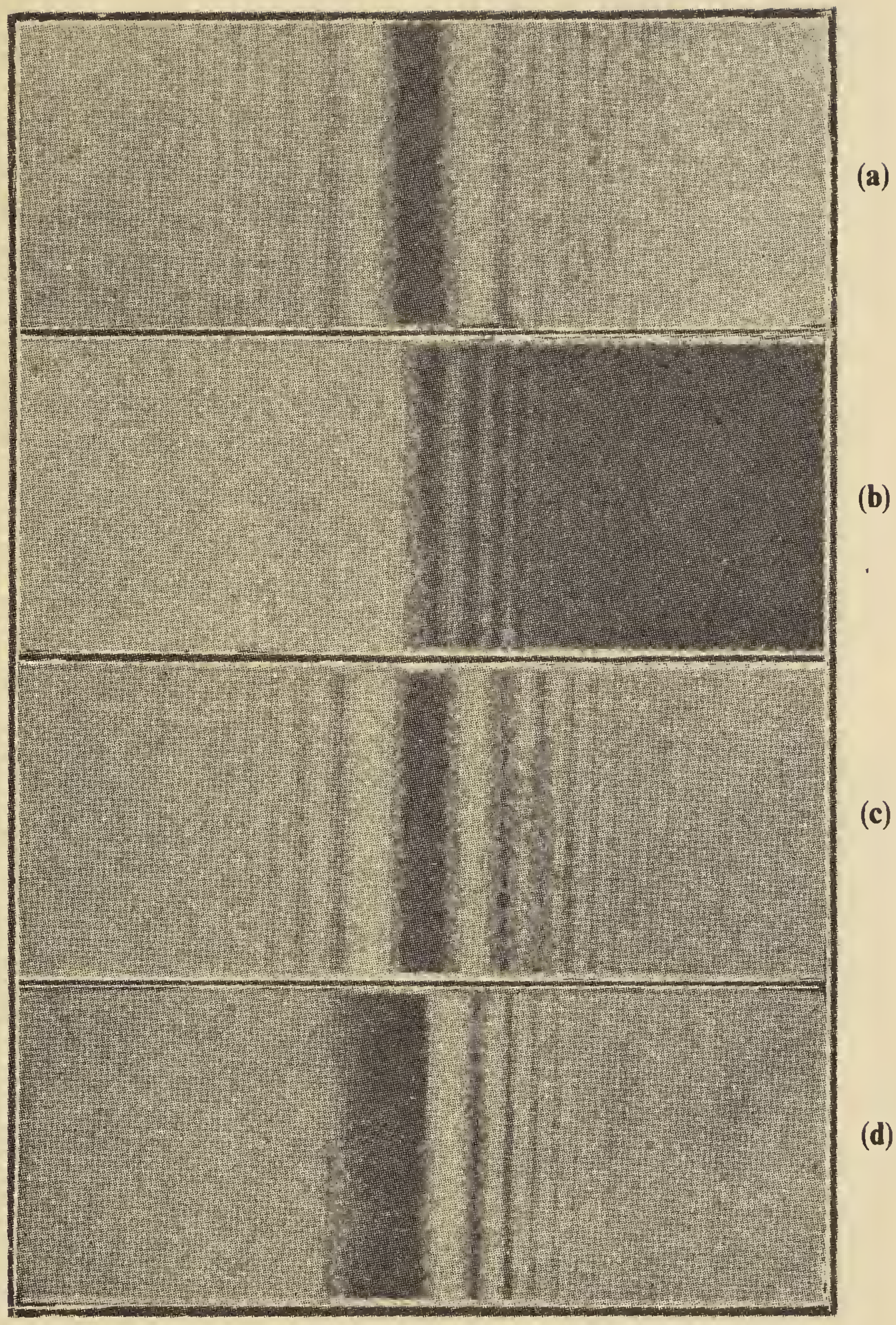



**Figure 50.** Fresnel patterns of semi-transparent laminae.

Haidinger's rings, and by the uniform colour and brightness of large areas of the surface of the sheet as seen by the reflected light of a mercury lamp. Whenever a variation of brightness or colour appears, it is sudden, indicating a sharply defined boundary at which a change of thickness occurs. These changes of thickness become vividly apparent when the mica is examined by the method of the knife-edge or Foucalt test. They then appear as *bright* lines, often exhibiting brilliant colours.* Observed in the same test but with a symmetrical aperture, the striae are seen as *dark* lines bordered by bright coloured fringes on either side,

*C V Raman and P N Ghosh, *Nature* (*London*), 1918, **102**, 205; see also P N Ghosh, *Proc. R. Soc. London*, 1919, **A96**, 257.

indicating a reversal of phase of the edge waves analogous to that we have already noticed in the case of other diffracting boundaries (see figure 51). Microscopic examination shows that the laminar boundaries may be single or multiple.* and the phenomena which the striae exhibit are found to depend on their precise nature in this respect. Single striae are extremely sharp laminar edges and are, therefore, well suited for a critical study of the diffraction effects due to such edges. They diffract light through large angles†, exhibiting colour and polarisation effects which, in some respects, are analogous to and in other respects differ from those observed with sharp metallic edges. Examined by direct light in critical focus under high powers of the microscope, the edges appear as dark lines bordered by bright fringes,‡ but as we shall see presently, the observed facts are not reconcilable with elementary notions of diffraction theory. These circumstances lend interest to a detailed study of the phenomena and justify an attempt to explain them on the basis of more exact theory.§

**Figure 51.** Mica striae in the Foucalt test.

*P N Ghosh, *Proc. Indian Assoc. Cultiv. Sci.*, 1920, **6**, 51.
†I R Rao, *Indian J. Phys.*, 1928, **2**, 365.
‡N K Sur, *Proc. Indian Assoc. Cultiv. Sci.*, 1922, **7**, 125.
§C V Raman and I R Rao, *Proc. Phys. Soc. London*, 1927, **39**, 453.

When a sheet of mica is held in the path of a beam of light and the field of view behind it is examined through a lens, the striae render themselves evident by the diffraction fringes of the Fresnel class to which they give rise. From the character of these fringes, it is evident that they arise principally from the difference in phase of the waves transmitted on either side of the boundary, though the difference in amplitude consequent on the difference in thickness also requires consideration. The central fringe in the Fresnel pattern is often brightly coloured. If, however, the difference in thickness be considerable, no colour is noticed, but the central fringe then appears darker than the rest of the field. When the slit of a pocket spectroscope is set transversely across the pattern, alternate dark and bright bands may be seen running obliquely through the spectrum at the centre of the fringe system. The bright bands determine the colour of the central fringe and if their number is not too great, it may be noticed that they correspond to the wavelengths at which the outer diffraction fringes are very weak. These oblique bands in the spectrum indicate that the position of the central dark fringe shifts laterally with the change of wavelength. Using white light and a stria giving a sufficiently large path difference, this asymmetry averages out and should cease to be evident in the absence of spectroscopic aid. In practice, however, the fringes are often more prominent on one side of the pattern than on the other. There is little doubt that this is due to a disturbing factor, namely, the finite width of the striae. Instead of a single sharp edge, we have a series of them like an echelon. In such a case, the diffraction fringes are stronger on the retarded side of the wave-front than on the other, a circumstance which is readily understood if we consider the form of the wave-front after its passage through the echelon.

The Foucalt test is the most suitable way of examining the light diffracted by the striae at comparatively small angles with the primary beam. The colours exhibited are complementary to those observed in the central fringe of the Fresnel patterns, and the changes produced by tilting the plane of the mica with reference to the incident light are also of a complementary character in the two cases. Thick striae appear white in the Foucalt test. Spectral examination, however, which discloses an alternation of intensity with wavelength. This is readily explained, as the difference in phase of the waves transmitted on either side of the boundary would have a maximum effect when it represents an odd number of half wavelengths and would have no effect when it represents an integral number of wavelengths; only the small differences in amplitude would then be left to give any observable diffraction effect, according to the elementary theory.

For examining the light diffracted at larger angles, it is convenient, as in the case of metallic edges, to illuminate the stria and view it through a low-power microscope focussed on it. It then appears as a bright line exhibiting colour, but the latter is found to alter with the angle of diffraction in a manner which depends on the character of the stria, viz., the phase-retardation on the two sides of the boundary and its micro-structure. The most striking and interesting results are those shown by single striae. Seen in a direction nearly coinciding with the

incident light, the colour is the same as when it is observed in the Foucalt test, viz., complementary to the colour of the central fringe in the Fresnel pattern. But on tilting the microscope away from this direction, the colour alters in a continuous sequence which is not symmetrical with respect to the direction of the incident light. The stria is brighter when viewed from the retarded side of the wave-front, but the colours are more striking on the other side; the stria appears achromatic when viewed rather obliquely from the retarded side of the wave-front.

It is found that the light diffracted by the individual striae is partially polarised. The percentage of polarisation increases with increasing angle of diffraction, the electric vector of the favoured component being *perpendicular* to the edge on the retarded side of the wave-front and parallel to it on the other side. The partial polarisation is, however, less marked on the retarded side of the wave-front, while on the other side, it is easily observed and in some cases is found to be nearly complete at an angle of diffraction of 90°. A quantitative comparison with the case of a steel razor blade shows, however, that the radiation by the laminar boundary is less perfectly polarised. Examination by a double image prism shows that the two images of the striae may exhibit a difference in colour as well as of intensity. More detailed studies of this effect and of the elliptic polarisation to be expected when the incident light is polarised in any azimuth are, however, lacking.

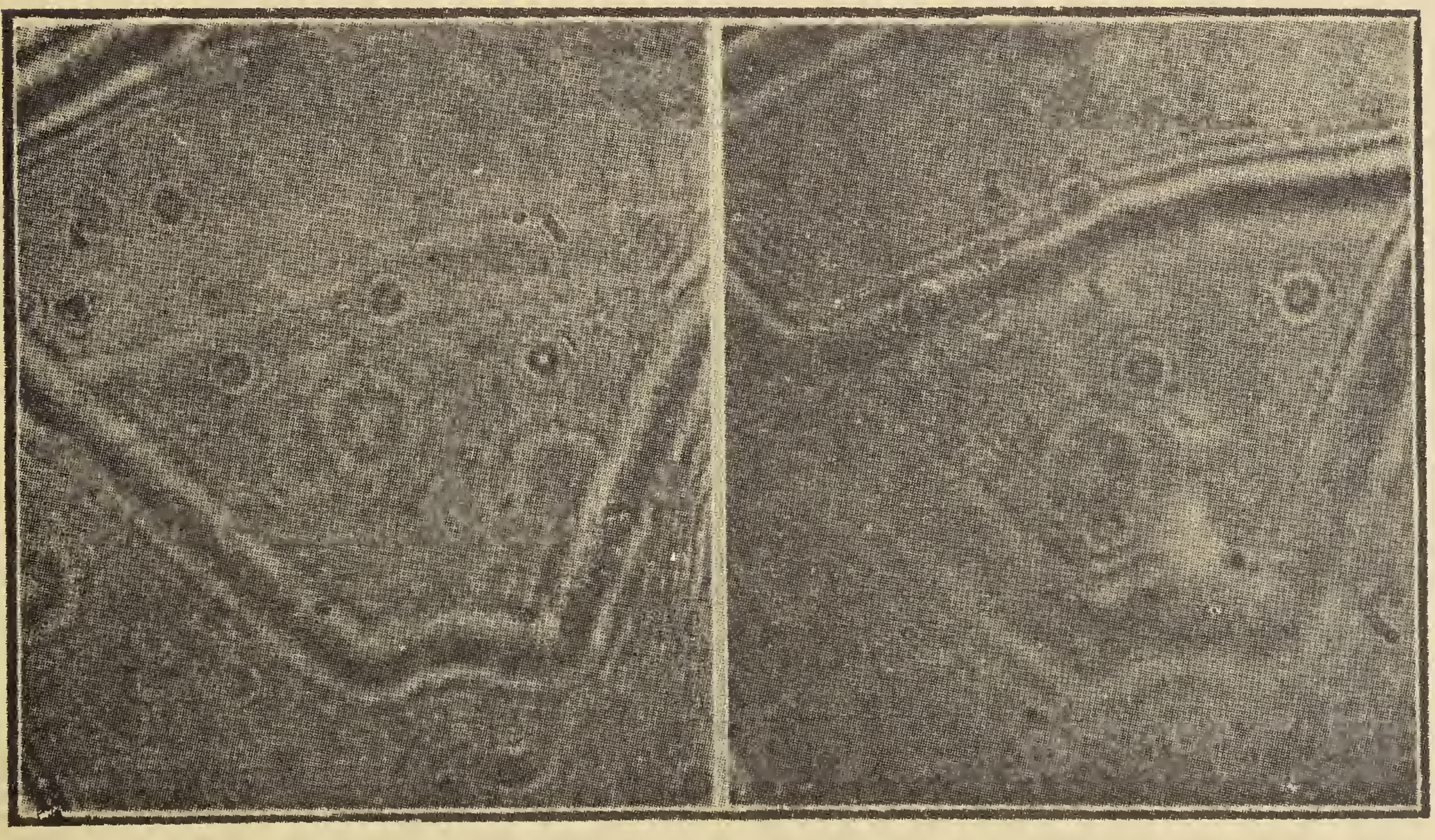
The visibility of the striae under the high powers of the microscope is a consequence of their being extremely sharp boundaries which diffract light. Under low powers, the mica striae appear as bright lines on a dark ground when viewed under oblique illumination and as dark lines on a bright ground when seen in direct light. The diffraction of light by a stria and its visibility under the microscope are thus closely connected. The stria, seen by direct light and out of focus, exhibits the usual Fresnel pattern. If the retardation be not too great, this has a coloured centre, and the fringes on either side of it have a spacing which progressively diminishes from the centre outwards. As the focus is approached, the fringes become narrower, and the character of the pattern also alters. At critical focus the stria appears as a perfectly black line with equally spaced dark and bright fringes bordering it, these being often more numerous and distinct on the thicker side of the stria than on its thinner side. To investigate whether this pattern is influenced by the phase difference on the two sides of the edge, a monochromator may be used as the light source, and the wavelength continuously varied. The surprising and interesting observation* is then made that apart from a narrowing down of the fringes with diminishing wavelength, the nature of the pattern *as seen at critical focus* does not notably alter as the wavelength of the light is changed over the whole range of the spectrum from red to violet (see figure 52). On the other hand, the Fresnel patterns of the same striae

*Previously unpublished observation by the author.

when seen out of focus show striking changes, appearing very prominently at some wavelengths, and nearly disappearing at the other lengths, these being different again for the different striae (see figure 53). It thus appears that the phase-relation of the wave-fronts on the two sides of a stria does not notably

5400 Å 6200 Å

**Figure 52.** Mica striae in the microscope (in focus).



5400 Å 6200 Å

**Figure 53.** Mica striae in the microscope (out of focus).

influence its microscopic aspects when seen in focus, and that the latter depends on the actual difference of thickness of the mica on the two sides of the stria and on its unresolved micro-structure. The asymmetry of the diffraction pattern at critical focus appears to be variable. In some cases, hardly any fringes are seen on the thinner side of the mica, while in others, several may be seen, though generally fewer than on the thicker side.

Very significant also are the effects noticed when the microscope is put *slightly* out of focus one way or the other. In the case in which the pattern at critical focus is strongly asymmetrical, it is found that when the microscope objective is slightly pushed towards the mica, a bright band of light forms on the thicker side of the pattern and moves into the thinner side. This effect is analogous to the well known Becke phenomenon, and the observations indicate that it is a consequence of the unsymmetrical diffraction of light through large angles by a laminar boundary, for which direct evidence is forthcoming, as already set out. The nature of the pattern seen at critical focus also alters and in an unsymmetrical way when the incidence of the light on the mica is made oblique. The fringes crowd up towards one side of the edge, becoming very fine and numerous and remaining well-defined, while on the other side, they recede from the edge, and become fewer, broader and more diffuse. The effects are reversed when the incidence is altered from one side to the other, the fringes being better seen in either case when they are on the thicker side of the mica.

It is evident from the observed polarisation of the light diffracted through large angles, the variations of its colour and intensity and especially from the microscopic phenomena described above, that a theory of laminar diffraction which considers only the differences in phase and amplitude of the waves transmitted on either side of the boundary can, at best, give only a very imperfect account of the phenomena. By proceeding somewhat on the same lines as those adopted in the case of sharp metallic edges earlier in this lecture and considering also the waves reflected at the boundary, the position may be somewhat improved. In particular, we may get at least a general indication of the nature of the polarisation effects to be expected. Such a treatment, however, fails to explain the *asymmetry* with respect to the direction of the incident rays which is an essential and important feature in the observations. It is evident, therefore, that a satisfactory theory of laminar diffraction is, as yet, lacking.

*Talbot's and Powell's bands*: These bands are observed with white light in a spectroscope under certain conditions when one half of the aperture of the instrument is covered by a retarding plate. The theory of these bands accordingly depends on the nature of the Fraunhofer diffraction pattern due to a rectangular aperture, one half of which is covered by a retarding plate. This is evidently the same as the pattern due to the separate halves of the aperture, but modified by their mutual interference. The intensity in the pattern may be found in the usual way by integrating over the area of the aperture, and comes out (omitting a

constant factor) as

$$4a^2\frac{\sin^2\eta}{\eta^2}\cdot\cos^2(\delta-\eta),$$

where $\eta = \pi a \sin\theta/\lambda$, $a$ being the half width of the aperture, $\theta$ the angle of diffraction, and $2\delta$ the retardation produced by the plate. In the particular case when $\delta$ is zero or any multiple of $\pi$, the pattern is identical with that of the complete aperture of width $2a$. More generally, the pattern is crossed by the interference bands expressed by the factor $\cos^2(\delta-\eta)$, the values of $\theta$ for which the resulting intensity is zero being given by

$$\sin\theta = (\mu-1)t/a-(n+\tfrac{1}{2})\lambda/a,$$

$t$ being the thickness of the retarding plate and $\mu$ its refractive index. The angular separation of the interference bands is thus inversely proportional to the aperture. Considering the particular interference band appearing within the central fringe of the pattern, the shift of its angular position with a change of wavelength $d\lambda$ is given by

$$\lambda\frac{d\theta}{d\lambda} = -\left(\mu-1-\lambda\frac{d\mu}{d\lambda}\right)t/a,$$

and is thus proportional to the thickness of the plate and inversely proportional to the aperture.

This shift of the interference fringes with wavelength may be exhibited by setting the focussed pattern for white light *transversely* on the slit of a spectrograph. The interferences then appear as bands *obliquely* traversing the spectrum,* one such band being visible within the central fringe at any given wavelength (see figure 54). The inclination of the interference bands to the direction of the spectrum is determined by the ratio of the shift $d\theta/d\lambda$ parallel to

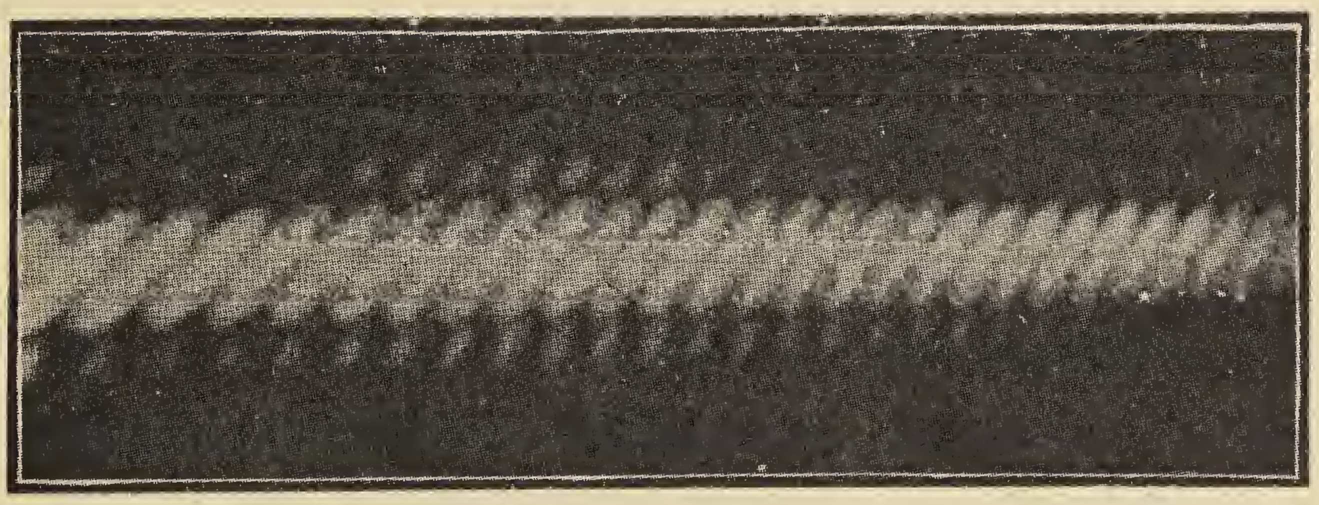


**Figure 54.** Spectral analysis of laminar diffraction.

*N K Sethi, *Phys. Rev.*, 1920, **16**, 519.

the slit and of the shift $d\psi/d\lambda$ perpendicular to it due to the dispersion of the instrument. The separation of the successive interferences along the spectrum is determined by the quantity

$$\lambda/d\lambda = \left(\mu - 1 - \lambda\frac{d\mu}{d\lambda}\right)t/\lambda = -a\frac{d\theta}{d\lambda},$$

and is thus inversely proportional to the thickness of the plate. If $\lambda/d\lambda$ exceeds the resolving power of the spectrograph, the bands would naturally cease to be observed. The visibility of the interferences produced by a thick retarding plate thus provides a test of the resolution available. It can easily be arranged that the retardation giving the diffraction pattern arises from the difference of the refractive indices of two media. For instance, a glass plate of thickness $t$ and refractive index $\mu_g$ may be immersed in a cell of liquid of refractive index $\mu_l$ covering half the aperture; the path retardation would then be $(\mu_g - \mu_l)t$ and the shift of the interferences with wavelength would be given by

$$\lambda\frac{d\theta}{d\lambda} = -\left[\left(\mu_g - \lambda\frac{d\mu_g}{d\lambda}\right) - \left(\mu_l - \lambda\frac{d\mu_l}{d\lambda}\right)\right]t/a.$$

Thus, when the fringes given by this arrangement are set transversely on the slit of a spectrograph, whether the interference bands in the spectrum slope up or slope down, is determined by the sign of the difference between the quantities $(\mu - \lambda(d\mu/d\lambda))$ for the two media which are their group refractive indices and not by the difference of their wave refractive indices $\mu$. Only at the particular wavelength for which the group indices for the glass and the liquid are the same, would the fringes run horizontally in the spectrum. On either side of this wavelength, the bands would slope in opposite directions (see figure 55).

If the central fringe of the diffraction pattern in white light is set on the spectrograph *parallel* to the slit and not transversely, the interference bands would now appear crossing the spectrum in the direction of the slit. Their

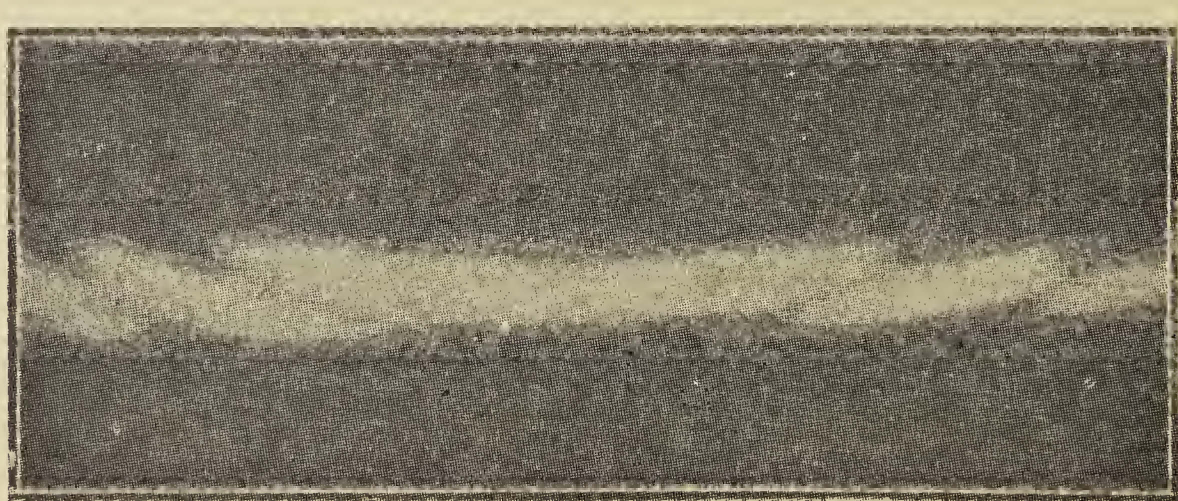


**Figure 55.** Spectral analysis of laminar diffraction.

separation along the length of the spectrum would, however, be the same as before, and their visibility would be subject to the same condition, namely, the suffciency of the resolving power of the instrument. If now the slit of the spectrograph is opened wide, very remarkable changes would be noticed in the visibility of the interferences, depending upon which side of the aperture forming the diffraction pattern is covered by the retarding plate.* In one position of the retarding plate, the visibility of the fringes in the spectrum rapidly diminishes to zero when the slit is opened and after some feeble reappearances finally vanishes. In the other position of the retarding plate, the slit may be opened wide, and though this necessarily diminishes the purity of the spectrum, the interference bands continue to be seen in it, and indeed in favourable cases suffer no diminution in their visibility. These results are readily understood when we consider the two directions of march of the interferences, proportional respectively to $d\theta/d\lambda$ and $d\psi/d\lambda$ which are superposed in the spectrum. In one position of the retarding plate, these movements are in the same direction. Hence, the displacements are added and at any given point of the spectrum, the maxima and minima of illumination are superposed to an increasing extent with the opening of the slit. The rapid diminution of the visibility to zero after minor reappearances follows as a necessary consequence. In the other position of the retarding plate, the two directions of march of the fringes are opposed and if they are numerically equal, compensate each other. The interferences, therefore, remain fixed in the spectrum and suffer no diminution in their visibility when the slit is opened wide.

The distribution of intensity in the central fringe of the pattern is shown in figure 56 for a series of values of $\delta$ and exhibits the lateral movement of the interferences by the slope of the line $A_1$ $A_2$ $A_3$ $A_4$ $A_5$ joining the zeroes of illumination in the successive curves. If, as remarked above, this slope is exactly compensated by the dispersion of the spectrograph, the interference fringes are rendered stationary and are, therefore, seen with perfect visibility. From the diagram it is also evident that if the dispersion of the spectrograph is diminished to about one-half of that required for such perfect visibility, the minimum $A_5$ in the fifth curve would be superposed on the maximum $B_1$ of the first curve, and the visibility of the fringes would, therefore, be completely destroyed. On the other hand, if the dispersion is greater than that required for perfect visibility, the straight line drawn through the minimum $A_5$ would slope over to the left and traverse the region outside the central fringe where the intensities are much smaller. The interferences would, therefore, continue to be visible in the spectrum though with diminished visibility.

It may be remarked that the foregoing discussion practically covers the theory of Talbot's and Powell's bands. We have only to remark that in the usual form of

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*N K Sethi, *Philos. Mag.*, 1921, **41**, 218.

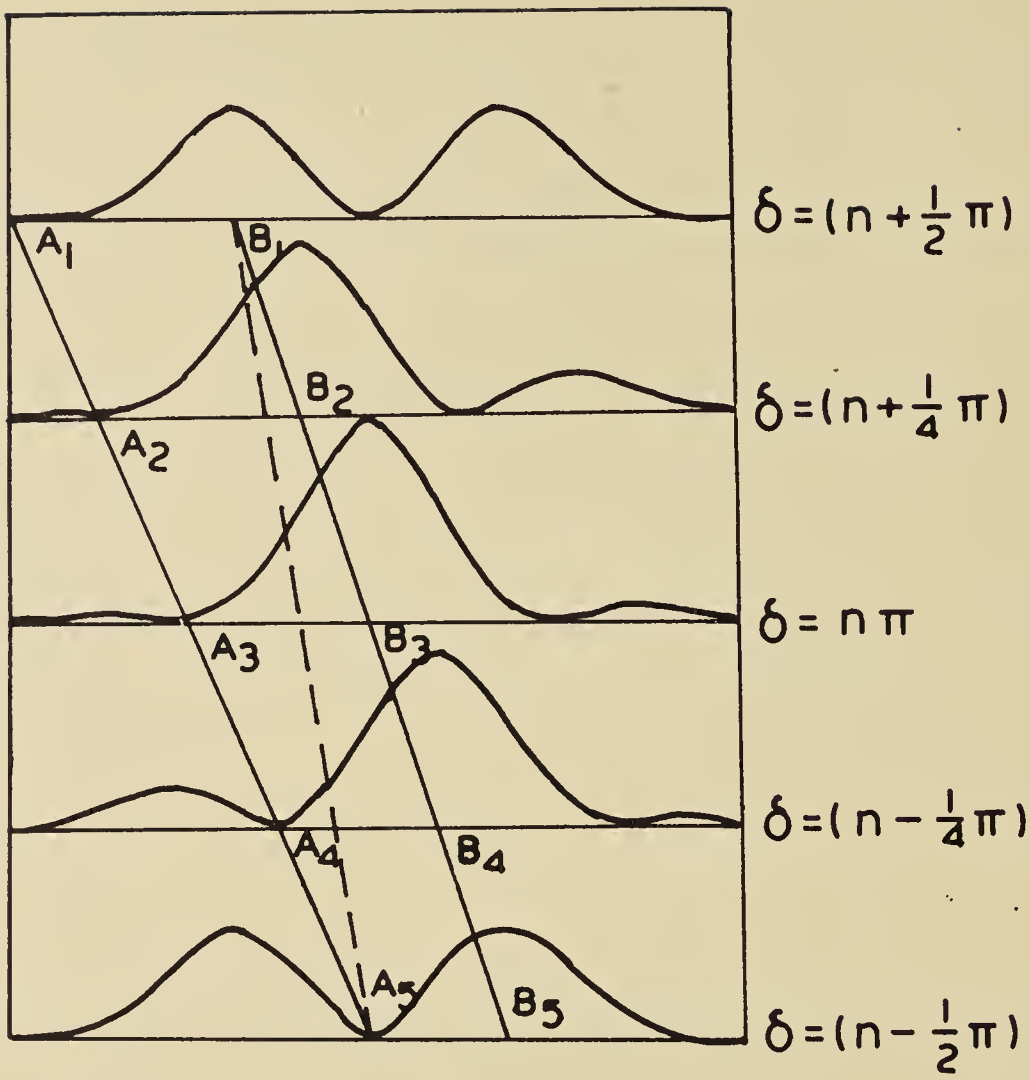


**Figure 56.** Graphs of function $(\sin^2\eta/\eta^2)\cos^2(\delta-\eta)$.

these experiments, the retarding plate is inserted within the spectrograph, which itself simultaneously forms the diffraction pattern and disperses it into a spectrum. The conditions for obtaining Talbot's bands under these conditions immediately follow from the foregoing theory and may be stated very simply. *The visibility of Talbot's bands is perfect when the resolving power of the spectrograph at full aperture is twice the minimum required for separating the bands. The bands are invisible when the resolving power of the instrument is only sufficient to separate them.* The conditions for observing the bands are much more flexible in the form of the experiment discussed above in which the laminar diffraction pattern is first formed outside the instrument and then focussed on the slit of the spectrograph. The focal length of the lens which forms the pattern and its aperture which is half covered by the retarding plate are entirely at our disposal and may be quite different from those of the dispersing instrument. It is, therefore, possible to obtain what are essentially Talbot's bands with maximum visibility under almost any desired conditions.

Perhaps the most remarkable feature of Talbot's bands is that they involve an alteration of the colour sequence in the spectrum of white light.* Under favourable conditions, a spectrum traversed by the bands appears as an echelon

*N K Sethi, *Philos. Mag.*, 1921, **41**, 211.

of colour, presenting as many discrete steps as there are bands instead of the progressive change of hue normally seen in the spectrum. This effect is best exhibited by the bands when the conditions are slightly different from those required for their perfect visibility. Referring again to figure 56, it will be noticed that the line $B_1$ $B_2$ $B_3$ $B_4$ $B_5$ drawn through the successive maxima of intensity has a slightly greater slope than the line $A_1$ $A_2$ $A_3$ $A_4$ $A_5$. If, therefore, the dispersion of the spectrograph is about four-fifths of that required for perfect visibility, the intensity maxima in the bands come into coincidence in the spectrum. The dark bands are then not perfectly black, but the proportions of the various wavelengths contributing to the intensity between one minimum and the next remain constant. The result is that the band exhibits a uniform hue, and when we cross from one band to the next, there is a jump in the colour. The effect is most striking when the number of colour steps in the whole spectrum is small, say 3 or 4 or 5. A spectrum of this kind may be obtained by forming the diffraction pattern with a thin plate of mica and resolving it by a prism or grating with relatively small dispersion. With the ordinary arrangements for observing Talbot's bands, the colour jumps may be effectively demonstrated even with as many as 25 or 30 bands in the spectrum, by arranging to move the latter over a slit behind which the eye is placed so as to view the surface of the dispersing prism or grating.

Before leaving this subject, attention may be drawn to the fact that the dispersive power of the retarding plate appears explicitly in the theory of Talbot's and Powell's bands. The significance of this is that, as in all interference experiments with non-homogeneous light, the observed phenomena are determined by the group velocity and not by the wave-velocity of light in the material media. This is very prettily illustrated with Powell's bands when the strongly dispersive mixture of benzene and carbon disulphide is employed to fill the cell in which the retarding glass plate is immersed.* The refractive index of the mixture may be steadily diminished by addition of benzene. Fairly thick plates may be used as the retardations involved depend only on the differences between glass and liquid. The point at which there is equality of refractive index between glass and liquid may be nicely judged by viewing a source of light obliquely through the edge of the plate. It is found that the bands are visible throughout the spectrum at that stage, and continue to be visible until the refractive index is further lowered and the group indices for the glass plate and the liquid mixture are equalised for some particular point in the spectrum. This corresponds to the wavelength at which the bands curve round in figure 55, and as it advances further into the spectrum, the bands on one side of it disappear. If, now, the position of the plate in the cell is reversed, the bands become visible in the part of the spectrum in which they were previously invisible, and *vice versa*.

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*N K Sethi, *Phys. Rev.*, 1920, **16**, 519.

It may be mentioned that spectra crossed by interferences analogous in principle to Talbot's bands and Powell's bands may be obtained when we have a succession of light beams differing in path by equal amounts (instead of only two) interfering with each other. These may be obtained by using a number of retarding plates in echelon order, or by multiple reflection between parallel surfaces as in a Fabry–Perot etalon or a Lummer–Gehrcke plate. In the latter case, it is necessary to immerse the plate in a dispersive medium.*

*Oblique reflection and refraction*: The surface of separation between two media differing in their properties is the seat of familiar optical phenomena, viz., reflection, refraction and total reflection. These appear in the electromagnetic theory of light as consequences of the difference in dielectric constant of the two media. By considering the conditions which have to be satisfied at the boundary, which is assumed to be of unlimited area, the laws of reflection and refraction and the intensities of the reflected and refracted beams may be deduced. In the case of total reflection, the theory also leads to the conclusion that there is a superficial disturbance in the second medium. In practice, however, the surface of separation between the two media is of finite extension, and it follows that reflection, refraction and total reflection are necessarily accompanied by diffraction phenomena. We shall now proceed to consider the special features arising in these cases.

The diffraction patterns resulting from the reflection of light at a plane optical surface are easily observed.† A prism is set on the table of a spectrometer, and the slit of the collimator is viewed by reflection at one of the surfaces of the prism through the observing telescope. As the incidence of the light on the surface is made more oblique, the image of the slit broadens into diffraction pattern, the extension of which depends on the wavelength of the light employed, the width of the prism face and the angle of incidence. At moderate incidences, the pattern is indistinguishable from that of a rectilinear slit held normally in the path of a parallel beam of light. With increasing obliquity, however, the pattern progressively changes in character and becomes unsymmetrical (see figure 57). The fringes increase in width and at an accelerated rate as we pass from one side of the central fringe to the other. It is evident that on the side of the pattern where the fringes are narrower, there is no specifiable limit to their number, while on the side where they are broader, their number is finite. This limitation of the number of fringes on one side arises from the fact that the pattern does not extend beyond its intersection with the plane of the reflecting surface. Another noteworthy features is that the corresponding bands on either side of the central band are of very

*N K Sethi, *Phys. Rev.*, 1921, **18**, 389.
†C V Raman, *Philos. Mag.*, 1906, **12**, 494.

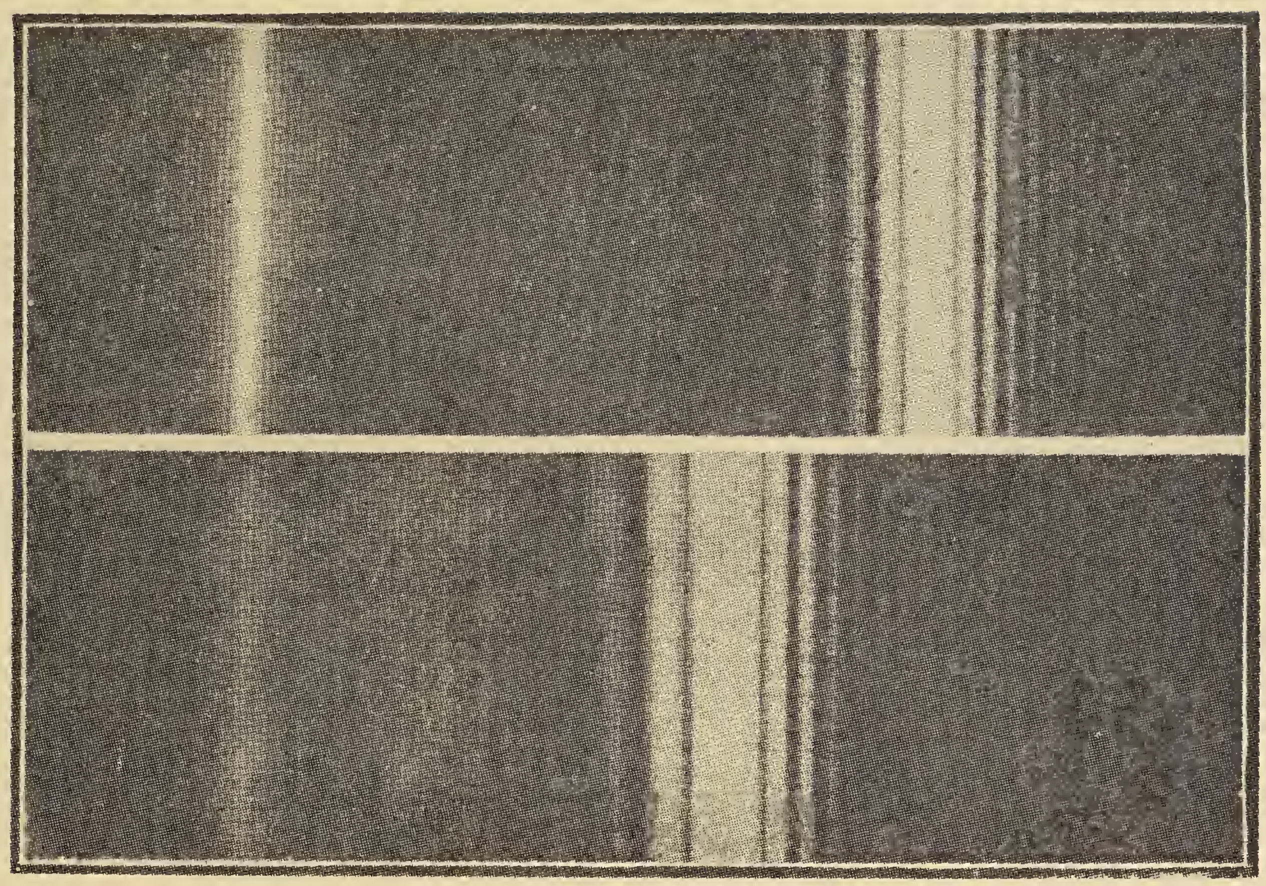

**Figure 57.** Diffraction patterns of obliquely reflected light.

different intensities. The fringes on the side where they are broader and fewer are much fainter than the corresponding narrow bands on the other side. This difference becomes more conspicuous when the bands compared are respectively nearer and farther from the limit of the pattern. It is also evident that on the side of the pattern remote from this limit, the successive fringes fall off in intensity more slowly than in the normal diffraction pattern of a rectangular aperture.*

Very similar effects are also noticed in oblique refraction at a plane surface. It is well known that an ordinary prismatic spectroscope may be adjusted to give a large dispersion by placing the prism on the table of the instrument in such a position that the light falls at nearly the critical angle of incidence on its second face and emerges in a direction almost parallel to it. There is, however, no gain of resolving power by placing the prism in this position, as the image in the focal plane of the observing telescope is greatly broadened by diffraction. The image is also strongly curved, the deviation being appreciably different for the rays which have passed in slightly different planes through the prism. Owing to the large dispersion produced by the prism, it is necessary to use monochromatic light to observe the diffraction pattern of the obliquely emergent light in the viewing telescope.

Except for the curvature of the fringes and their greater width, the diffraction photographs reproduced in figure 58 for two different angles of emergence are

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*C V Raman, *Philos. Mag.*, 1909, **17**, 204.

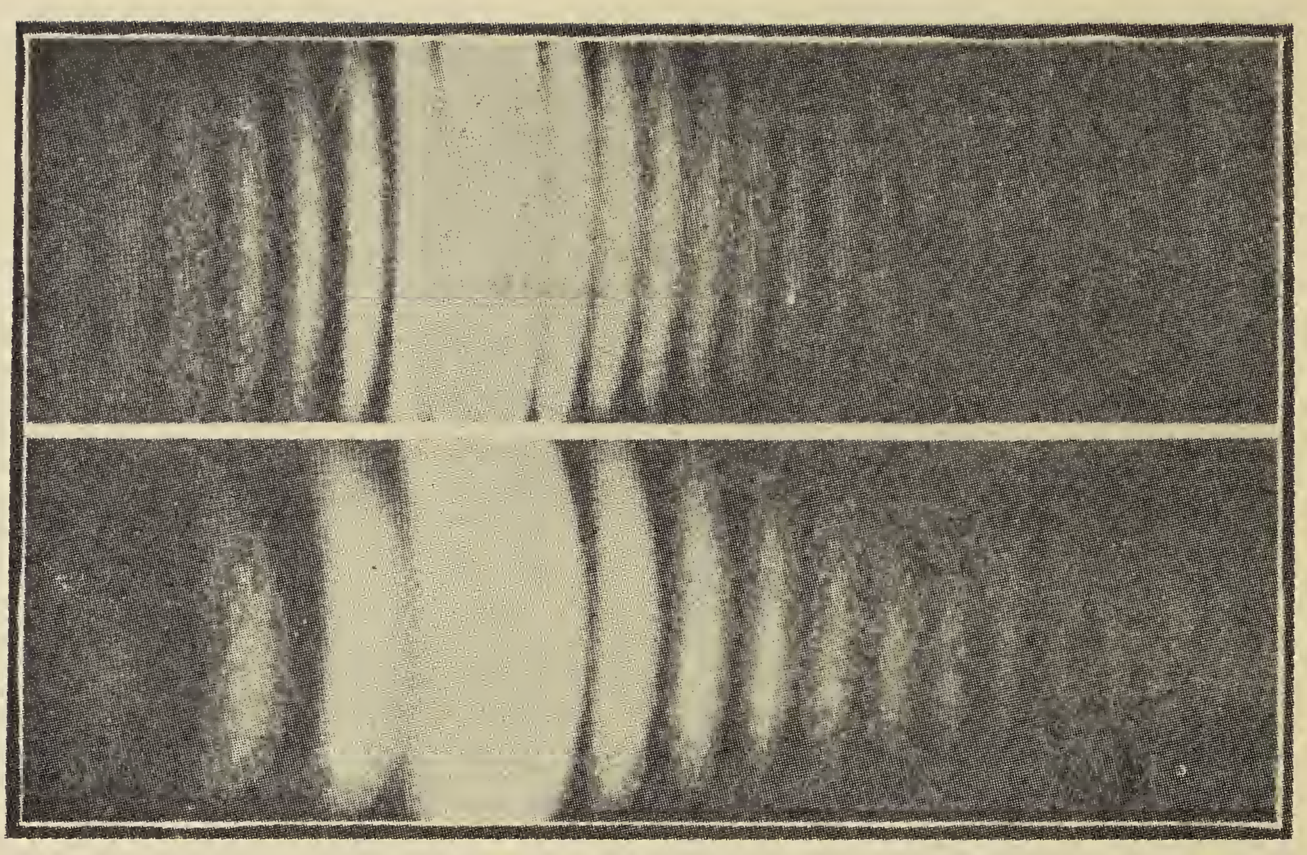

**Figure 58.** Diffraction patterns of obliquely refracted light.

very similar to the patterns for oblique reflection discussed above and illustrated in figure 57. The fringes show a progressive increase in width from one side of the pattern to the other. Their number on one side is limited by the fact that the plane of the surface of the prism limits the extension of the pattern in that direction. There is also a striking difference in the intensity of the corresponding fringes on either side, the wider fringes being also the fainter. The great number of narrow fringes visible on one side of the pattern indicates that on this side they diminish in intensity more slowly than in the normal diffraction pattern of a rectangular aperture.

With exactly the same arrangements as those used in obtaining the diffraction photographs reproduced in figure 58, if the incidence is increased beyond the critical angle, all the fringes on one side of the pattern and the central fringe move out and disappear, but those on other side persist.* As the angle of incidence is further increased, more fringes move out of the field, but other fringes move into it, with the result that the general appearance of the pattern remains much the same except for the diminished intensity and width of the fringes. Indeed, it is clear that the diffraction phenomena at incidences less and greater than the critical angle form a continuous sequence. The slow diminution in the intensity of the successive fringes observed in figure 59 has evidently the same origin as the corresponding feature in figure 58.

The patterns illustrated in figures 57, 58 and 59 are explicable as interferences of the radiations diffracted by the edges of the surface at which reflection or

*C V Raman, *Philos. Mag.*, 1925, **50**, 812.

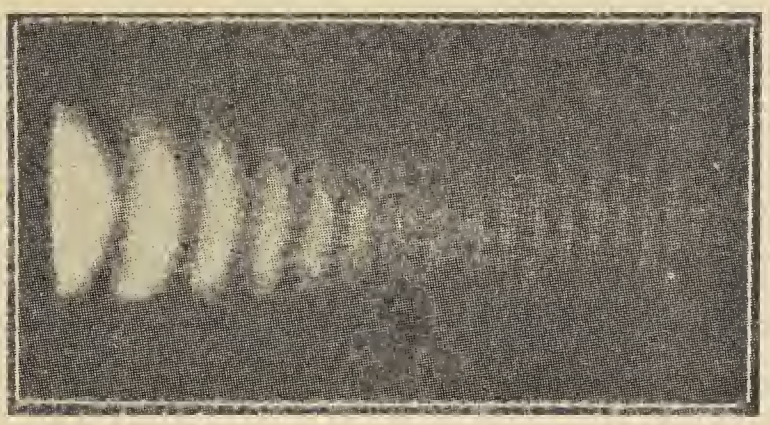

**Figure 59.** Diffraction pattern in the second medium in total reflection.

refraction occurs. Alternatively, we may consider them as interferences of the secondary radiations having the elements of area of the reflecting or refracting surface as their origin. Accepting the latter view, we obtain by integration over the surface the expression for the intensity in the pattern, namely,

$$a^2/f\lambda \cdot \cos^2\psi \cdot \sin^2\xi/\xi^2$$

where

$$\xi = \pi a(\sin\psi - \sin\psi_0)/\lambda \text{ (for reflection)}$$
$$\xi = \pi a(\sin\psi - \mu\sin\psi_0)/\lambda \text{ (for refraction).}$$

$\psi_0$ and $\psi$ are the angles of incidence and diffraction respectively. The zeroes of intensity appear at the values of $\psi$ found by making $\xi = \pm\pi, \pm 2\pi, \pm 3\pi$, etc.

A remark is necessary regarding the factor $\cos^2\psi$ appearing in the foregoing expression for the intensity in the pattern. It results from assuming that the amplitude in the hemispherical secondary waves from the elements of the surface is proportional to the cosine of the angle of diffraction, in other words, that it is greatest at the vortex of a hemisphere and zero at its base. The introduction of this obliquity factor in the law of the secondary wave is found to be necessary for an explanation of the observed distribution of intensity in the patterns. Photometric studies* have confirmed its correctness in the case of reflection† as well as of refraction‡ at individual plane surfaces, as also at a whole series of surface elements lying in a plane and forming a diffraction grating. That the inclusion of the obliquity factor $\cos^2\psi$ in the expression for the intensity in the pattern is necessary is also indicated by a consideration of the total energy appearing in it. To show this, we may take the case of a perfectly reflecting surface on which light is obliquely incident. The energy received by the surface is that passing through

*C V Raman, *Philos. Mag.*, 1911, **21**, 618.
†S K Mitra, *Philos. Mag.*, 1918, **35**, 112.
‡B N Chakravarty, *Proc. R. Soc. London*, 1921, **A99**, 503.

an area $a \cos\psi_0$ of the incident beam, and the same should appear in the diffraction pattern. The energy actually appearing in the latter according to our formula is

$$\int \frac{a^2}{f\lambda}\cdot\cos^2\psi\cdot\frac{\sin^2\xi}{\xi^2}\cdot f\cdot d\psi,$$

and this may be written in the form

$$\int \frac{a}{\pi}\cdot\cos\psi_0\cdot\frac{\cos\psi}{\cos\psi_0}\cdot\frac{\sin^2\xi}{\xi^2}\cdot d\xi.$$

This permits of being equated to $a \cos\psi_0$. For, the integral of $\sin^2\xi/\xi^2$ with respect to $\xi$ when a sufficient number of fringes is present on both sides of the pattern is $\pi$; in such an integration, we may without sensible error write $\psi$ equal to $\psi_0$, its value for the central fringe which contains the largest part of the total energy. On the other hand, when some of the fringes in the part of the pattern where $\psi > \psi_0$ have disappeared, the loss of their contribution to the integral would be compensated for by the increased intensity of fringes for which $\psi < \psi_0$.

It is worthy of remark that the diffraction of light obliquely emergent from a plane surface plays an essential role in the operation of the well known and valuable instrument known as the Lummer–Gehrcke plate. It is not unusual to find this described as an interferometer. The theory of the instrument as actually employed is, however, far from being that of a simple interference plate. In practice (see figure 60), the light is admitted into it through a reflecting prism cemented or optically attached to one end, and the reflection at the external surface of the plate is thus avoided. This makes the plate a "direct-vision" instrument and results in a large gain of illumination. As a result of this arrangement, also, we have *exactly* similar patterns on both sides of the plate, and the complementary character of the results as observed by transmitted and reflected light is no longer in evidence. In other words, *the Lummer plate functions as a diffraction grating and not as a simple interference plate.* Corresponding to a definite direction of incidence within the plate, the direction of emergence of the light may coincide with any one of several orders of interference. That this should happen is not surprising, as the individual beams emerging from the plate very obliquely are necessarily of limited aperture and are, therefore, immensely widened by diffraction. Though, in practice, an extended source of light is

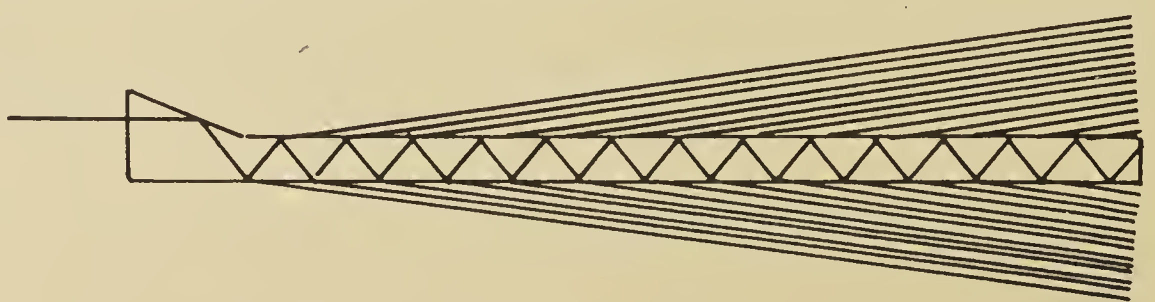

**Figure 60.** Diagram of the Lummer–Gehrcke plate.

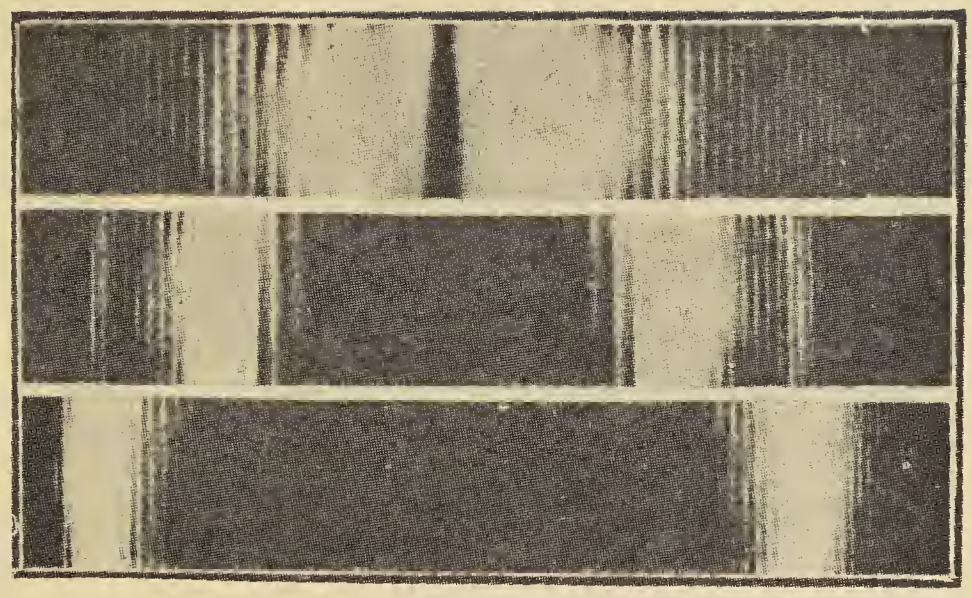

**Figure 61.** Lummer patterns at different incidences.

employed, this is not essential and the pattern may be observed even with a narrow slit, and indeed even when the light is incident within the plate at more than the critical angle. This is shown by the photographs reproduced in figure 61 which were obtained with the full aperture of the plate and with various incidences, using the 4358 A.U. radiations of the mercury lamp.

Figure 61 shows that the position of the interferences remain unaffected, though the distribution of intensity amongst them alters, when the angle of incidence of the light is varied. This circumstance which enables an extended source of light to be used for observing the interferences follows as a consequence of the Fermat principle of stationary path. If $\theta$ and $\phi$ are the glancing angles of incidence and emergence of the light beams for a given order of interference, $\lambda$ and $\mu$ being respectively the wavelength of the light and the refractive index of the plate, it is readily shown that $d\phi/d\lambda$ has the same value whether we regard the arrangement as an interference plate or as a diffraction grating. In the former case, we vary both $\theta$ and $\phi$ and maintain the geometric relation $\cos\phi = \mu\cos\theta$ between them. In the latter case, we keep $\theta$ constant and vary $\phi$ maintaining the path-difference between the successive diffracted beams as a constant multiple of the wavelength. In either case, we obtain the same result, namely,

$$\sin\phi\frac{d\phi}{d\lambda} = \frac{1}{\lambda\cos\theta}\left(\mu\sin^2\theta - \lambda\frac{d\mu}{d\lambda}\right).$$

The aggregate aperture of the emerging pencils is $l\sin\phi$, where $l$ is the length of the plate; the ratio of this to the wavelength determines the angular width of the diffraction maxima and, therefore, also the resolving power of the instrument. The latter may be evaluated by writing $d\phi = \lambda/l\sin\phi$ in the preceding formula, and we thus obtain

$$\frac{\lambda}{d\lambda} = \frac{l}{\lambda\cos\theta}\left(\mu\sin^2\theta - \lambda\frac{d\mu}{d\lambda}\right).$$

*The phenomena of total reflection*: Total reflection was first explained on wave-principles by Huygens. Assuming that secondary wavelets issue from the surface into both media, he showed that when the incidence is beyond the critical angle, no common envelope can be drawn to the wavelets in the second medium and no resultant wave can, therefore, emerge into it. Supplementing this argument by the principle of interference, it can be rigorously proved that there should be a superficial disturbance in the second medium. It will be useful first to show this in an elementary way by the familiar method of the Fresnel zones. To mark out the form of the zones on the surface, we consider some particular point of observation in the second medium and drop a perpendicular from it on the surface. Around the foot of this perpendicular as centre, circles are drawn of which the distances from the point of observation increase successively by units of half a wavelength. Parallel and equidistant straight lines are similarly drawn perpendicular to the plane of incidence and indicating the points on the surface at which the phase of the incident plane waves differs by half a period. Both the circles and straight lines are serially numbered, and by adding these numbers at the points of intersection, the points of constant total path may be found and curves drawn through these to represent the Fresnel zones on the surface for the particular point of observation. The form of the zones thus derived and the changes in their configuration with the angle of incidence and the position of the point of observation enable us to obtain a general and comprehensive view of the case.* The zones are closed curves only when the angle of incidence is less than the critical angle; at and beyond the critical incidence, they open out and assume approximately hyperbolic forms. It is evident that there are no poles or points of stationary phase on the surface at such incidences, and the disturbance in the second medium is, therefore, a residual or diffraction effect.

Considering the disturbance at some point fairly close to the surface, it is evident that the contribution to this from distant parts of the surface is insignificant. For, the obliquity factor being the cosine of the angle which the diffracted ray makes with the normal to the surface, the effect of such parts of the surface becomes vanishingly small. On the other hand, the parts of the surface near the points of observation have the maximum value of the obliquity factor, namely, unity. In this region, remarkable changes in the form of the Fresnel zones are observed if the point of observation is at or near the surface. Figure 62 shows the form of the zones for a case in which the incidence is at 60°, the critical angle being 45°, while the point of observation is on the surface itself. Figure 63 shows the zones for the same case when the point of observation is at a distance of $4\lambda$ from the surface. A sudden discontinuity in the width of the Fresnel zones to the

*C V Raman, *Proc. Indian Assoc. Cultiv. Sci.*, 1926, **9**, 271 and 330, (The second reference contains some remarks on the sharpness of the boundary of critical reflection as affected by diffraction when the aperture is limited.)

right and left of the origin will be noticed in figure 62. Accordingly, there must be a large resultant effect at this point. In figure 63, the discontinuity has disappeared, there being now a steady increase in the width of the zones as we pass from one side of the origin to the other. Accordingly, the effects of the Fresnel zones should cancel each other out by interference. The configuration of the Fresnel zones thus clearly indicates that there is a strong superficial disturbance

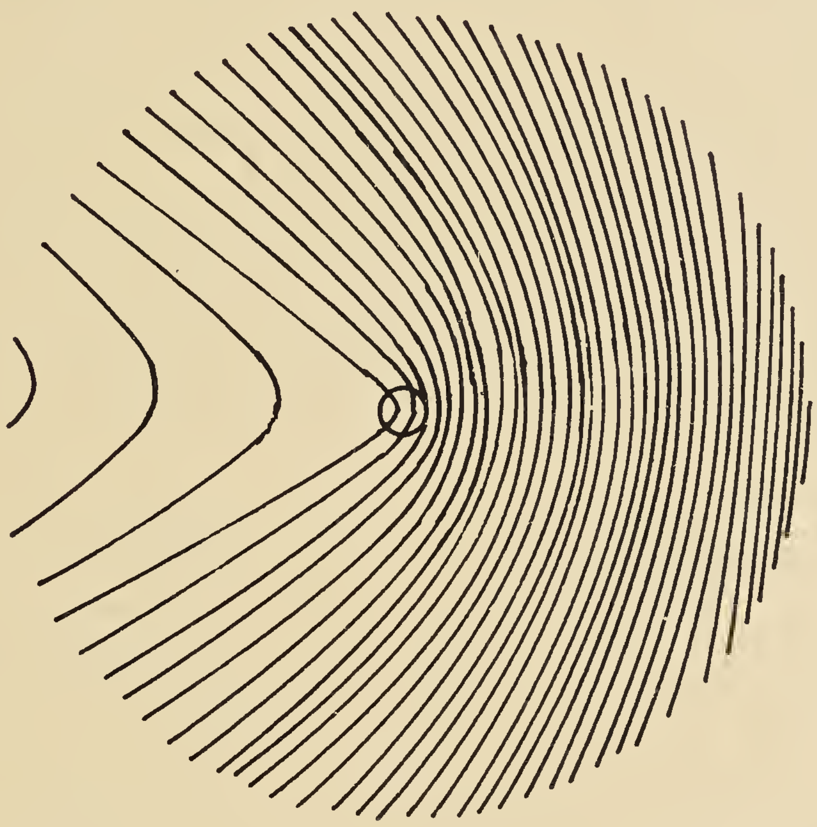

**Figure 62.** Form of Fresnel zones on surface. Incidence at 60° (critical angle 45°); point of observation on surface.

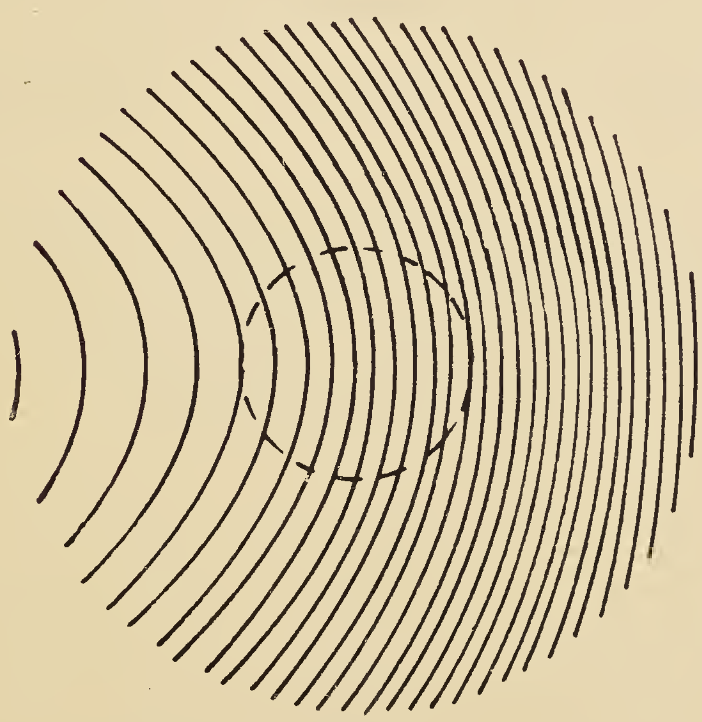

**Figure 63.** Form of Fresnel zones; incidence at 60° (critical angle 45°); point of observation $4\lambda$ from surface.

in the second medium, and that this diminishes rapidly as we move away from the surface. The further we are from the critical incidence, the more quickly does this diminution occur.

When the point of observation is sufficiently removed from the surface, the contribution from the neighbouring parts of the surface becomes entirely negligible. On the other hand, the effects arising at the margin of the illuminated area of the surface then come into prominence. The magnitude of these effects depends on two factors, namely, the width of the uncompensated zones at the edges, and the obliquity factor. These factors work in opposite directions, since the width of the zones would be greatest when the direction of observation is parallel to the surface, while in this direction the obliquity factor actually vanishes. The most suitable way of observing the edge effects is to view them obliquely from a point entirely outside the surface and in a direction nearly parallel to it. Both the rear and front edges then appear as fine luminous lines and are seen to be of equal intensity. The interference of the edge radiations gives rise to a diffraction pattern which can be recorded photographically with sufficiently long exposures* (see figure 59).

Returning to the effects observed at a point $P$ near the surface, we may derive an expression for the superficial wave in the second medium. The perpendicular drawn from $P$ to the surface is taken as $z$-axis, its foot as the origin, and the plane of incidence of the light as the $XZ$ plane. The light vector in the first medium along the surface may be assumed to be of the form

$$A\sigma_0 \cdot \cos(Q_0 - 2\pi x \sin\psi_0/\lambda), \quad \text{or simply} \quad A\sigma_0 \cdot \cos Q$$

for brevity, $\sigma_0$ being an undetermined constant. The surface is divided up into elements of area $\rho \mathrm{d}\rho \mathrm{d}\phi$ in polar co-ordinates, $r$ the distance of the element from $P$ being $(z^2 + \rho^2)^{1/2}$. The effect of the secondary wave from such an element at $P$ is

$$A\sigma_0 \cdot \frac{\rho \mathrm{d}\rho \mathrm{d}\phi}{\lambda r} \cdot \cos(Q - 2\pi\mu \cdot \rho \sin\psi_0 \cos\phi/\lambda - 2\pi r/\lambda).$$

The obliquity factor is taken as unity, since the effect at $P$ is mainly due to elements on the surface near the origin. Integrating with respect to $\phi$ between the limits 0 and $2\pi$, we obtain the resultant effect at $P$ as

$$A\sigma_0(C\cos Q + S\sin Q)$$

*C V Raman, *Philos. Mag.*, 1925, **50**, 812.

where $C$ and $S$ respectively stand for

$$C = \frac{2\pi}{\lambda}\int_0^\infty J_0\left(\frac{2\pi\mu\rho\sin\psi_0}{\lambda}\right)\cos\frac{2\pi r}{\lambda}\frac{\rho d\rho}{r}$$

$$S = \frac{2\pi}{\lambda}\int_0^\infty J_0\left(\frac{2\pi\mu\rho\sin\psi_0}{\lambda}\right)\sin\frac{2\pi r}{\lambda}\frac{\rho d\rho}{r}.$$

These are well known integrals.* When $\mu\sin\phi_0 > 1$, $S$ vanishes and the effect at $P$ reduces† to

$$A\sigma_0(\mu^2\sin^2\psi_0 - 1)^{-1/2}\cos Q\cdot\exp\left(-2\pi z\sqrt{\mu^2\sin^2\psi_0 - 1}/\lambda\right)$$
$$= A\sigma\cos Q\cdot\exp\left(-2\pi z\sqrt{\mu^2\sin^2\psi_0 - 1}/\lambda\right)$$

thus appearing as a superficial wave $A\sigma\cos Q$ at the surface and travelling along the $x$-axis, the amplitude of which decreases exponentially with the distance $z$ from the surface. The disturbance at the surface in the first medium is due jointly to the incident and totally reflected waves. If these are individually

$$A\cos(Q - \delta/2) \quad \text{and} \quad A\cos(Q + \delta/2),$$

their resultant is $2A\cos\delta/2\cos Q$. The continuity of the disturbance at the surface parallel to the $y$-axis requires that $\sigma = 2\cos\delta/2$, thus showing that the amplitude $\sigma$ of the superficial wave and the phase change $\delta$ in total reflection are closely connected with each other. To evaluate $\sigma$ and $\delta$, we require a second relation. This is obtained by considering the continuity of the disturbance parallel to the $x$-axis on the two sides of the boundary. If the electric vector in the incident waves is parallel to the $y$-axis, then the magnetic vector $H_x$, i.e., $\partial E_y/\partial z$ must be the same on the two sides of the boundary. If, similarly, the magnetic vector in the incident waves is parallel to the $y$-axis, then the electric force $E_x$ must be the same on the two sides; in other words

$$\left(\frac{\partial Hy}{\partial z}\right)_1 = \mu^2\left(\frac{\partial Hy}{\partial z}\right)_2.$$

Applying these conditions, the values of $\sigma$ and $\delta$ in these two cases are respectively

*Bateman's *Electrical and Optical Wave-Motion*, p. 72; and Watson's *Treatise on Bessel Functions*, 1922, p. 416.
†C V Raman, *Trans. Opt. Soc. Am.*, 1927, **28**, 149.

found to be

$$\sigma_s^2 = \frac{4\mu^2\cos^2\psi_0}{\mu^2-1}, \quad \tan\tfrac{1}{2}\delta_s = \frac{\sqrt{\mu^2\sin^2\psi_0 - 1}}{\mu\cos\psi_0}$$

$$\sigma_p^2 = \frac{4\cos^2\psi_0}{(1-\mu^2)+(\mu^4-1)\sin^2\psi_0}, \quad \tan\tfrac{1}{2}\delta_p = \frac{\sqrt{\mu^4\sin^2\psi_0 - \mu^2}}{\cos\psi_0}.$$

It is readily verified that at the critical incidence, $\delta_s = \delta_p = 0$ and $\sigma_s = \sigma_p = 2$. In other words, the amplitude of the superficial disturbance which is then a maximum is the arithmetic sum of the amplitudes in the incident and reflected waves. At grazing incidence, $\delta_s = \delta_p = \pi$, and $\sigma_s = \sigma_p = 0$. In other words, the surface is a nodal plane and the superficial wave vanishes. At intermediate incidences, $\delta_p$ is greater than $\delta_s$ and this results, when the incident light is plane polarised in an azimuth inclined to the plane of incidence, in the reflected light being elliptically polarised.

The existence of a superficial wave in the second medium may be demonstrated in several ways. A direct method which has the advantage of enabling us to determine the distribution of intensity in depth as well as direction of energy flow and the state of polarisation, is to use the well known property possessed by a sharp metallic edge of diffracting a stream of radiation falling upon it.* A fresh safety razor-blade is held normal to the surface of a totally reflecting prism and with its edge exactly parallel to it. A fine slow-motion of the kind provided in interferometers enables the blade to be moved forward or backward by fractions of a wavelength. The razor edge is viewed through a microscope focussed on it. If the axis of the microscope is in the plane of incidence of the light, and the razor-blade is perpendicular to it, the edge when slowly advanced to within a very small distance of the surface, becomes visible as a fine luminous line. The intensity is greatest when the axis of the microscope is as nearly as possible parallel to the surface. The distance from the surface within which the luminosity of the edge is perceptible is a measure of the thickness of the superficial disturbance. It is known that a diffracting edge is only luminous when seen along the surface of a cone having the edge as axis and the ray incident on it as a generating line. Hence the observations indicate that the direction of energy-flow in the superficial disturbance is in the plane of incidence and parallel to the surface. When the incidence is just at the critical angle, the intensity of the superficial wave is found to be a maximum and comparable with that of the incident and reflected waves. As the incidence is increased, the intensity falls off very quickly. The decrease of intensity with increasing distance from the surface is also rapid. When the incidence is not much greater than the critical angle, say about 50° for glass, the

*C V Raman, *Trans. Opt. Soc. Am.*, 1927, **28**, 149.

luminosity is perceptible when the edge is within a wavelength or so from the surface. For larger angles of incidence, the decrease is much more rapid, and the luminosity is perceptible only when the edge is practically in actual contact with the prism. When the incident light is polarised, the edge radiation is found to be partially polarised, the stronger component of the electric vector being perpendicular to the edge. This is in accord with the theoretical expectations.

*Diffraction of light by curved surfaces*: A convex rounded edge on which light is incident reflects the light falling upon it, and the superposition of this on the exterior diffraction increases its apparent intensity, and makes it perceptible over larger angles. On the other hand, the light bending into the region of shadow is diminished in intensity by reason of the curvature of the edge and its visibility is restricted to smaller angles. This effect may be observed with edges having a curvature which may lie within wide limits and is expressible in microns or centimetres or even metres. The angles involved and the magnitude of the effects would, however, naturally depend on the radius of curvature. A typical experiment demonstrating the effect under consideration is the comparison of the intensities of the bright spots seen at the centre of a shadow of a circular disk and of a spherical ball of the same radius.* To make the comparison significant, the disk should have a sharp edge, while the ball should be an accurately made sphere with a highly polished surface. The ball and disk may be set side by side and held in the path of a beam of light from a point-source of light. The spots at the centre of their respective shadows may then be directly viewed and their intensities photometrically compared.

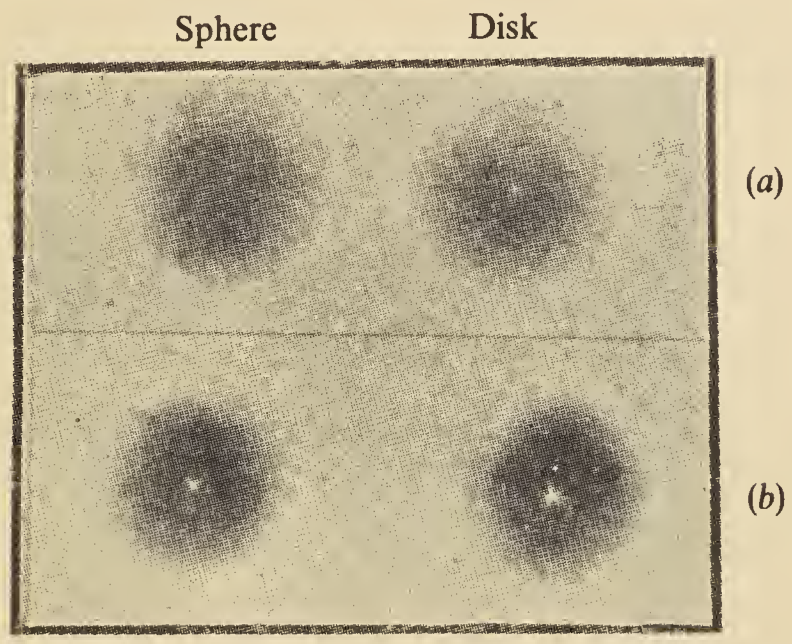


**Figure 64.** Diffraction by sphere and disk of equal diameter (1·5 cm) and at (*a*) 40 cm, and (*b*) 120 cm distances from them.

*C V Raman and K S Krishnan, *Proc. Phys. Soc.*, 1926, **38**, 350.

A great difference is noticed in the brightness of the two spots. This difference increases as we approach the objects (figure 64); the intensity of the spot for the sphere falls off more quickly than for the disk, becoming a very small fraction of it. At a distance of 120 centimetres, the difference of the intensities is still perceptible, while at greater distances, the two spots tend gradually to approach equality of brightness.

The diffraction of light by convex cylindrical edges similarly shows interesting features. If the radius of curvature of the edge is of the order of a few centimetres, the phenomena in the vicinity of the edge may be conveniently studied with a distant slit parallel to the cylinder as the source of light and a microscope for viewing the fringes.* The effects are, however, more striking when a strip of mirror glass, 3 centimetres wide and 75 centimetres long, is bent into a cylindrical shape of large radius of curvature. The diffraction fringes produced by it can be seen directly on a screen or viewed with an ordinary magnifier.† The general nature of the case will be evident when we consider the rays of light which pass by the cylinder without meeting its surface and those which fall on it and are reflected. These form the two branches of a cusped wave-front which is fully developed at the edge of the cylinder grazed by the incident rays. The two branches interfere, giving fringes parallel to the edge, their number and visibility being greatest when the plane of observation is that of the edge itself. As we move away from this plane, the intensity of the reflected rays diminishes more quickly on account of their divergence. The visibility of the fringes, therefore, falls off until, finally, they are scarcely distinguishable from the usual type of diffraction bands along the boundary of the shadow of a straight edge. The law of the spacing of these fringes may be readily deduced. If $a$ is the radius of the cylinder, and $d$ the distance of the plane of observation from its edge, the distance $x$ of the maxima and minima of the illumination from the edge of the geometric shadow may be found by eliminating $\varepsilon$ from the two equations

$$x = 2\varepsilon d + 3\varepsilon^2 a/2 \quad \text{and} \quad n\lambda = 4\varepsilon^2 d + 4\varepsilon^3 a.$$

When $d$ is much larger than $a$, we have $x = \sqrt{n\lambda d}$, and the positions of the fringes differ only slightly from those in the diffraction pattern due to a straight edge for which we have $x = \sqrt{(n - \frac{1}{4})\lambda d}$. Indeed except in the vicinity of the cylinder, the spacing of the fringes is scarcely different from that due to a sharp straight edge, the principal difference being in the greater number and the visibility of the fringes. Phenomena of the same nature are also observed in the exterior diffraction by

*N Basu, *Philos. Mag.*, 1918, **35**, 79.

†T K Chinmayanandam, *Philos. Mag.*, 1919, **37**, 9. The same procedure may be used for observing the optical analogue of the whispering gallery effect with a concave surface.

reflecting obstacles of other forms having a convex surface, e.g., cones, spheres or ellipsoids.*

The rapidly diminishing intensity of the light entering the region of shadow and its restriction to smaller angles with increased radius of curvature of the surface are facts of observation which require explanation. Some light is thrown on the matter by a consideration of the facts regarding the diffraction of light by metallic screens dealt with earlier in this lecture. An examination of the formulae shows that the nature of the results to be expected is greatly influenced by the angle of incidence of the light on the surface of the screen.† When the incidence is sufficiently oblique, both components of the electric vector in the diffracted rays parallel to the surface of the screen become of equal intensity, and are quite small. This indication of theory is in accord with observation,‡ and may be expected to be true for all reflecting surfaces whether metallic or not. In our present problem, we are concerned with the diffraction of light which is incident very obliquely or actually grazes the surface of the obstacle. Hence, unless the radius of curvature of the obstacle is very small, the polarisation effects would be negligible and both components of the diffracted light would be weak along its surface. The greater the radius of curvature, the longer the arc of the surface which the diffracted ray has to graze before it can emerge at any desired angle. Hence, the attenuation of

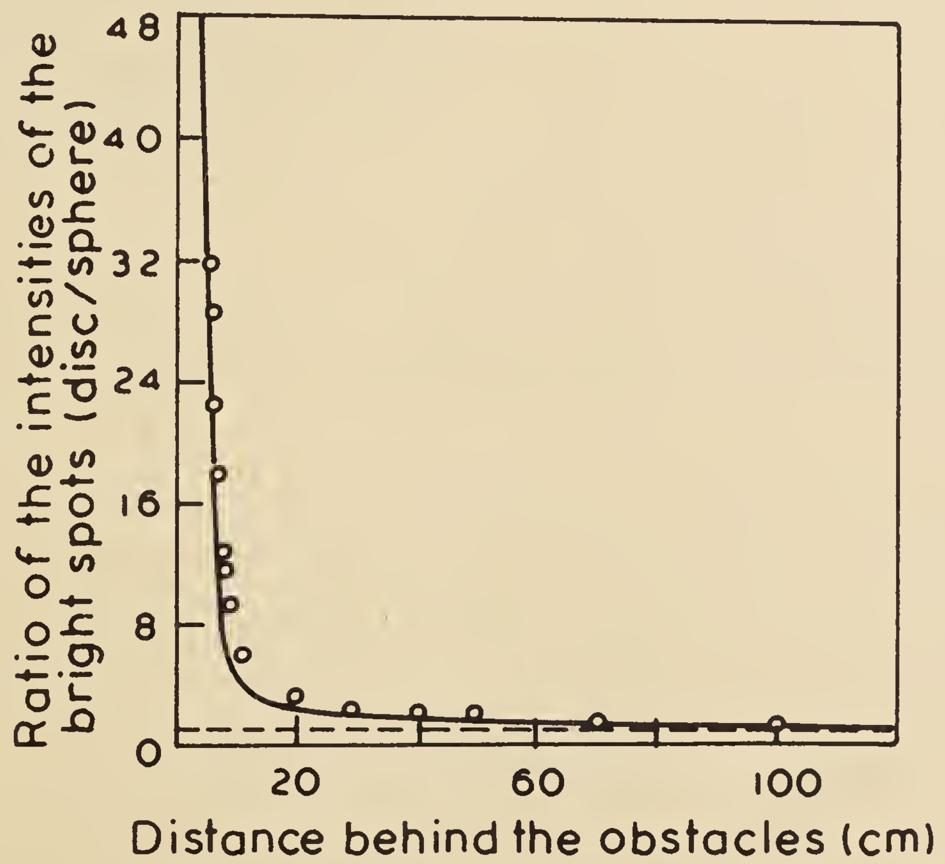


**Figure 65.** Comparison of theoretical attenuation factor and experimental data.

*A B Datta, *Bull. Calcutta Math. Soc.*, 1922.

†C V Raman and K S Krishnan, *Proc. R. Soc. London*, 1927, 116, 254.

‡S K Mitra, *Philos. Mag.*, 1919, **37**, 50.

the diffracted light must increase rapidly with the radius of curvature. The problem here considered is evidently analogous to that of the bending of electric waves around the surface of the earth. The attenuation factor for the amplitude in this case* is $\exp(-0{\cdot}70(2\pi a/\lambda)^{1/3}\cdot\theta)$, where $a$ is the radius of the earth, $\lambda$ the wavelength and $\theta$ the angle which the waves have to creep round. The formula indicates a rapid fall in the intensity of the radiation as it bends round, if the radius of curvature of the edge is large compared with the wavelength, and this is in accord with actual experience in the optical problem.

The measurements† made of the relative intensity of the bright spots in the shadow of a sphere and a disk of equal diameter at different distances along the axis enable a quantitative test to be made of the attenuation formula quoted above. The arc over which the diffracted waves have to creep may be taken as zero for the sharp-edged disk. For the sphere, it is the arc on the surface between the circles of contact with the tangent cones drawn to it respectively from the point-source of light and from the point of observation. Figure 65 shows the theoretical attenuation curve for this case, the experimental data being indicated by dots. The general agreement leaves little doubt that the explanation of the facts which has been put forward is on the right lines.

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*Riemann–Weber, *Differential–Gleichungen Phys.*, 1927, **2**, 594.
†C V Raman and K S Krishnan, *Proc. Phys. Soc.*, 1926, **38**, 350.

## Lecture III

# Coronae, haloes and glories

IN the present lecture, we shall consider the phenomena which arise from the diffraction of light simultaneously by a great many particles or obstacles, the size of these being sufficiently large to permit of an elementary approach to their explanation. Many such phenomena are known, and it is of advantage to consider them together in a general survey, so that the common principles underlying all such cases may be brought into relief. The optical character of the particles, their size, shape and number, the manner in which they are disposed and orientated in space, and the particular circumstances of observation may all influence the results. Included within the survey are some natural phenomena which may be observed in the earth's atmosphere when particles of water or ice are present in it and are suitably illuminated by the rays of the sun or the moon.

*Diffraction by a cloud of particles*: Secondary radiations derived from the same primary source, and therefore having specifiable phase-relations with it and with each other, would evidently be capable of interference. Hence, when a cloud of particles is present in a light-field and the radiations diffracted by the individual particles are superposed at any given point of observation, interferences would arise. Their character would be determined by the phase-differences, in other words, by the optical paths traversed from the original source to the individual diffracting particles and thence to the point of observation. Considering first a case in which the line joining the primary light source with the point of observation passes *through* a cloud of particles, it is evident that the optical paths would differ infinitely little for all particles lying on this line or in its immediate vicinity. On the other hand, the optical path would alter in a rapidly increasing measure with the actual position of the particle as it lies further and further away from this line. Thus, in general, *except along the direction of propagation of the*

*light rays from the original source*, the distribution of the individual diffracting particles in space is a controlling factor in determining the optical effect produced by a cloud of such particles.

Considering the effects produced by the cloud in any direction other than that of the primary rays, we shall assume that the particles are distributed at random and execute *rapid uncorrelated movements* within the cloud. It is obvious that in such circumstances, the interferences between the effects of the individual particles would be unobservable. We may then assume the observed intensity in the field to be a summation of the intensities of the individual effects. If all the $n$ particles were similar and produced similar effects at any point of the field, the total observed intensity would be $n$ times the effect of an individual particle. On the other hand, *if the particles occupied stationary positions* within the cloud, the situation would be entirely different. However numerous the particles might be, and howsoever they might be distributed within the cloud, the phase-relations between them would be determinate, and hence the interferences between the individual effects should be observable. We have to evaluate the result of such interferences to find the optical effect due to the entire cloud of particles.

The problem which thus arises of finding the effect of $n$ superposed radiations of equal amplitude but of differing phase may be dealt with graphically by means of a two-dimensional diagram. Choosing a given point $O$ as origin, we draw a set of $n$ *radii vectores* of equal length $A$ representing the amplitudes of the $n$ superposed radiations; their relative phases would be given by the angles which these make with each other. It is evident that the resultant obtained by the summation of the vectors so drawn would depend on the manner in which the terminal points of the radii are distributed around the circle on which they lie. If, for example, the phases are all identical, the vectors would all be superposed, and the resultant amplitude would be $nA$ and the resultant intensity $n^2A^2$. If, on the other hand, the $n$ vectors divide the circle into $n$ equal arcs, the resultant amplitude and intensity would both be zero. If now, we consider the case in which the $n$ phases are distributed at random, it is obviously impossible to specify what either the amplitude or the phase of the resultant would be *in any particular trial.* The diagram, however, gives some indications of a general character regarding what we may expect to find. If the number $n$ be sufficiently large, the most probable location of the points on the circle in a random distribution would evidently be a sensibly uniform one. Hence, the most probable resultant intensity would be the same as for a perfectly regular distribution, namely, zero. It is evident also that the resultant intensity averaged over a large number of trials would be $nA^2$. This follows immediately, if we suppose that the phases vary continuously and rapidly with time, so that the $n$ intensities, each of which is $A^2$, become additive.

Thus, *for a random distribution of phases, the most probable resultant intensity is zero, while the average intensity in a large number of trials would be* $nA^2$. For a

more complete description of the case, we have to find an expression for the probability that the intensity has a specified value I in different trials. It is easily verified that this is given by the exponential probability formula

$$\mathrm{d}W = \exp(-I/nA^2)\cdot d(I/nA^2).$$

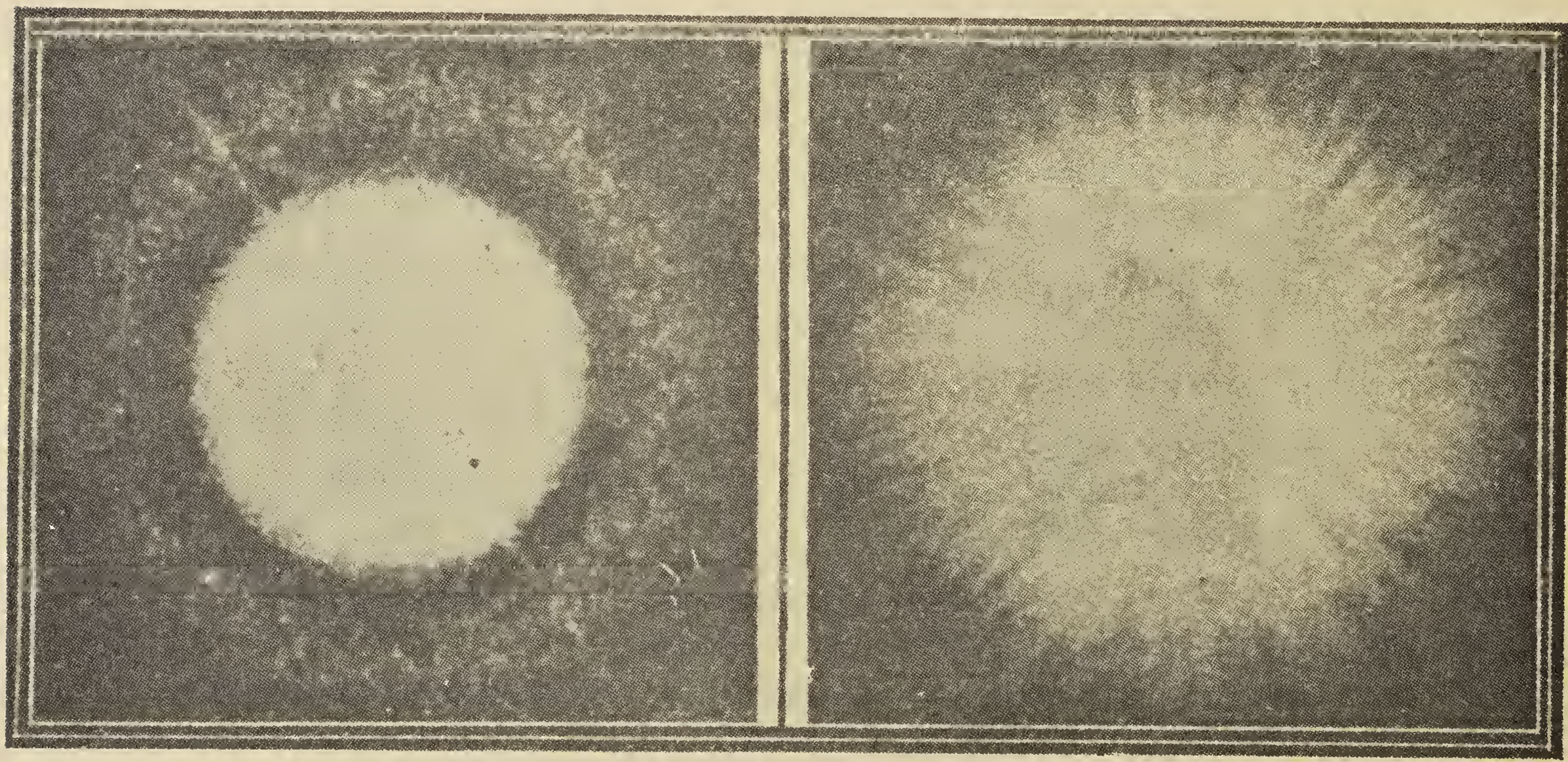
The formula agrees with what the graphical treatment suggests; it shows that the probability is a maximum for zero intensity and that it diminishes continuously and ultimately vanishes with increasing values of the intensity. Further, the integration of $\mathrm{d}W$ over all possible values of $I$ gives unity as it should, and the average intensity found by integrating $I\mathrm{d}W$ over all possible values of $I$ is $nA^2$, as already found. Hence, the formula $\mathrm{d}W = \exp(-f)\mathrm{d}f$ correctly gives the probability of any given value of the observed intensity expressed as a fraction $f$ of the average intensity. We note also that the formula agrees with that given by a detailed consideration of the problem on the basis of the general theory of probability.* It is important to notice that the chance of finding any particular resultant intensity decreases continuously as it increases, and that the average intensity is very far indeed from being the most probable intensity. Indeed, the average is determined entirely by the cases in which the resultant intensity is greater than the most probable value which is zero.

A point source of monochromatic light viewed through a cloud of particles would appear surrounded by a corona or halo due to diffraction by the particles. The radiations diffracted by the particles and reaching the retina of the eye and focussed thereon are superposed and would thus be capable of interfering with each other. The foregoing discussion shows that if the particles are all similar and are disposed at random in space, the intensity in the corona would only *statistically* be a summation of the intensities of the diffraction patterns produced by the individual particles. While the general features of the pattern due to each separate particle would be recognisable in the aggregate effect, the latter is essentially different in detail. Instead of a continuous distribution of intensity, we have a violently fluctuating one which, in general terms, may be described as a dark field on which appear a great many points of illumination irregularly distributed and of varying brightness. The illumination at such points arises from the accidental agreements of phase of the effects of the diffracting particles, while the dark field results from the general cancellation of their effects by mutual interference. *Each such point in the corona exhibiting an observable intensity is, therefore, essentially an optical image of the original source produced by the entire*

*Rayleigh, I, *Philos. Mag.*, 1880, **10**, 73, *Scientific Papers*, **1**, p. 491.

*cloud of particles functioning as a randomly distributed set of secondary sources of light.**

As we shall show later, the theoretical conclusions set out above are fully supported by the experimental results (see figure 66a). It is important to remark that, in practice, cases may also arise in which the diffracting particles are not distributed at random in space. The distribution may either present a closer approach to uniformity, or may tend in the opposite direction, the individual particles clustering together to form large groups. The optical effects would in either case differ from those observed with a random spacing of the particles. In the limiting case of a perfectly uniform distribution, the particles would in effect constitute a diffraction grating. We would then get sharply-defined and intense diffraction spectra located at regular intervals in a dark field. The transitional cases, where the distribution of the particles in space is neither completely random nor completely uniform, are of particular interest. The phenomena observed in all such cases may be included under the general descriptive term of "diffraction haloes", the expression "corona" being reserved for the case of randomly spaced particles. As examples of such haloes, we may turn to figure 23 on page 432 of the second lecture, in which the effect of viewing a source of light through a thin piece of mother-of-pearl was illustrated. As was remarked on



(a) (b)

**Figure 66.** Diffraction corona due to lycopodium spores showing (**a**) granular structure in monochromatic light, and (**b**) radial streaks in white light.

*G N Ramachandran, *Proc. Indian Acad. Sci.*, 1943, **A18**, 190.

page 432, the size and shape of the crystallites of aragonite, and their spacing and orientation within the mother-of-pearl determine the character of these haloes, and as will be evident from the three examples reproduced, these features and the resulting haloes are very different in the three great classes of mollusca. We shall meet with other cases of the production of diffraction haloes later in the present lecture.

*Coronae due to water droplets*: The well known coronae or disks of light with marginal coloured rings seen surrounding the sun or the moon when viewed through thin clouds are amongst the most familiar phenomena of meteorological optics. What we see in such cases is evidently the cloud itself which becomes visible by reason of the light incident on it and diffracted through various angles by the particles of which it is composed. The optical character of the phenomena, as well as the form and level of the clouds exhibiting them, make it clear that the coronae with vividly coloured rings owe their origin to minute spherical droplets of water contained in the clouds. Thin clouds consisting of small particles of crystalline ice do exhibit observable disks surrounding the sun or moon when seen through them. But these are usually of smaller size and have a quite different and characteristic distribution of intensity. They are also much less vividly coloured than the coronae arising from water clouds. Indeed, a cirrus haze can just as readily be recognised by the diffuse illumination observed in the vicinity of the sun or the moon as by the familiar halo due to refraction by the ice-crystals seen at an angle of 22° from the luminary. It should be remarked also that clouds do sometimes display marked iridescence in circumstances which indicate that their temperature must be well below the freezing point of water. Such iridescence is often observed at quite large angles with the sun, though, of course, a complete corona is not usually then seen. Whether such iridescent clouds consist of crystalline particles of ice is a debatable question. The vividness of the colours suggests that the particles are probably supercooled droplets of water, or possibly even an amorphous form of solidified water. The retention of an amorphous structure and of non-crystalline shape by droplets of water when supercooled is a well-established fact of observation under laboratory conditions, and it is permissible, therefore, to suppose that it can also occur in nature.

Coronae can also be artificially produced and observed in the laboratory over a wide range of droplet size, and they are actually more striking than the coronae seen in nature, the colours of which are somewhat diluted by the finite angular dimensions of the sun or the moon. As is well known, a sudden expansion of moist air, if of sufficient magnitude, results in the formation of a cloud consisting of minute droplets of water. The condensation usually occurs around "nuclei" of some sort, and the number of droplets formed and therefore also their size depends on the number of such nuclei present. The size of the droplets as indicated by their rate of free fall, as also by the optical effects which we shall presently consider exhibits a remarkable uniformity. It may be regulated within

wide limits by varying the amount of the expansion and the number of nuclei present. Very beautiful and interesting effects are observed when such clouds are viewed under strong illumination, or if a bright source of light is seen through such a cloud. By using an electric arc as the source of light and projecting an image of it as seen through the cloud chamber on the screen, the coronae due to water droplets can be shown as a beautiful lecture demonstration. Using a red glass as a monochromatising filter, or with a quartz mercury lamp and single ray filter, the coronae may be readily photographed.

The particles of water in a cloud are very numerous and yet not so numerous as to occupy an appreciable fraction of the volume of the air. They are obviously distributed at random in the space, and presumably execute small irregular movements. It follows that though the droplets are all illuminated by the same original source of light, we may nevertheless regard them as practically independent sources of diffracted radiation. The justification for this is that the phases of the diffracted radiations in any assigned direction from the different droplets are totally unrelated. An exception must, however, be made in considering the rays diffracted in the same direction as the rays incident on the drops; for, in this direction the optical paths for all the drops are identical, and hence their amplitudes must be added to find their resultant effect. In all other directions, we may add the intensities of the diffracted radiations from the drops and expect the results to be in accord with the facts.

The appearance of the coronae in the experiments is found to vary in a remarkable manner with the size of the droplets. The central disk of the corona as seen with the finest droplets is not white but shows vivid colour varying with their size; as the drop size is altered progressively, there is a recognisable cycle of changes in the colour observed. The sequence of changes observed with increasing size of the droplets is not a mere progressive diminution in the angular diameter of the corona as seen in monochromatic light. A periodic alteration in the diameter and intensity of the coronal disk is noticeable, while from the published photographs* it is evident that the relative intensities and positions of the outer rings vary notably when the drop size is altered.

The experimental facts thus compel us to reject the usual explanation of coronae which is based on the assumption that the droplets may be regarded as opaque spheres. The starting point for a more satisfactory theory is a consideration of the phase-changes resulting from the passage of plane waves through a transparent sphere of liquid. In the limiting case when the refractive index of the liquid $\mu$ is only slightly greater than unity, the waves pass through the sphere without any change of amplitude but with a change of phase $\xi \cos \varepsilon$ where $\xi$ is $4\pi a(\mu - 1)/\lambda$; $a$ is the radius of the sphere and $\varepsilon$ is half the supplement of the

*M N Mitra, *Indian J. Phys.*, 1928, 3, 175. The photographs reproduced in figure 67 are due to Mr H Ramachandran.

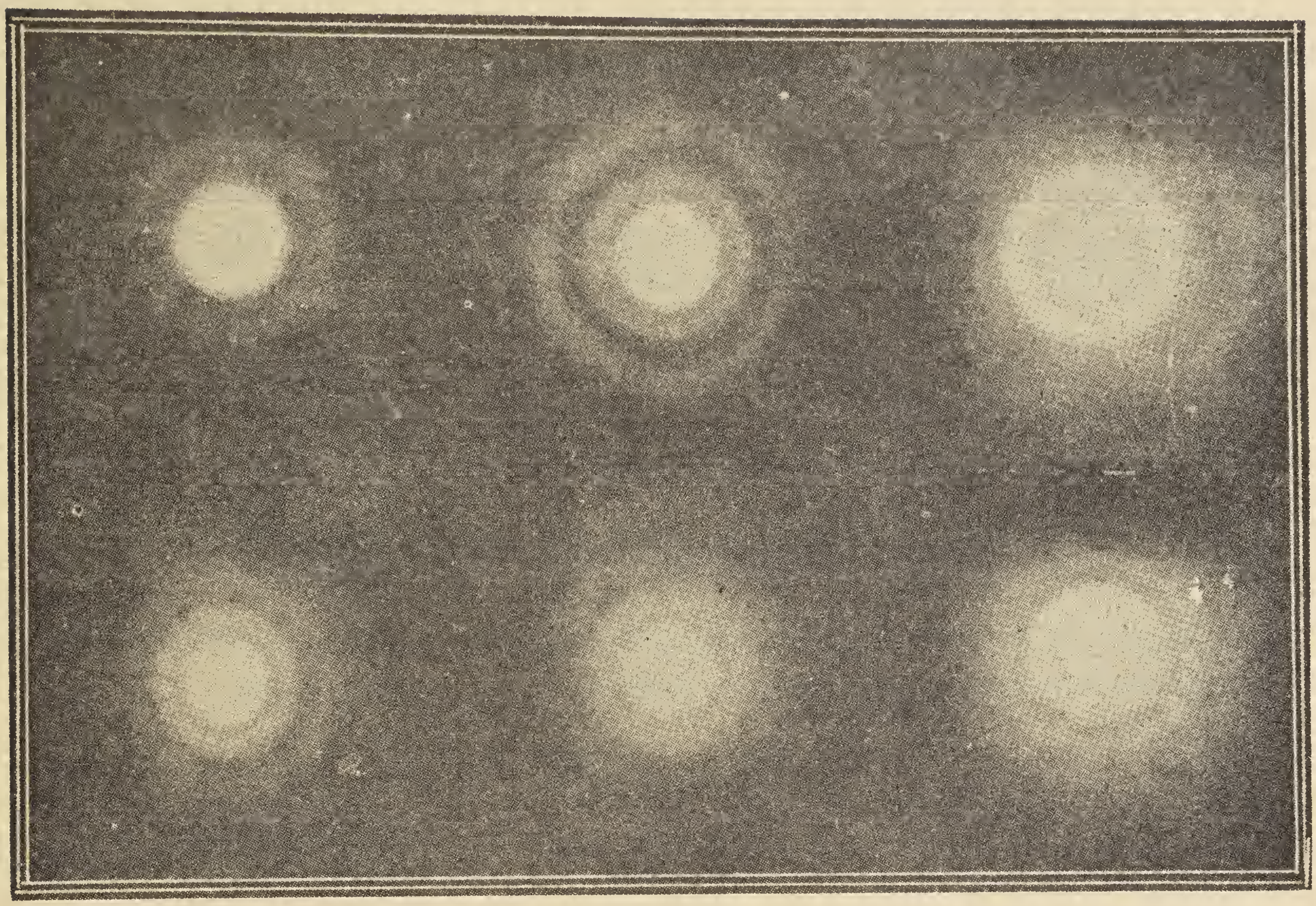


**Figure 67.** Coronae due to water droplets of different sizes.

angle subtended at the centre by the path inside the sphere, being zero for a ray passing centrally and $\pi/2$ for a marginal ray grazing the surface. The wave-front on emergence would thus exhibit *a dimple* having the same radius as the drop and a depth equal to the maximum retardation it produces. If $(\mu - 1)$ be not small, this simple picture would not be accurate, as the wave-front on emergence from the drop would exhibit both amplitude and phase changes. We may, however, adopt it as the basis for an approximate theory which, though it could scarcely be expected to give a complete account of the facts, should nevertheless go far towards doing so.

If the dimples in the wave-front be removed, and the resulting holes filled up, the diffracted radiations would disappear. It follows that the effect of a drop may be found by *subtracting* from the optical effect of the dimple in the wave-front, the effect produced by plane waves of light passing through a circular aperture of the same radius in an opaque screen. The relation between the amplitude and phases of the two effects which are thus superposed determines the observed phenomena, and it is evident that the interference between them is responsible for the observed cycle of changes in the appearance of the corona with increasing drop size.

The detailed calculations are made on much the same lines as for a simple circular aperture. Besides the phase-change $\xi \cos \varepsilon$, we have also to consider the

phase-difference between the different parts of the wave-front introduced by the observation of their resultant at a great distance $d$ and at an angle $\beta$ with the incident rays. This may be written as $\eta \sin\varepsilon \cos\alpha$, where $\varepsilon$ is the angle already introduced, and $\alpha$ is the azimuthal angle defining the position of an element of area, viz., $a^2 \sin\varepsilon \cos\varepsilon \, d\varepsilon \, d\alpha$, in the wave-front emerging from the drop. $\eta$ stands for $2\pi a \sin\beta/\lambda$. The disturbance in the direction $\beta$ due to the light which has traversed the drop is given by the integral

$$\int_0^{\pi/2}\int_0^{2\pi} \frac{a^2}{\lambda d} \sin(Z - \xi\cos\varepsilon + \eta\sin\varepsilon\cos\alpha)\sin\varepsilon\cos\varepsilon \, d\varepsilon \, d\alpha.$$

On integration with respect to $\alpha$, this yields

$$\frac{2\pi a^2}{\lambda d}\int_0^{\pi/2} J_0(\eta\sin\varepsilon)\sin(Z - \xi\cos\varepsilon)\cos\varepsilon\sin\varepsilon \, d\varepsilon.$$

If we put $\mu = 1$, $\xi$ vanishes, and the integral reduces, as it should, to the effect of a simple circular aperture of radius $a$, namely,

$$\frac{2\pi a^2}{\lambda d}\sin Z \cdot \frac{J_1(\eta)}{\eta},$$

and, as already remarked, the contribution of the drop to the corona is found by deducting this from the foregoing integral.

In the exact forward direction, $\beta$ is zero and $\eta$ vanishes. The foregoing integral can then be completely evaluated, and after the deduction indicated is made, it gives for the amplitude the expression*

$$\frac{2\pi a^2}{\lambda d}\sin Z\left(\frac{\cos\xi}{\xi^2} + \frac{\sin\xi}{\xi} - \frac{1}{2} - \frac{1}{\xi^2}\right) + \frac{2\pi a^2}{\lambda d}\cos Z\left(\frac{\cos\xi}{\xi} - \frac{\sin\xi}{\xi^2}\right).$$

The intensity of forward scattering is thus

$$I_f = \frac{4\pi^2 a^4}{\lambda^2 d^2} F(\xi),$$

where

$$F(\xi) = \left[\left(\frac{\cos\xi}{\xi^3} + \frac{\sin\xi}{\xi} - \frac{1}{2} - \frac{1}{\xi^2}\right)^2 + \left(\frac{\cos\xi}{\xi} - \frac{\sin\xi}{\xi^2}\right)^2\right].$$

The discussion hereafter follows, in the main, two papers by G N Ramachandran.† $F(\xi)$ is plotted against $\xi$ in figure 68 exhibiting the manner in which the forward intensity varies with the size of the droplets. The curve starts from the origin, increases in direct proportion to $\xi^2$, reaches a maximum and then oscillates, finally tending to a value 1/4. For very small particles, the intensity

*T A S Balakrishnan, *Proc. Indian Acad. Sci.*, 1941, **A13**, 188.

†G N Ramachandran, *Proc. Indian Acad. Sci.*, 1943, **A17**, 171 and 202.

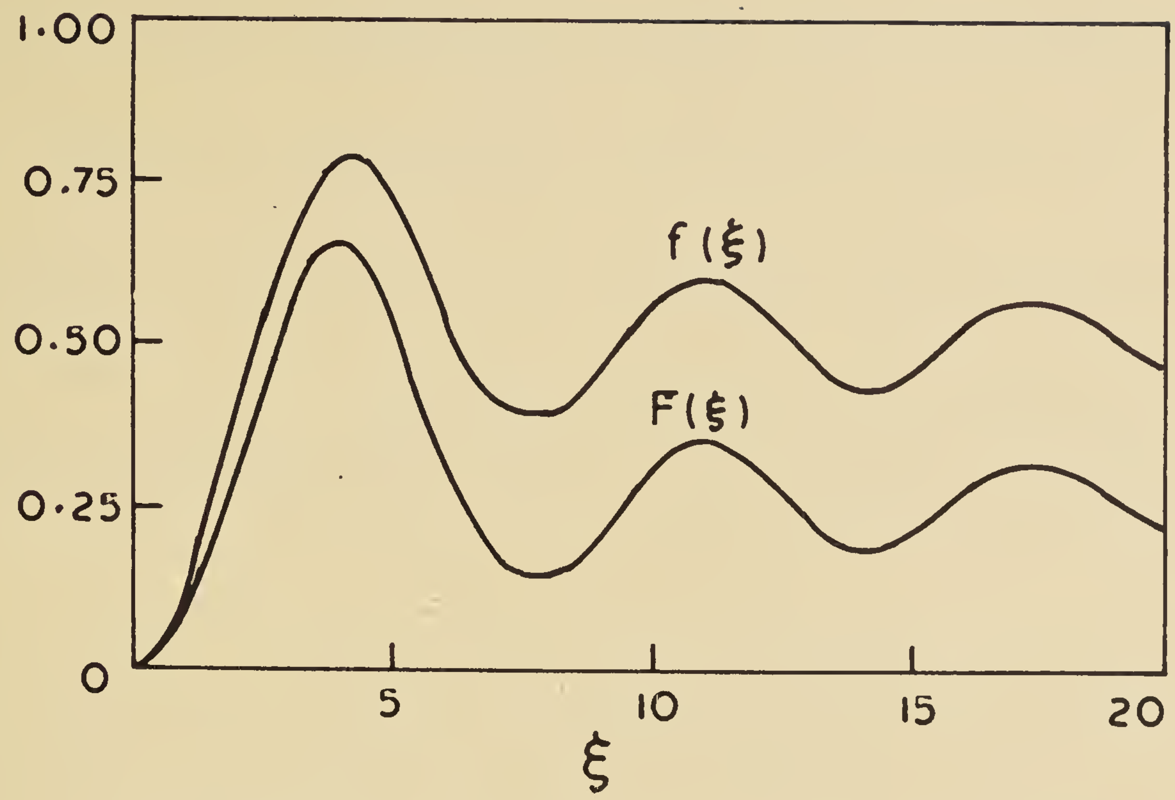



**Figure 68.** Graph showing the variation of $F(\xi)$ and $f(\xi)$ with $\xi$.

reduces to the expression

$$\frac{64\pi^4}{9d^2}(\mu-1)^2\frac{a^6}{\lambda^4}.$$

This is identical with Rayleigh's well-known formula for the blue of the sky, except that in our formula we have a factor $4(\mu-1)^2$ instead of $(\mu^2-1)^2$ to which it is nearly equal if $\mu$ does not differ much from unity as is assumed in the theory. Thus, in the initial stages, for very small droplets, the theory predicts a preferential scattering of the smallest wavelengths, which is a readily observable phenomenon. For larger particles, the intensity reaches a maximum and then oscillates. Over this range, we may neglect terms in $F(\xi)$ involving higher powers of $1/\xi$ than the first and write

$$I_f = (4\pi^2 a^4/\lambda^2 d^2)(1/4 - \sin\xi/\xi).$$

It is evident from this that the light would show a cyclic change of colours with increasing particle size. Finally, for very large particles, the expression becomes equal to $(\pi^2 a^4/\lambda^2 d^2)$, agreeing with the intensity at the centre of the diffraction pattern due to a circular aperture of the same radius.

It is evident also that the superposition upon the primary waves of the radiations scattered forward by the particles in a thin cloud of thickness $dl$ must result in the alteration of both the amplitude and the phase of the latter in its passage through the layer. The former would depend upon the coefficient of $\sin z$ in the expression for the forward scattering and the latter on the coefficient of $\cos z$. Writing the diminution of the amplitude due to the particles in the layer as

$kdl$, and the retardation as $(n-1)dl$, we deduce that

$$k = 2\pi N a^2\left(\frac{1}{2}+\frac{1}{\xi^2}-\frac{\sin\xi}{\xi}-\frac{\cos\xi}{\xi^2}\right) = 2\pi N a^2 f(\xi) \quad \text{(say)}$$

$$n-1 = N\lambda a^2\left(\frac{\sin\xi}{\xi^2}-\frac{\cos\xi}{\xi}\right),$$

where $N$ is the number of particles per unit volume.

The intensity of the incident beam falls off in its passage through the cloud, and after passing through a length $l$ it may be represented by

$$I_l = I_0 \exp(-2kl).$$

$I_0$ being the intensity of the incident beam. The attenuation coefficient $2k$ is thus proportional to $f(\xi)$. The course of this function with increasing $\xi$ is also plotted in figure 66, and is seen to be similar to that of the forward intensity, giving rise to periodic changes in the colour of the transmitted beam also. When $\xi$ is not small, higher powers of $1/\xi$ may be neglected, and the attenuation coefficient becomes $4\pi N a^2 l(1/2-\sin\xi/\xi)$, finally tending to a value $2\pi N a^2$ which is the same as if the particles were opaque. *Also, the value of the attenuation coefficient is double what would be given by simple geometric considerations, and this may be explained as due to diffraction which introduces an extra loss of energy.*

Since the attenuation coefficient and the intensity of forward scattering undergo similar variations as $\xi$ increases, it is clear that the colour of the source as seen through the cloud must be complementary to that of the light scattered forward as can be observed, for instance, when the sun is seen through the puffs of steam emitted by a locomotive. However, with thick and long columns of cloud, the scattered light itself will be attenuated, and the phenomena thereby modified.

The refractive index of the cloud also undergoes oscillations, alternately becoming greater and less than unity, as the size of the particle steadily increases. In the limiting case of large particles, it becomes practically unity. This is readily explained by the fact that large droplets transmit little light, and such opaque particles can produce change in refractive index.

So far, we have been considering only the light coming out in the forward direction. When we turn to the diffraction in other directions, it is found that the integral for the amplitude cannot be evaluated completely; but it can be expressed in the form of a series. The method to be adopted depends upon whether $\xi$ is small or large. In the case where coronas are observed, $\xi$ is sufficiently large, and the evaluation may be done by writing $x=\xi\cos\varepsilon$, and by repeatedly integrating by parts with respect to $x$. We then obtain a series, which, on omitting terms containing higher powers of $1/\xi$ than the first, reduces to

$$\frac{2\pi a^2}{\lambda d}\left[\sin Z\left(\frac{\sin\xi}{\xi}-\frac{J_1(\eta)}{\eta}\right)+\cos Z\frac{\cos\xi}{\xi}\right].$$

The contribution of the drop to the intensity is therefore

$$\frac{4\pi^2 a^4}{\lambda^2 d^2}\left[\frac{J_1^2(\eta)}{\eta^2}-\frac{2\sin\xi}{\xi}\cdot\frac{J_1(\eta)}{\eta}+\frac{1}{\xi^2}\right].$$

The intensity depends both on $\xi$ and $\eta$, and hence on the size of the droplets and on the angle of diffraction. Since the function $\sin\xi/\xi$ oscillates and diminishes progressively as $\xi$ increases, the intensity of the corona would fluctuate as $a/\lambda$ alters, the fluctuations diminishing in extent as the size of the droplets increases. The coronal disc would, in consequence, exhibit colours which are most vivid with the smallest drops, and go through cycles with their saturation progressively diminishing as the drops become larger. The ratio of $\sin\xi/\xi$ to $J_1(\eta)/\eta$ increases rapidly as we move away from the centre of the corona. Hence, the colours would be more prominent towards the margin of the central disc than at its centre. The presence of the function $J_1(\eta)/\eta$ gives rise to alternate bright and dark rings, which can be observed in monochromatic light, but the positions of these rings would be greatly influenced by the value of $\sin\xi/\xi$ and the whole appearance of the corona would be different from that of the diffraction pattern of an opaque circular disc. The cyclic changes in this function $\sin\xi/\xi$ also give rise to an alternate contraction and expansion of the ring system. In the limit when $\xi$ is sufficiently large, the corona, or at least the central part of it for which $\eta$ is not too great, tends to become similar to that given by a set of opaque spheres.

*Colours of mixed plates*: Very beautiful phenomena are shown by the heterogeneous films known as "mixed plates". Though they differ essentially from the coronae due to water-droplets discussed in the foregoing pages in their nature and origin, there are, nevertheless, some features common to the two cases which justify their being considered in this lecture: To obtain the "mixed plates", a few drops of egg albumen are spread between two plates of glass about ten centimetres square in size and a centimetre thick. The plates are then separated and put back together a few times and slid over each other with a circular movement. The material is thus worked up into a film of uniform thickness which, when seen under the microscope, appears as a thin layer of liquid enclosing a large number of air-bubbles. These vary in size and are irregularly arranged and often depart considerably from a circular shape, but except in special circumstances, show no bias towards elongation in any particular direction. Gorgeous colours are shown by such films when they are freshly prepared and are not too thick. On being allowed to stand, the albumen in the film begins to dry up and forms hexagonal networks between the two plates. The character of the optical phenomena then completely alters.

The colours of mixed plates may be studied in two distinct ways which are roughly analogous to the Haidinger and Newtonian methods of viewing the interferences of transparent plates. The first method is to prepare a film of uniform thickness between flat plates and to view the source of light through the

films with the eye placed close behind it and adjusted for distant vision. The second method is to form a mixed plate between two glass lenses in the manner of Newton's rings and to view the illuminated film with the eye placed behind the plate at a suitable distance. As the effects alter with the angles of incidence and observation, the source of light should in either case be of small angular area. An aperture in a screen backed by a filament lamp or a mercury arc may be employed and a dark field of observation should be provided around it. In the first method of observation, the eye observes the light diffracted by the film simultaneously over a wide range of angles. In the second method, films of different thickness are seen simultaneously at nearly the same angle of diffraction; this angle, of course, may be varied by moving the eye laterally. In either case, the angle of incidence of the light on the film may be varied by tilting the plate with reference to the direction of the source.

The characteristic effect* of which the explanation largely covers the whole theory of mixed plates is the diffraction halo seen around a bright source of light viewed normally through the film (see figure 69). This halo consists, in monochromatic light, of a series of circular rings, alternately bright and dark, which concentrically surround the source. The rings are narrowest nearest the centre of the halo and widen as we proceed towards its outer margin. If white light is used, the outermost ring is practically achromatic and is followed within by coloured rings. A thick plate shows numerous close rings, while a thin plate shows fewer rings which are wide apart. The rings move inwards when the thickness of the film

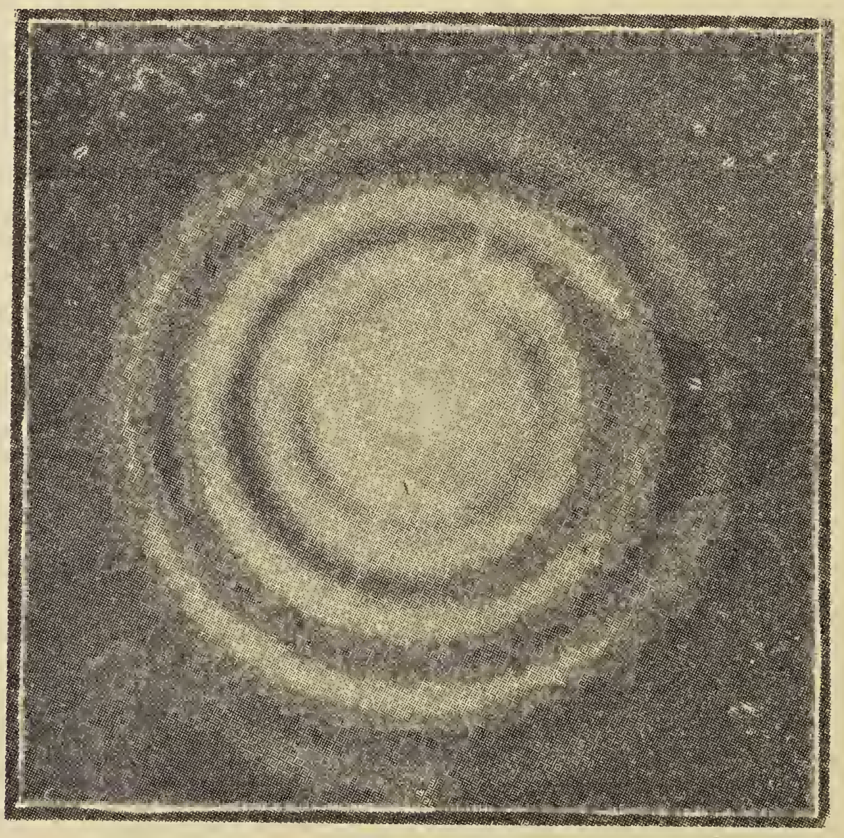


**Figure 69.** Diffraction halo of mixed plates.

*C V Raman and B Banerji, *Philos. Mag.*, 1921, **41**, 338.

is reduced. Thus, the thinner the plate, the more striking are the colours shown by the rings nearest the centre of the halo. A source of white light viewed through the film appears dimmed in intensity and exhibits a hue complementary to the colour of the part of the halo actually overlying it. A monochromatic light source fluctuates in intensity when the thickness of the films through which it is viewed is altered, appearing brightest when the halo has a dark ring at the centre, and feeblest when the source is overlaid by a bright ring. The rings near the centre of the halo show peculiar variations in their visibility depending on the thickness of the plate through which the source is viewed, sometimes being scarcely observable and sometimes very vivid and clear. Such fluctuations are not shown by the outer rings in the halo. The observations indicate that there is a second ring-system of small angular extension superimposed upon the main system and affecting its visibility when the two sets of rings are not in coincidence in any particular direction.

It is clear from the facts already stated, that the character of the halo is determined by the thickness of the liquid-air film and not by the size or shape of the air bubbles in it. It is also evident that the halo registers the characteristics of the diffracted radiation from the laminar edges in the film.* Each line element of the edge diffracts light principally in a plane normal to its own direction; the part which proceeds towards the air-side of the boundary may be referred to as exterior diffraction, and the parts towards the liquid side as interior diffraction. The existence of both types of diffraction in equal intensity but with opposite phases at small angles with the incident beam is shown by the Foucalt test. As in the case of the striae in mica, the laminar boundaries in mixed plates appear as brilliantly coloured *double lines* when the light is blocked out at the focus, the colour being complementary to that of the central fringe in the Fresnel diffraction patterns (figure 70).

Though the interior and exterior diffractions by the laminar boundary are symmetric *at small angles*, they cease to be so at larger angles. The interior diffraction is much more intense and is visible over a wide range of angles, whereas the exterior diffraction rapidly diminishes in intensity and vanishes when the angle of diffraction exceeds a few degrees. This is readily seen on illuminating the film and viewing it obliquely through a microscope. The two halves of the edge of each bubble appear of very different intensities and indeed one half very quickly vanishes, while the other half remains visible but shortens into a crescent as the obliquity is increased.† The reason for these facts is obvious when we consider the form of the laminar boundaries which owing to the action of surface tension have a specific shape independent of the size of the bubble, namely, a

*C V Raman and B Banerji, *Philos. Mag.*, 1921, **41**, 860.

†I R Rao, *Indian J. Phys.*, 1927, **2**, 167. Some photographs illustrating the effect are reproduced with this paper.

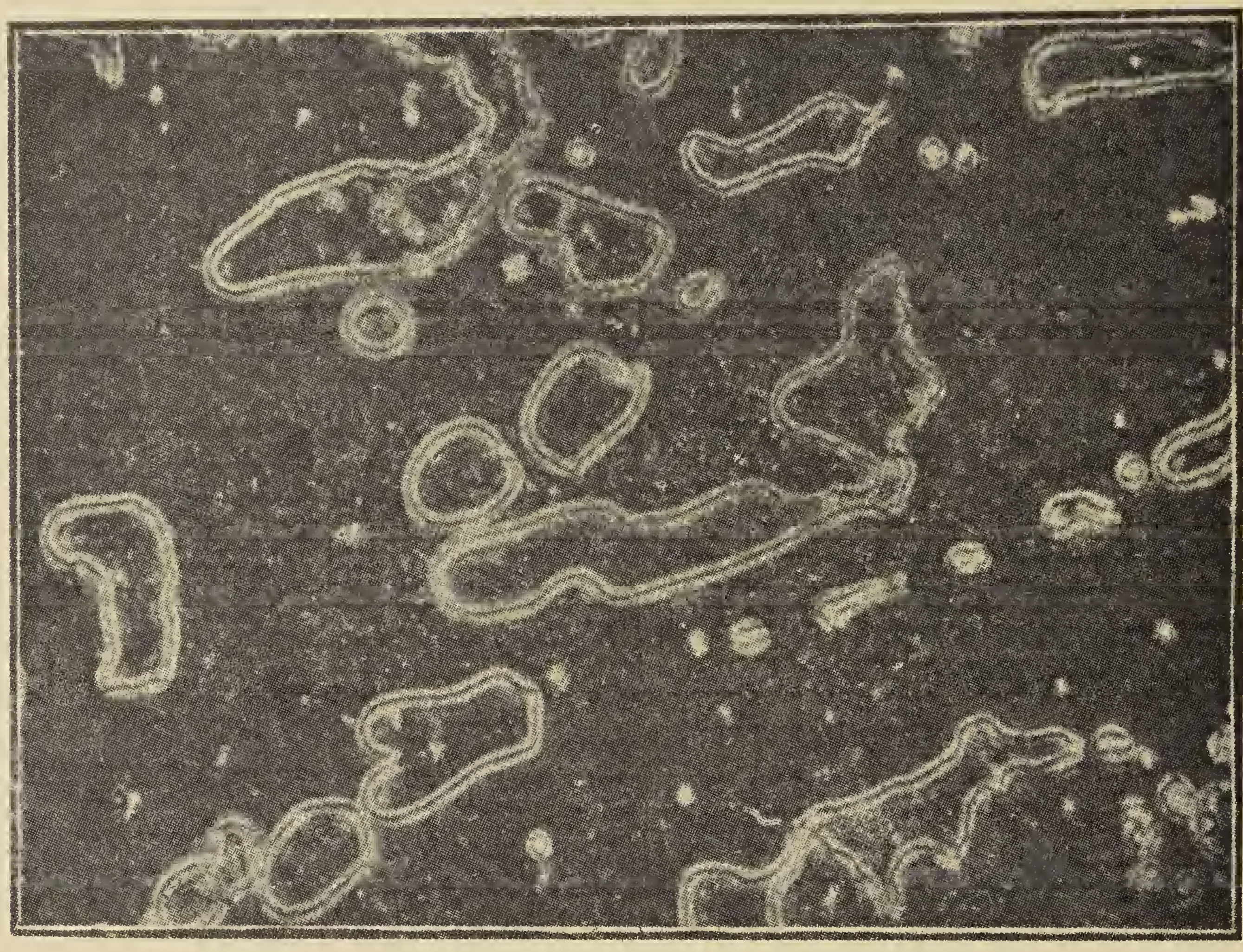


**Figure 70.** Mixed plates in the Foucalt test.

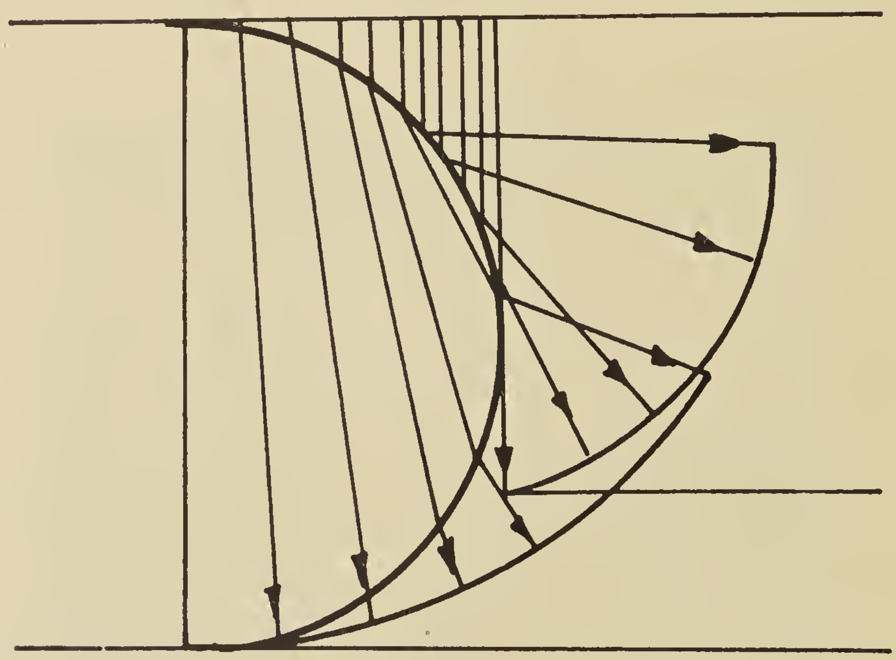


**Figure 71.** Form of wave-front in mixed plates.

semi-circular arc whose diameter is equal to the thickness of the film. The manner in which the wave-front of the light is modified in its passage through the film is indicated in figure 71. The wave incident on the curved liquid-air boundary is in part twice refracted and in part totally reflected at the interface. The twice-refracted part forms a curved continuation of the wave-front which has passed through the air, while the totally reflected part forms a cusp-like appendage to the

wave-front which has passed through the liquid. The appearance of the diffracted radiation towards the interior is thus strongly favoured, while towards the exterior it is greatly weakened. The explanation of the asymmetry of diffraction indicated by figure 71 is completely confirmed by viewing the edge under sufficiently high powers of the microscope. The emergence of the refracted and reflected rays from distinct points on the meniscus and their approach to each other with increasing obliquity can actually be observed when the films are fairly thick. The disappearance of the exterior diffraction at larger angles is found to occur more rapidly with films of greater thickness.

It is thus evident that the interior and exterior diffractions by the laminar boundaries appear superposed in the halo. The two are of the same intensity at small angles of diffraction, but at larger angles the interior diffraction is much more intense and principally determines the observed phenomena. In either case, the edge radiations are derived simultaneously from the two parts of the wave-front which have passed through the film. If we ignore the light transmitted or reflected by the liquid by the liquid meniscus, the edge radiations from the wave-fronts which emerge from liquid and air respectively would interfere under a path difference

$$(\mu - 1)d - \tfrac{1}{2}d\cdot\sin\theta - \tfrac{1}{2}\lambda$$

for interior diffraction, and under a path difference

$$(\mu - 1)d + \tfrac{1}{2}d\sin\theta + \tfrac{1}{2}\lambda$$

for exterior diffraction, $\theta$ being the angle of diffraction, $\mu$ the refractive index of the liquid and $d$ is the diameter of the meniscus, which is also the thickness of the film. When $\theta$ is sufficiently small, $\tfrac{1}{2}d\sin\theta$ may be neglected and the expressions show that the colour of the diffracted light would be complementary to the interference colour of the light transmitted through the film. For larger angles of diffraction, the path difference increases for exterior diffraction and diminishes for interior diffraction. But at such angles, the effects due to the meniscus become of great importance in interior diffraction. Here the case may be treated as practically one of interference between the rays which are totally reflected and those twice refracted at the meniscus. Their path difference is easily shown to be

$$d(1 - \mu\cdot\sin i)(\mu\cos i - \sqrt{1 - \mu^2\sin^2 i}) - \delta,$$

where $i$ is the angle of incidence at the meniscus of the light which is twice refracted, and $\delta$ is the correction for the change of phase in total reflection. The angle of diffraction $\theta$ of the light emerging from the film is given by the formula

$$\sin\theta = \mu\sin 2(r - i), \quad \text{where } \sin i = \mu\sin r.$$

For small values of $i$ and $\theta$, it is readily shown that the path difference given by this formula is sensibly the same as in the one given above, namely,

$$\alpha(\mu - 1) - \tfrac{1}{2}d\sin\theta - \tfrac{1}{2}\lambda.$$

For larger values of $i$ and $\theta$, the path difference falls off more rapidly, finally vanishing when $i$ is equal to the critical angle for the liquid and $\sin\theta = \mu \sin 2i$. The corresponding direction of emergence of the light from the film would be outside the observable limit of the diffraction halo.

The diffraction halo as observed thus consists of two sets of rings, the intensities of which in any direction are superposed. In one of them, the path difference of the interfering rays diminishes with increasing angle of diffraction and finally vanishes in the direction of the achromatic ring. In the other set of rings which has a relatively small angular extension, the path difference becomes larger with the increasing angle of diffraction. The superposition of the two sets of rings whose angular positions are not the same thus leads to fluctuations in their visibility at small angles. From the formulae, the angular positions of the rings due to interior diffraction can be calculated and compared with observation and a satisfactory agreement is found.* The formula also enables a calculation to be made of the diameters of the dark and bright rings localised on a film of non-uniform thickness at any given angle of observation, and the particular angle at which a blurring of the rings would occur for a given thickness of the film. In every case the theory is confirmed by the actual measurements. Since the phase change occurring in total reflection is different for light polarised in and at right angles to the plane of incidence, there should be a corresponding small difference in the positions of the rings in the two cases. Even this fine point in the theory is confirmed by observation.† It is noticed that a plate which is too thick to show colours when viewed normally shows them if seen obliquely. Further, a film which shows colours when viewed normally appears achromatic when observed obliquely. These facts receive a satisfactory explanation on the theory.

It may be ramarked that the edge of each bubble in the film gives the complete diffraction halo, the diameter of the rings being, however, independent of the size of the bubble. The intensity of the halo in any particular direction depends on the aggregate length of the laminar edges running in the perpendicular direction. Hence, if the bubbles show a bias towards elongation in any particular direction, the halo appears intensified in the transverse direction, the rings, however, remaining circular.

An easy extension of the theory enables the oval haloes observed with obliquely held plates and the corresponding phenomena with non-uniform plates to be explained. As already mentioned, dry films exhibit phenomena of a quite different nature. For these and other details, reference may be made to the original papers.‡

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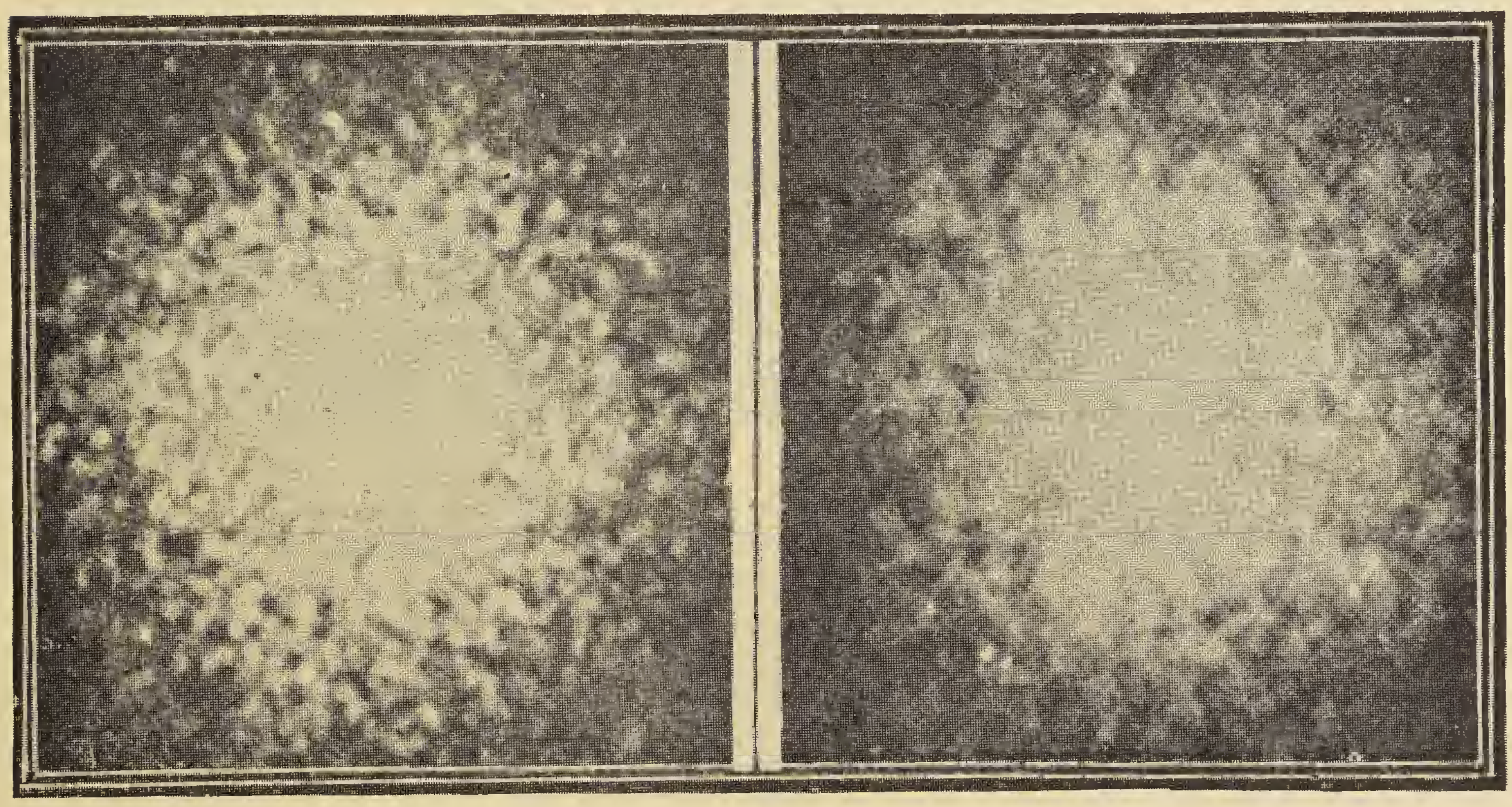
*C V Raman and K. Seshagiri Rao, *Philos. Mag.*, 1921, **42**, 679.

†It follows that if the incident light be plane-polarised in an arbitrary azimuth, the light diffracted at the boundary would, in general, be elliptically polarised.

‡See also K Seshagiri Rao, *Proc. Indian Assoc. Cultiv. Sci.*, 1923, **8**, 243. In this paper, the intensity distribution in the diffraction halo of mixed plates and the phenomena presented by dry films are discussed.

*Intensity fluctuations in coronae:* We shall now proceed to a closer examination of the nature of the diffraction pattern produced by a randomly distributed cloud of particles. As remarked earlier, such a pattern is *statistically* a summation of the effects of the individual particles but differs from them vastly in detail. Figure 66(a) on page 502 exhibits the central region of the corona observed around a *monochromatic* source of light of small angular extension, when viewed through a glass plate lightly dusted with lycopodium. The central disk of the corona is over-exposed in the photograph and shows no detail, but the granular structure of the pattern is seen very clearly in the first ring surrounding it. *Each of the bright spots in the field is a focussed image of the original source of light, formed by the joint action of the diffracting particles and the lens of the photographic camera.* This is verified by varying the size or shape of the source of light and noting its effect on the appearance of the pattern. It is then noticed that all the bright spots in the field alter in the same way and have the same form as the source. This is illustrated in figure 72 which shows the central disc of the corona photographed with a smaller exposure and on a larger scale than in figure 66(a), so as to clearly bring out the structure of the pattern. A small circular aperture and another in the form of a somewhat larger equilateral triangle were used as sources in photographing the two patterns reproduced. The circular and triangular shapes of the individual spots appearing in figures 72(a) and (b) can easily be recognised. The triangles in figure 72(b) appear inverted on the plate with respect to the source, as they should be in the images formed by a converging lens.*



**(a)** **(b)**

**Figure 72.** Central disc of corona in monochromatic light with (**a**) a circular pin-hole and (**b**) a triangular aperture as source.

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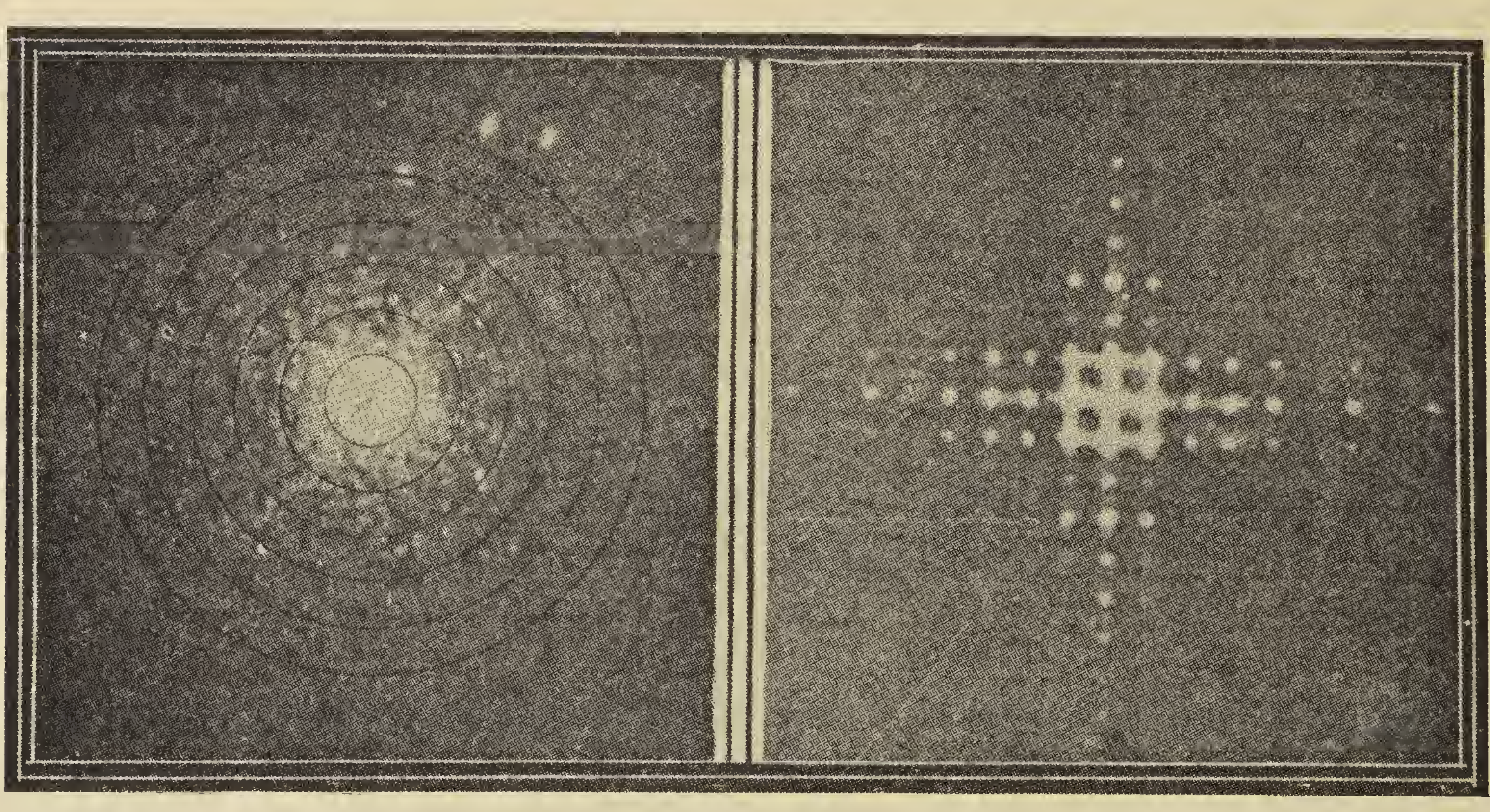
*G N Ramachandran, *Proc. Indian Acad. Sci.*, 1943, **A18**, 190.

It is familiar knowledge that a regularly spaced arrangement of apertures or obstacles can function as a diffraction grating and in combination with a lens give focussed spectra which in effect are monochromatic images of the source of light employed. The patterns reproduced in figure 72 show that a perfectly random arrangement of diffracting apertures or obstacles can also give well-defined images; the superiority of the regular grating is that it gives fewer and correspondingly more intense images in easily calculable positions instead of a great many feeble and irregularly spaced ones. The results are readily understood, since the optical effect in the focal plane of the lens can always be regarded as due to a plane wave of appropriate amplitude covering the entire area of the lens and travelling in such a direction that it comes to a focus at the point under consideration. The definition of the image of the source appearing at such point would be determined in every case by the configuration of the boundary of the lens and not by the disposition of the individual apertures or obstacles over its area. That the images formed by a random distribution of diffracting particles are not inferior in definition to those given by a regular diffraction grating is



(a) (b)

**Figure 73.** Comparison of corona with diffraction spectra given by a grating: (**a**) corona and (**b**) diffraction spectra.

illustrated in figures 73(a) and (b). These reproduce respectively the central part of the corona observed through a glass plate dusted with lycopodium and the diffraction spectra given by a fine sieve of metallic wires. A fine pin-hole illuminated by the 5461 Å radiation of a mercury lamp was the source and the optical conditions were also otherwise completely identical in the two cases.

The relation between the structure of the corona and the distribution of the diffracting particles on the plate can be illustrated in various ways. If, for example, the plate is moved in front of the eye, keeping the latter fixed on the source, the

ring-system does not undergo any change, but the fine structure of the corona appears to move relative to the pattern of rings in *the same direction* as the motion of the plate. *Vice versa*, if one moves the eye, keeping the plate fixed, all the while looking at the source, the structure of the corona appears to move in *the opposite direction*. If the plate is rotated, the structure rotates in the same direction. The prettiest effects are those observed when a very small aperture is held immediately before the eye so as to limit the effective area of the lycopodium-dusted plate held in front of it. As the plate is moved relative to the aperature, different areas of the former become operative, and the spots in the corona appear and vanish at random positions in the field, thus simulating the effects seen in a spinthariscope.

The theoretical law of distribution of intensities resulting from random interferences which was derived earlier, viz., that $dW = p(f)\,df = \exp(-f)\,df$ has been tested* making use of the photograph reproduced in figure 73(a) for counting the spots and classifying them according to their observed intensities. The average intensity in a corona falls away from the centre in the proportion $J_1^2(x)/x^2$, where $x = 2\pi a \sin\phi/\lambda$, $a$ being the radius of the particles, $\phi$ the angle of diffraction, and $\lambda$ the wavelength of the light. To take account of this factor, the parts of the corona where the spots could be clearly seen were divided into five annular regions as marked in the photograph; the spots in each annulus were counted and classified in a scale of intensities established with the aid of the diffraction pattern of the grating photographed under strictly comparable conditions [figure 73(b)]. Using this tabulated list, the average intensity for each region was computed, and thus the values of $p(f)$ and $f$ were determined for that

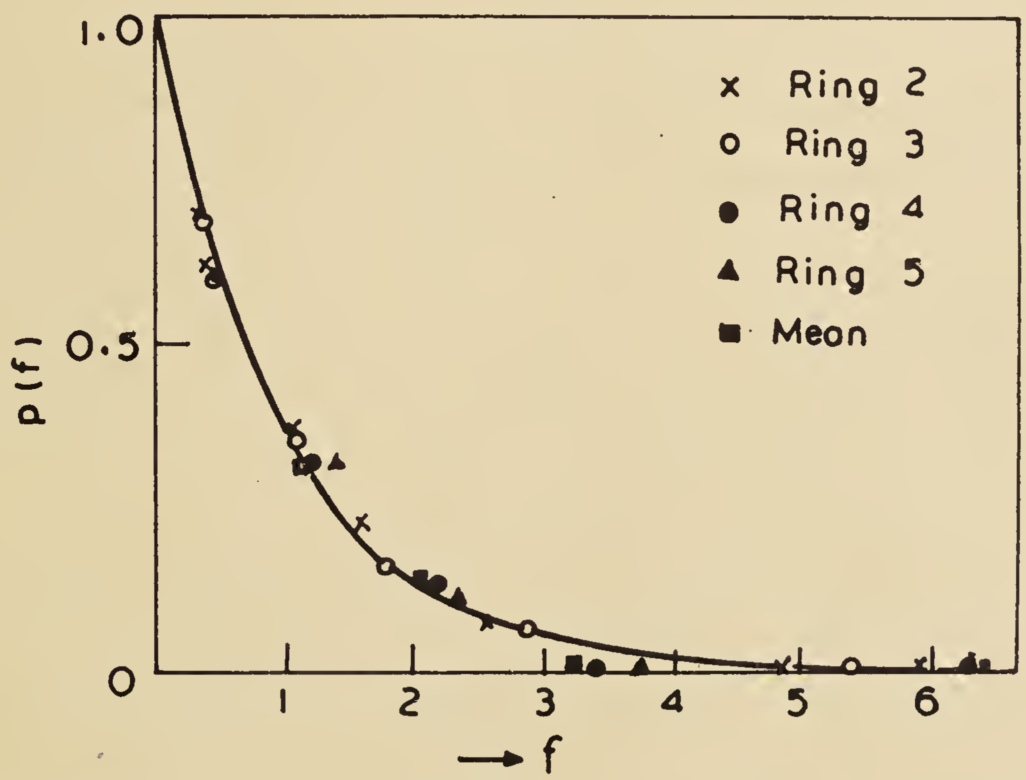



**Figure 74.** Verification of the statistical law of intensity fluctuation.

*G N Ramachandran, *Proc. Indian Acad. Sci.*, 1943, **A18**, 190.

region. These were plotted in figure 74 for four regions (the innermost one being too dense to facilitate counting), the continuous curve in the figure being the one calculated from theory. A final average for the four regions was obtained by dividing the intensities of the spots in each by the mean value of $J_1^2(x)/x^2$ for it, and the values of $p(f)$ thus obtained appear represented by black dots in figure 74. It will be seen from this figure that the experimental values fit the exponential formula indicated by the theory remarkably well.

*The phenomenon of the radiant spectrum:* Since, as we have seen, the bright spots in a diffraction corona as seen with monochromatic light are real images of the source employed, it follows that when white light is used instead, each bright spot would be drawn out into a spectrum, the different radiations appearing at distances from the centre of the pattern proportional to their respective wavelengths. This explains why in such circumstances, coronae exhibit numerous long coloured streamers or spectra located at random but directed radially outwards from the centre of the pattern. The streamers are most clearly seen in the outer parts of the corona, traversing its marginal rings and extending to the farthest visible limits of its extension. The streamers are distinguishable also in the central disc of the corona [see figure 66(b) on page 502], but their radial distribution and their colours are least conspicuous near the centre of the pattern. It is interesting to observe the coronae through a filter which transmits only two well-separated regions in the spectrum, e.g., the red and green regions. The entire pattern then appears filled with red and green spots; every green spot is accompanied by a red one, the two being along the same radius and the red spot at a distance from the centre greater than that of the green spot in the proportion of the two wavelengths. It is also of interest to view the diffraction corona through a dispersing prism held in front of the eye. The source of light then itself appears drawn out into a spectrum, and the radiant spectra are drawn out or shortened and also tilted one way or another, according to the direction in which they run. As the result of these changes, the "achromatic centre" of the diffraction pattern from which the coloured streaks appear to diverge is shifted away from its original position to a point lying well beyond the violet end of the spectrum into which the light-source is itself seen dispersed.

A familiar example of "radiant spectra" are those noticed when a small intense source of light is viewed directly by the normal eye against a dark background. Long coloured streamers of light are seen to diverge from the source in all directions, and faint coloured haloes also appear encircling the source near the outer limit of the streamers. Curious movements are also noticed within these streamers which may be controlled to some extent by fixing the eye on the source. The fact that the streamers disappear and are replaced by numerous bright points of illumination when a monochromatic source is used instead of white light, clearly indicates that we are here dealing with diffraction effects analogous to

those discussed in the foregoing pages.* The diffracting structures are evidently those present in the refractive media of the eye, including especially the cornea, and the crystalline lens, and possibly also the vitreous and aqueous humors. To give rise to such effects, it is not necessary that the diffracting particles should be opaque or spherical or of uniform size. Even small differences of refractive index in regions of appropriate size should be sufficient to give the observed phenomena. The angular dimensions of the brightest region of the diffraction corona are in accord with the supposition that it owes its origin to the known structure of the cornea of the eye, while it appears probable that the outer coloured haloes arise from the fibrous structure of the crystalline lens around its margin.

Holding a dispersing prism in front of the eye and viewing a bright source of light through it, the radiant streamers now appear to diverge from a point well beyond the violet end of the spectrum into which the light-source is itself dispersed. This effect, noticed long ago by Brewster, is clearly analogous to that observed with diffraction coronae and discussed above.† It is, of course, necessary that the prism used should have clean and well-polished surfaces so that it does not itself give rise to disturbing effects of a similar nature.‡

*Observation of Brownian movements without a microscope:* As illustrated by figures 72(a) and 73(a) appearing on earlier pages, the corona due to a cloud of diffracting particles exhibits strongly marked local variations of intensity. These variations are determined by the distribution of the diffracting particles in space, and if this alters with time, there would necessarily be corresponding changes in the corona. If the movements of the particles are large and rapid, all trace of visible structure would disappear from the field. If, however, the movements are sufficiently small and slow, it should be possible to follow the changes in the corona from instant to instant and thus obtain visual evidence that the diffracting particles are in motion.

As is well known, the individual particles in colloidal suspensions and emulsions execute "Brownian movements", which are most lively when the particles are very small and are suspended in an inviscid fluid. For our present purpose, it is necessary to select a substance in which the particles are of fair size so that the coronal disc is of sufficient intensity and also exhibits a visible structure. Fresh milk is the most easily available material satisfying this requirement. When a little of it is flowed on to a clean glass plate and then allowed to drain away as completely as possible, a thin film remains firmly adherent to the plate. A small aperture illuminated by a mercury arc lamp and viewed through

*C V Raman, *Philos. Mag.*, 1919, **38**, 568.
†C V Raman, *Philos. Mag.*, 1922, **43**, 357.
‡C V Raman, *Nature* (*London*), 1922, **109**, 175.

such a film exhibits an extended field of diffuse illumination surrounding it. Fixing the attention over a limited area of the field, it is noticed that this exhibits a structure which is not static but is continually changing. Bright points of illumination continually appear in the field and others disappear. These changes become less rapid and ultimately stop when the film is dry; the structure of the field is then completely static.*

*Unpublished observations by the author and G N Ramachandran.

# Index to Co-authors

## Volume III. Optics



# Subject Index

## Volume III. Optics

(For the monograph on Physical Optics, see under Lectures on physical optics)

## Name Index

### Volume III. Optics

## List of papers published from C V Raman's laboratory

### Volume III. Optics

1. Banerjee S K, 1918, On some phenomena observed in the Focualt's test, *Astrophys. J.*, **48** 50
2. Basu Nalini Mohan, 1918, Diffraction of light by cylinders of large radius, *Philos. Mag.*, **35** 79
3. Chinmayanandam T K, 1918, On Haidinger's rings in mica, *Proc. R. Soc.*, **95** 176
4. Chinmayanandam T K, 1918, Diffraction of light by an obliquely held cylinder, *Phys. Rev.*, **12** 314
5. Mitra Sisir Kumar, 1918, On the asymmetry of illumination curves in oblique diffraction, *Philos. Mag.*, **35** 112
6. Banerjee S K, 1919, On the radiation of light from the boundaries of diffracting apertures, *Philos. Mag.*, **37** 112
7. Chinmayanandam T K, 1919, On the flow of energy in the electromagnetic field surrounding a perfectly reflecting cylinder, *Philos. Mag.*, **37** 9
8. Ghosh P N, 1919, On the diffraction theory of microscopic vision, *Phys. Rev.*, **14** 497
9. Ghosh P N, 1919, On the colours of the striae in mica and the radiation from laminar diffracting boundaries, *Proc. R. Soc. London*, **A96** 257
10. Mitra S K, 1919, On large angle diffraction by apertures with curvilinear boundaries, *Philos. Mag.*, **38** 289
11. Mitra S K, 1919, On the Sommerfeld's treatment of the problem diffraction by a semi-infinite screen, *Philos. Mag.*, **37** 50
12. Prasad Chandi, 1919, On the theory of superposed diffraction fringes, *Phys. Rev.*, **13** 27
13. Mitra Sisir Kumar, 1920, On a new geometrical theory of the diffraction figures observed on the heliometer, *Proc. Indian Assoc. Cultiv. Sci.*, **6** 1
14. Sethi Nihal Karan, 1920, On wave-propagation in optically heterogeneous media and the phenomena observed in Christiansen's experiment, *Proc. Indian Assoc. Cultiv. Sci.*, **6** 121
15. Sethi Nihal Karan, 1920, On the theory of Powell's bands and the group velocity in dispersive media, *Phys. Rev.*, **16** 519
16. Chuckerbutti B N, 1920–21, On the colours of tempered steel and other tarnished metal surfaces, *Proc. Indian Assoc. Cultiv. Sci.*, **7** 75
17. Sethi Nihal Karan and Sogani C M, 1920–21, Quetelet's rings in mica, *Proc. Indian Assoc. Cultiv. Sci.*, **7** 61
18. Chinmayanandam T K, 1921, On the flow of energy near an optical focus, *Proc. Indian Assoc. Cultiv. Sic.*, **7** 21
19. Chuckerbutti B N, 1921, On the diffraction of light incidental at nearly critical angle on the boundary between two media, *Proc. R. Soc. (London)*, **A99** 503
20. Ghosh P N, 1921, Some phenomena of laminar diffraction observed with mica, *Proc. Indian Assoc. Cultiv. Sci.*, **6** 51
21. Ghosh P N, 1921, Some optical notes, *Proc. Indian Assoc. Cultiv. Sci.*, **7** 53
22. Sethi Nihal Karan, 1921, On Talbolt's bands and the colour sequence of the spectrum, *Philos. Mag.*, **41** 211
23. Sethi Nihal Karan, 1921, A new method of observing Talbolt's bands, *Philos. Mag.*, **41** 218
24. Sethi Nihal Karan, 1921, On the diffraction of light by cylinders and spheres immersed in a medium of nearly equal refractive index, *Philos. Mag.*, **42** 669

25. Sethi Nihal Karan, 1921, On Talbot's bands and the theory of Lummer-Gehrcke interferometer, *Phys. Rev.*, **18** 389
26. Sethi Nihal Karan, 1921, Some observations on interference phenomena in non-homogeneous light, *Proc. Indian Assoc. Cultiv. Sci.*, **7** 37
27. Chuckerbutti B N, 1922, On the colours of tempered steel and other tarnished metal surfaces, *Proc. Indian Assoc. Cultiv. Sci.*, **7** 75
28. Chuckerbutti B N, 1922, On the deformation of the ring and brushes observed through a spath hemitrope, *Philos. Mag.*, **43** 560
29. Das P, 1922, Caustics formed by diffraction, *Cal. Math. Soc. Bull.*, **13** 35
30. Dutta A B, 1922, On the diffraction of light by a reflecting cone, *Cal. Math. Soc. Bull.*
31. Dutta A B, 1922, Formation of optical images by diffracting boundaries, *Cal. Math. Soc. Bull.*
32. Mitra S K, 1922, On diffraction of light by apertures having the form of a segment of a circle, Asutosh Mookerjee Jubilee Volume.
33. Roy B B, 1922, Optical analogue of the whispering gallery effect, *Cal. Math. Soc. Bull.*, **12** 225
34. Sur N K, 1922, On laminar diffraction and the theory of microscopic vision, *Proc. Indian Assoc. Cultiv. Sci.*, **7** 125
35. Krishnaiyar N C, 1923, On the fluorescence of didymium in glass, *Proc. Indian Acad. Sci.*, **8** 217
36. Ramdas L A, 1923, Colours of chlorate of potash, *Proc. Indian Assoc. Cultiv. Sci.*, **8** 231
37. Seshagiri Rao K, 1923, On the colours of mixed plates, *Proc. Indian Assoc. Cultiv. Sci.*, **8** 243
38. Ray Bidhu Bhushan, 1923, The scattering of light by liquid droplets, and the theory of coronas, glories, and iridescent clouds, *Proc. Indian Assoc. Cultiv. Sci.*, **8** 23
39. Sur Nalina Karan, 1923, Fresnel's laws of reflection from note on a departure, *Phys. Rev.*, **21** 699
40. Narayanan A L, Gunnaiya M A and Rao K R, 1924, Absorption and dispersion of thallium vapours, *Proc. R. Soc. London*, **106A** 596
41. Sethi Nihal Karan, 1924, Effect of a retarding plate on white light interferometer fringes, *Phys. Rev.*, **23** 69
42. Chuckerbutti B N, 1925, On the colours shown by Nobili's rings, *Proc. Indian Assoc. Cultiv. Sci.*, **9** 83
43. Ghosh R N, 1925, Duration of fluorescence of mercury vapour, *Phys. Rev.*, **26** 376
44. Narayanan A L and Rao K R, 1925, On absorption of light by vapours of Pb, Sn, Bi, Sb, Mg, *Philos. Mag.*, **50** 645
45. Sur N K and Ghosh R N, 1925, On absorption spectrum of pottasium vapour at high temperatures, *Philos. Mag.*, **49** 60
46. Banerjee K, 1926, On the permanent deformation produced by contact of solids, *Indian J. Phys.*, **1** 59
47. Sogani C M, 1926, On the optical properties of chromatic emulsions, *Philos. Mag.*, **1** 321
48. Sur N K, 1926, On the arc spectrum of lead, *Philos. Mag.*, **2** 633
49. Sur N K, 1926, On the water-spark absorption spectrum of ion, *Philos. Mag.*, **1** 433, 704
50. Sur N K and Majumdar K, 1926, On the absorption spectra of alluminium and cobalt, *Philos. Mag.*, **1** 451
51. Datta S K, 1927, On Brewster's bands, Part II, *Trans. Opt. Soc.*, London **28** 214
52. Datta S K, 1927, On Brewster's bands Part II, *Nature*, **120** 173
53. Krishnan K S, 1927, Magnetic double refraction in paramagnetic gases, *Indian J. Phys.*, **1** 245
54. Ramakrishna Rao I, 1927, Studies in laminar diffraction I. Colour of mixed plates, *Indian J. Phys.*, **2** 167
55. Mishra Manindra Nath, 1927–28, Photographs of coronas in monochromatic light, *Proc. Indian Assoc. Cultiv. Sci.*, **3** 175
56. Ramdas L A, 1927–28, The spectrum of potassium excited during its spontaneous combustion with chlorine, *Proc. Indian Assoc. Cultiv. Sci.*, **3** 31
57. Sur N K 1927, On the origin of terms of the spectrum of cobalt, *Philos. Mag.*, **4** 36
58. Ramakrishna Rao I, 1928, Studies in laminar diffraction, Part II. Laminar boundaries in mica, *Indian J. Phys.*, **2** 365

59. Rao K R and Narayanan A L, 1928, On series in the spark spectra of germanium, *Proc. R. Soc. London*, **119A** 607
60. Sirkar S C, 1928, The Kerr effect in viscous liquids due to radio frequency oscillating field, *Indian J. Phys.*, **3** 409
61. Ghosh P N and Mahanti P C, 1929, Raman effect in carbondioxide, *Nature*, **124** 92
62. Ghosh P N and Chatterji B D, 1929, High-frequency discharge of organic vapours, *Nature*, **124** 654
63. Krishnan K S, 1929, Are black soap films birefringent?, *Indian J. Phys.*, **4** 385
64. Ramanandham M, 1929, Magnetic birefringence in solutions and its relations to crystal structure and properties, *Indian J. Phys.*, **4** 109
65. Ramanandham M, 1929, Magnetic birefringence in liquids of the aliphatic series, *Indian J. Phys.*, **4** 15
66. Rao K R, 1929, The arc spectrum of germanium, *Proc. R. Soc. London*, **124A** 465
67. Rao K R, 1929, Regularities in the arc spectrum of arsenic, *Proc. R. Soc. London*, **125A** 238
68. Ghosh P N, Mahanti P C and Mukherjee B C, 1930, Vibrational quantum analysis of the blue-green bands of magnesium oxide, *Phys. Rev.*, **35** 149
69. Krishnan M S and Mahadevan C, 1930, Pleochroic "giant" haloes in cordierite, *Proc. Indian Assoc. Cultiv. Sci.*, **5** 669
70. Sirkar S C, 1930, On the Laue photographs of iridescent crystals of potassium chlorate, *Proc. Indian Assoc. Cultiv. Sci.*, **5** 337
71. Chinchalkar S W, 1931, Magnetic birefringence and molecular anisotropy, *Indian J. Phys.*, **6** 563
72. Chinchalkar S W, 1931, A new type of magnetic birefringence, *Indian J. Phys.*, **6** 581
73. Chinchalkar S W, 1931, Magnetic double refraction in aliphatic liquids, *Indian J. Phys.*, **6** 165
74. Mahajan L D, 1931, The effect of light on the surface tension of Boys' soap solution, *Proc. Indian Assoc. Cultiv. Sci.*, **6** 147
75. Narasimham K L, 1931, Optical properties of some aromatic crystals, *Indian J. Phys.*, **6** 233
76. Pillai P K, 1931, Thermal variation of Faraday rotation, *Indian J. Phys.*, **6** 573
77. Ramdas L A and Venkiteswaran P, 1931, Spectrum of glow-worm, *Nature*, **128** 726
78. Ramaswamy S, 1931, On the transmission of light by thin films of metal, *Proc. R. Soc. London*, **A131** 307
79. Rao K R, 1931, The third spectrum of arsenic (As IV), *Proc. R. Soc. London*, **A134** 604
80. Chinchalkar S W, 1932, Magnetic birefringence in solutions of sodium chlorate and sodium bromate, *Proc. Indian Assoc. Cultiv. Sci.*, **7** 317
81. Krishnan M S and Mahadevan C, 1932, Giant pleochroic haloes, *Nature*, **129** 135
82. Mukhopadhyay B, 1932, Transmission of light through suspensions of powdered crystals, *Indian J. Phys.* **7** 307
83. Pillai P K, 1932, A note on the variation of the Faraday effect with concentration, *Proc. Indian Assoc. Cultiv. Sci.*, **7** 87
84. Chinchalkar S W, 1933, Magnetic birefringence in liquid mixtures, *Indian J. Phys.*, **7** 491
85. Krishnan K S and Dasgupta A C, 1933, The pleochroism and the birefringence of the $NO_3$ ion in crystals, *Proc. Indian Assoc. Cultiv. Sci.*, **8** 43
86. Raghavendra Rao B V, 1933, The first spark spectrum of gold Au II, *Proc. R. Soc. London*, **142A** 118
87. Raghavendra Rao B V, 1934, Examination of molecularly scattered light with a Fabry–Perot etalon: Part I. Liquid benzene, *Proc. Indian Acad. Sci.*, **A1** 261
88. Bhagavantam S, 1935, A suggested new interpretation of the structure of band-spectra, *Proc. Indian Acad. Sci.*, **A2** 92
89. Bhatt N B, 1935, High frequency spectrum of mercury vapour, *Proc. Indian Acad. Sci.*, **A1** 891
90. Chinchalkar S W, 1935, Magnetic birefringence in solutions of paramagnetic salt of rare earths, *Philos. Mag.*, **20** 856
91. Chinchalkar S W, 1935, Magnetic birefringence in solutions of organic substances, Part I, *Proc. Indian Acad. Sci.*, **A2** 525

92. Pisharoty P Rama, 1935, Laminar diffraction and the Becke phenomenon, *Proc. Indian Acad. Sci.*, **A2** 14
93. Raghavendra Rao B V, 1935, Doppler effect in light scattering in liquids, Part II. Polarisation of the transversely scattered radiations, *Proc. Indian Acad. Sci.*, **A2** 236
94. Raghavendra Rao B V, 1935, The Doppler effect in light scattered by liquids, Part I. Variation with temperature, *Proc. Indian Acad. Sci.*, **A1** 765
95. Raghavendra Rao B V, 1935, Examination of molecularly scattered light with a Fabry–Perot etalon Part II. Liquids, toluene and carbon tetrachloride, *Proc. Indian Acad. Sci.*, **A1** 473
96. Ramaswamy K L, 1935, Refractive indices and dispersions of volatile compounds of fluorine and boron: carbon tetrafluoride, nitrogen trifluoride, fluoroform, carbon-nitrogen compound of fluorine $(CF_3N)_2$, boron hexahydride and nitrogenous boron hydride $B_3N_3H_6$, *Proc. Indian Acad. Sci.*, **A2** 630
97. Thatte V N, 1935, Magnetic double refraction and light scattering in fused nitrates, *Proc. Indian Acad. Sci.*, **A2** 244
98. Vaidya W M, 1935, The flame spectra of some aromatic compounds, *Proc. Indian Acad. Sci.*, **A2** 352
99. Venkateswaran C S, 1935, The fluorescence of ruby, saphire and emerald, *Proc. Indian Acad. Sci.*, **A2** 459
100. Rajagopalan V S, 1935, On the structure and optical characters of the Nacre in iridescent shells I, *Proc. Indian Acad. Sci.*, **A3** 572
101. Born Max and Nagendra Nath N S, 1936, The neutrino theory of light II, *Proc. Indian Acad. Sci.*, **A4** 611
102. Born Max and Nagendra Nath N S, 1936, The neutrino theory of light, *Proc. Indian Acad. Sci.*, **A3** 318
103. Nagendra Nath N S, 1936, Neutrinos and light quanta, *Proc. Indian Acad. Sci.*, **A3** 448
104. Nagendra Nath N S, 1936, The visibility of ultrasonic waves and its periodic variations, *Proc. Indian Acad. Sci.*, **A4** 262
105. Nagendra Nath N S, 1936, The diffraction of light by high frequency sound waves—generalised theory (the asymmetry of the diffraction phenomena at oblique incidence), *Proc. Indian Acad. Sci.*, **A4** 222
106. Parthasarathy S, 1936, Diffraction of light by ultrasonic waves, *Proc. Indian Acad. Sci.*, **A3** 442
107. Parthasarathy S, 1936, Diffraction of light by ultrasonic waves II. Reflection and transmission phenomena, *Proc. Indian Acad. Sci.*, **A3** 594
108. Parthasarathy S, 1936, The visibility of ultrasonic waves in liquids, *Proc. Indian Acad. Sci.*, **A4** 555
109. Pisharoty P Rama, 1936, On the visibility of the ultrasonic waves, *Proc. Indian Acad. Sci.*, **A4** 27
110. Raghavendra Rao B V, 1936, Doppler effect in light scattering in liquids, Part III. Polarisation of light transversely scattered by formic and acetic acids, *Proc. Indian Acad. Sci.*, **A3** 607
111. Madhava Rao B S, 1937, On the fine structure of the Balmer lines, *Proc. Indian Acad. Sci.*, **A5** 56
112. Padmanabhan R, 1937, The fluorescence of acetone vapour, *Proc. Indian Acad. Sci.*, **A5** 594
113. Raghavendra Rao B V, 1937, Dispersion of sound velocity in liquids, *Nature*, **139** 885
114. Subbaramaiya D S, 1937, Diffraction of light by ripples on liquid surfaces—Part I, *Proc. Indian Acad. Sci.*, **A6** 333
115. Vaidya W M, 1937, The flame spectra of some aliphatic halides. Part I. Methyliodide, *Proc. Indian Acad. Sci.*, **A6** 122
116. Nagendra Nath N S and Muller Hans, 1938, Diffraction of light by supersonic waves in solids, *Nature*, **141** 37
117. Nilakantan P, 1938, Magnetic anisotropy and pleochroism of biotite mica, *Proc. Indian Acad. Sci.*, **A8** 39
118. Nagabhushana Rao K, 1938, Diffraction of light by ultrasonic waves, *Proc. Indian Acad. Sci.*, **A48** 124

119. Raghavendra Rao B V, 1938, Ultrasonic dispersion, *Nature*, **142** 121
120. Raghavendra Rao B V, 1938, Dispersion of acoustic velocity in liquids, *Proc. Indian Acad. Sci.*, **A7** 163
121. Thosar B V, 1938, On the fluorescent chromium ion in ruby, *Philos. Mag.*, **26** 878
122. Paranjpe G R, Naik Y G and Vaidya P B, 1939, Scattering of light by large water drops: Part I, *Proc. Indian Acad. Sci.*, **A9** 333–351
123. Paranjpe G R, Naik Y G and Vaidya P B, 1939, Scattering of light by large water drops: Part II, *Proc. Indian Acad. Sci.*, **A9** 352–364
124. Nagabhushana Rao K, 1939, Diffraction of light by supersonic waves, Part I, *Proc. Indian Acad. Sci.*, **A9** 422
125. Subba Ramaiah K, 1939, Studies on Liesegang rings, *Proc. Indian Acad. Sci.*, **A9** 467–478
126. Ananthanarayanan N, 1940, Diffraction of light by a thin metallic half-plane, *Proc. Indian Acad. Sci.*, **A10** 477
127. Ananthanarayanan N, 1941, Spectroscopic examination of the diffraction of light by a thin metallic half-plane, *Proc. Indian Acad. Sci.*, **A14** 85
128. Balakrishnan T A S, 1941, An elementary theory of the coronas of water droplets, *Proc. Indian Acad. Sci.*, **A13** 188–193
129. Raghavendra Rao B V and Subbaramaiya D S, 1941, An explanation of the diminished acoustic velocity in fluids at high frequencies, *Phys. Rev.*, **60** 613
130. Subrahmanian R V, 1941, The spectral character of the reflection by a regularly stratified medium: Part I, *Proc. Indian Acad. Sci.*, **A13** 467–482
131. Ramachandran G N, 1942, Reflection of light by a periodically stratified medium, *Proc. Indian Acad. Sci.*, **A16** 336–348
132. Satyanarayana B, 1942, The fluorescence of the uranyl compounds and the Raman spectrum of the uranyl ion, *Proc. Indian Acad. Sci.*, **A15** 414
133. Sunanda Bai K, 1942, The scattering of light in liquids, *Proc. Indian Acad. Sci.*, **A15** 338–348
134. Sunanda Bai K, 1942, Interferometric studies of light scattering in the alcohols, *Proc. Indian Acad. Sci.*, **A15** 349–356
135. Sunanda Bai K, 1942, Interferometric studies of light scattering in liquids: polarisation data, *Proc. Indian Acad. Sci.*, **A15** 357–361
136. Venkateswaran C S, 1912, Interferometric studies of light scattering in gases, *Proc. Indian Acad. Sci.*, **A15** 316
137. Venkateswaran C S, 1942, Interferometric studies of light scattering in mobile liquids, *Proc. Indian Acad. Sci.*, **A15** 322–337
138. Venkateswaran C S, 1942, Interferometric studies of light scattering in viscous liquids and glasses, *Proc. Indian Acad. Sci.*, **A15** 362–370
139. Venkateswaran C S, 1912, Interferometric studies of light scattering: Hypersonic velocities in liquids, *Proc. Indian Acad. Sci.*, **A15** 371–375
140. Ramachandran G N, 1943, On the transmission of light through a cloud of randomly distributed particles, *Proc. Indian Acad. Sci.*, **A17** 171–186
141. Ramachandran G N, 1943, The theory of coronae and iridescent clouds, *Proc. Indian Acad. Sci.*, **A17** 202–218
142. Ramachandran G N, 1943, Optical theory of chromatic emulsions and of the Christiansen experiment, *Proc. Indian Acad. Sci.*, **A18** 67–80
143. Ramachandran G N, 1943, Fluctuations of light intensity in coronae formed by diffraction, *Proc. Indian Acad. Sci.*, **A18** 190–200
144. Sunanda Bai K, 1943, Interferometric studies of light scattering in binary liquid mixtures: Part I, *Proc. Indian Acad. Sci.*, **A18** 210–217
145. Ramachandran G N, 1944, Diffraction coronae due to non-spherical particles, *Proc. Indian Acad. Sci.*, **A19** 123

146. Ramachandran G N, 1944, A new derivation of the Darwin–Prins formula of X-ray reflection, *Proc. Indian Acad. Sci.*, **A20** 100–106
147. Sunanda Bai K, 1944, Interferometric studies of light scattering in binary liquid mixtures: Part II, *Proc. Indian Acad. Sci.*, **A20** 18–23
148. Kathavate Y V, 1945, Geometric theory of Fresnel diffraction patterns: Part I. Basic ideas, *Proc. Indian Acad. Sci.*, **A21** 177–187
145. Kathavate Y V, 1945, Geometric theory of Fresnel diffraction patterns: Part II. Rectilinear boundaries, *Proc. Indian Acad. Sci.*, **A21** 188–193
150. Kathavate Y V, 1945, Geometric theory of Fresnel diffraction patterns: Part III. Boundaries having the shapes of the semi-circle and the segment of a circle, *Proc. Indian Acad. Sci.*, **A21** 194–198
151. Kathavate Y V, 1945, Geometric theory of Fresnel diffraction patterns: Part IV. Sectored discs and apertures, *Proc. Indian Acad. Sci.*, **A21** 198–202
152. Kathavate Y V, 1945, Geometric theory of Fresnel diffraction patterns: Part V. Elliptic obstacles and apertures, *Proc. Indian Acad. Sci.*, **A21** 203–207
153. Kathavate Y V, 1945, Geometric theory of Fresnel diffraction patterns: Part VI. Circular obstacles and apertures, *Proc. Indian Acad. Sci.*, **A21** 208–210
154. Kathavate Y V, 1945, The diffraction of light by an assembly of opaque circular disks, *Proc. Indian Acad. Sci.*, **A21** 233
155. Pant D D, 1945, Some studies in the spectra of single uranyl halides and other uranyl salts, *Proc. Indian Acad. Sci.*, **A22** 95–109
156. Pant D D, 1945, Analysis of the fluorescence bands of the uranyl salts, *Proc. Indian Acad. Sci.*, **A22** 110–122
157. Ramachandran G N, 1945, On the radiation from the boundary of diffracting apertures and obstacles, *Proc. Indian Acad. Sci.*, **A21** 165–176
158. Ramachandran G N, 1946, Haidingers rings divergent light, *Philos. Mag.*, **37** 700
159. Ramaseshan S, 1946, The Faraday effect in diamond, *Proc. Indian Acad. Sci.*, **A24** 104
160. Ramaseshan S, 1946, Determination of the magneto-optic anomaly of some glasses, *Proc. Indian Acad. Sci.*, **A24** 426–432
161. Krishnan R S, 1947, Thermal scattering of light in diamond, *Nature*, **159** 740
162. Radhakrishnan T, 1947, The dispersion, birefringence and optical activity of quartz, *Proc. Indian Acad. Sci.*, **A25** 260–265
163. Radhakrishnan T, 1947, Temperature variation of refractive indices of quartz, *Proc. Indian Acad. Sci.*, **A25** 382–388
164. Ramachandran G N, 1947, Photoelastic constants of diamond, *Proc. Indian Acad. Sci.*, **A25** 208
165. Ramachandran G N, 1947, Thermo-optic behaviour of solids, Part I. Diamond, *Proc. Indian Acad. Sci.*, **A25** 266–279
166. Ramachandran G N, 1947, Thermo-optic behaviour of solids, Part II. Fused quartz, *Proc. Indian Acad. Sci.*, **A25** 280–285
167. Ramachandran G N, 1947, Thermo-optic behaviour of solids, Part III. Fluorospar, *Proc. Indian Acad. Sci.*, **A25** 286–296
168. Ramachandran G N, 1947, Thermo-optic behaviour of solids, Part IV. Zinc-blende, *Proc. Indian Acad. Sci.*, **A25** 375–381
169. Ramachandran G N, 1947, Thermo-optic behaviour of solids, Part V. Alkali halides, *Proc. Indian Acad. Sci.*, **A25** 481–497
170. Ramachandran G N, 1947, Thermo-optic behaviour of solids, Part VI. Optical glasses, *Proc. Indian Acad. Sci.*, **A25** 498–514
171. Ramachandran G N, 1947, Birefringence of crystals and its temperature variation, Part I. Calcite and aragonite, *Proc. Indian Acad. Sci.*, **A26** 77–92
172. Ramachandran G N, 1947, Birefringence of crystals and its temperature variation, Part II. Sodium and pottasium nitrates, *Proc. Indian Acad. Sci.*, **A26** 114–120

173. Ramaseshan S, 1947, Faraday effect in some cubic crystals, *Proc. Indian Acad. Sci.*, **A25** 459–466
174. Ramachandran G N, 1948, Photoelastic constants of diamond—corrections, *Proc. Indian Acad. Sci.*, **A27** 421
175. Krishnan R S, 1948, Sir C V Raman and crystal physics, *Proc. Indian Acad. Sci.*, **A28** 258
176. Madhava Rao B S, 1948, Sir C V Raman as physicist and teacher, *Proc. Indian Acad. Sci.*, **A28** 254
177. Ramachandra Rao S, 1948, Sir C V Raman on his sixtieth birthday, *Proc. Indian Acad. Sci.*, **A28** 253
178. Radhakrishnan T, 1948, The dispersion formulae of the alkali halides, *Proc. Indian Acad. Sci.*, **A27** 165
179. Radhakrishnan T, 1948, The thermo-optic behaviour of mica, *Proc. Indian Acad. Sci.*, **A28** 447–453
180. Ramaseshan S, 1948, The Faraday effect and the magneto-optic anomaly of some cubic crystals, *Proc. Indian Acad. Sci.*, **A28** 360–369
181. Chandrasekharan V, 1950, Thermal scattering of light in crystals, Part II. Diamond, *Proc. Indian Acad. Sci.*, **A32** 379
182 Chandrasekharan V and Krishnan R S, 1950, Thermal scattering of light in quartz, *Nature*, **165** 406
183. Krishnan R S and Chandrasekharan V, 1950, Thermal scattering of light in crystals, Part I. Quartz, *Proc. Indian Acad. Sci.*, **A31** 427
184. Radhakrishnan T, 1950, Temperature variation of the refractive index of lithium fluoride, *Proc. Indian Acad. Sci.*, **A31** 224
185. Radhakrishnan T, 1950, The refraction of magnesium oxide and its temperature variation, *Proc. Indian Acad. Sci.*, **A31** 229
186. Ramachandran G N, 1950, Photo-elasticity of diamond, *Proc. Indian Acad. Sci.*, **A31** 171
187. Ramaseshan S, 1950, The magneto-optic anomaly of electrolytes in aqueous solution, I. Alkali halides, *Proc. Indian Acad. Sci.*, **A31** 149
188. Ramaseshan S, 1950, The magneto-optic anomaly of electrolytes in aqueous solution II, *Proc. Indian Acad. Sci.*, **A31** 172
189. Chandrasekhar S, 1951, The dispersion and thermo-optic behaviour of vitreous silica, *Proc. Indian Acad. Sci.*, **A34** 275
190. Chandrasekharan V, 1951, Thermal scattering of light in crystals, Part III. Theory for birefringent crystals, *Proc. Indian Acad. Sci.*, **A33** 183
191. Ramachandran G N and Chandrasekharan V, 1951, Photo-elastic constants of sodium chlorate, *Proc. Indian Acad. Sci.*, **A33** 199
192. Ramachandran G N, 1951, Theory of optical activity of crystals I. General ideas, *Proc. Indian Acad. Sci.*, **A33** 217
193. Ramachandran G N, 1951, The theory of optical activity of crystals II. Calculation of the rotatory power of $NaClO_3$ and $NaBrO_3$, *Proc. Indian Acad. Sci.*, **A33** 309
194. Ramdas A K, 1951, Thermo-optic behaviour of silicon carbide, *Proc. Indian Acad. Sci.*, **A34** 136
195. Ramaseshan S, 1951, Faraday effect and birefringence I, *Proc. Indian Acad. Sci.*, **A34** 32
196. Ramaseshan S, 1951, Faraday effect and birefringence II. Corundum, *Proc. Indian Acad. Sci.*, **A34** 97
197. Chandrasekhar S, 1952, The optical rotatory power of quartz and its variation with temperature, *Proc. Indian Acad. Sci.*, **A35** 103
198. Chandrasekhar S, 1952, Some remarks on the optical rotatory dispersion of quartz, *Proc. Indian Acad. Sci.*, **A36** 118
199. Ramdas A K, 1952, The thermo-optic behaviour of quartz, *Proc. Indian Acad. Sci.*, **A35** 89
200. Bhat M R, 1953, Some illustrations of the theory of the Christiansen optical filters, *Proc. Indian Acad. Sci.*, **A38** 67
201. Chandrasekhar S, 1953, A redetermination of the rotatory dispersion of sodium chlorate, *Proc. Indian Acad. Sci.*, **A37** 458

202. Chandrasekhar S, 1953, The optical rotatory dispersion of cinnabar, *Proc. Indian Acad. Sci.*, **A37** 697
203. Chandrasekhar S, 1954, The rotatory dispersion of benzil, *Proc. Indian Acad. Sci.*, **A39** 243
204. Chandrasekhar S, 1954, The temperature variation of the rotatory power of quartz from 30 to 40° C, *Proc. Indian Acad. Sci.*, **A39** 290
205. Pancharatnam S, 1954, On the pleochroism of amethyst quartz and its absorption spectra, *Proc. Indian Acad. Sci.*, **A40** 196
206. Pancharatnam S, 1955, Achromatic combinations of birefringent plates Part I. An achromatic circular polarizer, *Proc. Indian Acad. Sci.*, **A41** 130
207. Pancharatnam S, 1955, Achromatic combinations of birefringent plates Part II. An achromatic quarter-wave plate, *Proc. Indian Acad. Sci.*, **A41** 137
208. Pancharatnam S, 1955, The propagation of light in absorbing biaxial crystals I. Theoretical, *Proc. Indian Acad. Sci.*, **A42** 86
209. Pancharatnam S, 1955, The propagation of light in absorbing biaxial crystals II. Experimental, *Proc. Indian Acad. Sci.*, **A42** 235
210. Pancharatnam S, 1956, On the phenomenological theory of light propagation in optically active crystals, *Proc. Indian Acad. Sci.*, **A43** 247
211. Pancharatnam S, 1956, Generalized theory of interference and its applications Part I. Coherent pencils, *Proc. Indian Acad. Sci.*, **A44** 247
212. Pancharatnam S, 1956, Generalized theory of interference and its applications Part II. Partially coherent pencils, *Proc. Indian Acad. Sci.*, **A44** 398
213. Chandrasekhar S, 1957, The optical rotatory dispersion of quartz, *Proc. Indian Acad. Sci.*, **A45** 147
214. Pancharatnam, S, 1957, Generalized theory of interference and its applications III. Interference figures in transparent crystals, *Proc. Indian Acad. Sci.*, **A45** 402
215. Pancharatnam S, 1957, Light propagation in absorbing crystals possessing optical activity, *Proc. Indian Acad. Sci.*, **A46** 280
216. Jayaraman A, 1958, Unit cell parameters, space-group and optical properties of cobalt nitrate hexahydrate, *Proc. Indian Acad. Sci.*, **A47** 147
217. Pancharatnam S, 1958, The optic interference figures of amethystine quartz Part I, *Proc. Indian Acad. Sci.*, **A47** 201
218. Pancharatnam S, 1958, The optic interference figures of amethystine quartz Part II, *Proc. Indian Acad. Sci.*, **A47** 210
219. Pancharatnam S, 1958, Light propagation in absorbing crystals possessing optical activity-electromagnetic theory, *Proc. Indian Acad. Sci.*, **A48** 227

## Author index of papers from C V Raman's laboratory

Volume III. Optics

(The numbers against the names indicate the serial number in the list of papers)

# Consolidated list of C V Raman's scientific papers

## Volumes I–VI

## Volume I. Scattering of Light

1. The Doppler effect in the molecular scattering of radiation [1919 *Nature* (*London*) **103** 165]
2. On the transmission colours of sulphur suspensions [1921 *Proc. R. Soc. London* **A100** 102; with B B Ray]
3. A method of improving visibility of distant objects [1921 *Nature* (*London*) **108** 242]
4. The colour of the sea [1921 *Nature* (*London*) **108** 367]
5. The molecular scattering of light in liquids and solids [1921 *Nature* (*London*) **108** 402]
6. On the molecular scattering of light in water and the colour of the sea [1922 *Proc. R. Soc. London* **A101** 64]
7. Optical observations of the thermal agitation of the atoms in crystals [1922 *Nature* (*London*) **109** 42]
8. Anisotropy of molecules [1922 *Nature* (*London*) **109** 75]
9. Molecular structure of amorphous solids [1922 *Nature* (*London*) **109** 138]
10. Molecular diffraction of light [1922 The Calcutta University Press 103 pages]
11. Diffraction by molecular clusters and the quantum structure of light [1922 *Nature* (*London*) **109** 444]
12. Molecular aelotropy in liquids [1922 *Nature* (*London*) **110** 11]
13. Opalescence phenomena in liquid mixtures [1922 *Nature* (*London*) **110** 77]
14. Transparency of liquids and colour of the sea [1922 *Nature* (*London*) **110** 280]
15. Thermal opalescence in crystals and the colour of ice in glaciers [1923 *Nature* (*London*) **111** 13]
16. On the molecular scattering of light in dense vapours and gases [1923 *Philos. Mag.* **45** 113; with K R Ramanathan]
17. On the molecular scattering and extinction of light in liquids and the determination of the Avogadro constant [1923 *Philos. Mag.* **45** 625; with K Seshagiri Rao]
18. The molecular scattering of light in liquid mixtures [1923 *Philos. Mag.* **45** 213; with K R Ramanathan]
19. On the polarization of the light scattered by gases and vapours [1923 *Philos. Mag.* **46** 426; with K Seshagiri Rao]
20. The molecular scattering of light in carbon dioxide at high pressures [1923 *Proc. R. Soc. London* **A104** 357; with K R Ramanathan]
21. The scattering of light by anisotropic molecules [1923 *Nature* (*London*) **112** 165]
22. The structure of molecules in relation to their optical anisotropy [1924 *Nature* (*London*) **114** 49]
23. The scattering of light by liquid and solid surfaces [1923 *Nature* (*London*) **112** 281]
24. Relation of Tyndall effect to osmotic pressure in colloidal solutions [1927 *Indian J. Phys.* **2** 1]
25. The scattering of X-rays in liquids [1923 *Nature* (*London*) **111** 185]
26. The nature of the liquid state [1923 *Nature* (*London*) **111** 428]

27. The diffraction of X-rays in liquids, liquid mixtures, solutions, fluid crystals and amorphous solids [1923 *Proc. Indian Assoc. Cultiv. Sci.* **8** 127; with K R Ramanathan]

28. On the mean distance between neighbouring molecules in a fluid [1924 *Philos. Mag.* **47** 671]

29. The scattering of light by liquid boundaries and its relation to surface tension—Part I [1925 *Proc. R. Soc. London* **A108** 561; with L A Ramdas]

30. The scattering of light by liquid boundaries and its relation to surface tension—Part II [1925 *Proc. R. Soc. London* **A109** 150; with L A Ramdas]

31. The scattering of light by liquid boundaries and its relation to surface tension—Part III [1925 *Proc. R. Soc. London* **A109** 272; with L A Ramdas]

32. Die Zerstreuung des Lichtes durch dielektrische Kügeln (*German*) [1925 *Z. Phys* **33** 870]

33. On the thickness of the optical transition layer in liquid surfaces [1927 *Philos. Mag.* **3** 220; with L A Ramdas]

34. The birefringence of crystalline carbonates, nitrates and sulphates [1926 *Nature (London)* **118** 264]

35. The electrical polarity of molecules [1926 *Nature (London)* **118** 302; with K S Krishnan]

36. Magnetic double-refraction in liquids, Part I: Benzene and its derivatives [1927 *Proc. R. Soc. London* **A113** 511; with K S Krishnan]

37. Electric double-refraction in relation to the polarity and optical anisotropy of molecules, Part I: Gases and vapours [1927 *Philos. Mag.* **3** 713; with K S Krishnan]

38. Electric double-refraction in relation to the polarity and optical anisotropy of molecules, Part II: Liquids [1927 *Philos. Mag.* **3** 724; with K S Krishnan]

39. Disappearance and reversal of the Kerr effect [1928 *Nature (London)* **121** 794; with S C Sirkar]

40. Optique—La constante de birefringence magnetique du benzene (*French*) [1927 *C. R. Acad. Sci. Paris* **184** 449; with K S Krishnan]

41. Magnetic double refraction [1927 *Nature (London)* **119** 528; with I Ramakrishna Rao]

42. The magnetic anisotropy of crystalline nitrates and carbonates [1927 *Proc. R. Soc. London* **A115** 549; with K S Krishnan]

43. A theory of electric and magnetic birefringence in liquids [1927 *Proc. R. Soc. London* **A117** 1; with K S Krishnan]

44. A theory of the optical and electrical properties of liquids [1928 *Proc. R. Soc. London* **A117** 589; with K S Krishnan]

45. The Maxwell effect in liquids [1927 *Nature (London)* **120** 726; with K S Krishnan]

46. A theory of the birefringence induced by flow in liquids [1928 *Philos. Mag.* **5** 769; with K S Krishnan]

47. The scattering of light in amorphous solids [1927 *J. Opt. Soc. Am.* **15** 185]

48. The molecular scattering of light in a binary liquid mixture [1927 *Philos. Mag.* **4** 447]

49. A theory of light-scattering in liquids [1929 *Philos. Mag.* **5** 498; with K S Krishnan]

50. The theory of light-scattering in liquids [1929 *Philos. Mag.* **7** 160]

51. Optical behaviour of protein solutions [1927 *Nature (London)* **120** 158]

52. X-ray diffraction in liquids [1927 *Nature (London)* **119** 601; with C M Sogani]

53. X-ray diffraction in liquids [1927 *Nature (London)* **120** 514; with C M Sogani]

54. Thermal degeneration of the X-ray haloes in liquids [1927 *Nature (London)* **120** 770]

55. A critical-absorption photometer for the study of the Compton effect [1928 *Proc. R. Soc. London* **A119** 526; with C M Sogani]

56. Thermodynamics, wave-theory and the Compton effect [1927 *Nature* (*London*) **120** 950]
57. A classical derivation of the Compton effect [1928 *Indian J. Phys.* **3** 357]
58. A new type of secondary radiation [1928 *Nature* (*London*) **121** 501; with K S Krishnan]
59. A change of wavelength in light-scattering [1928 *Nature* (*London*) **121** 619]
60. A new radiation [1928 *Indian J. Phys.* **2** 387]
61. The optical analogue of the Compton effect [1928 *Nature* (*London*) **121** 711; with K S Krishnan]
62. A new class of spectra due to secondary radiation, Part I [1928 *Indian J. Phys.* **2** 399; with K S Krishnan]
63. The negative absorption of radiation [1928 *Nature* (*London*) **122** 12; with K S Krishnan]
64. Polarization of scattered light-quanta [1928 *Nature* (*London*) **122** 169; with K S Krishnan]
65. Molecular spectra in the extreme infrared [1928 *Nature* (*London*) **122** 278; with K S Krishnan]
66. The production of new radiations by light scattering—Part I [1929 *Proc. R. Soc. London* **A122** 23; with K S Krishnan]
67. Rotation of molecules induced by light [1928 *Nature* (*London*) **122** 882; with K S Krishnan]
68. Investigations of the scattering of light [1929 *Nature* (*London*) **123** 50]
69. The Raman effect: Investigation of molecular structure by light scattering [1929 *Trans. Faraday Soc.* **25** 781]
70. The molecular scattering of light, Nobel Lecture delivered at Stockholm, 11th December 1930
71. Colour and optical anisotropy of organic compounds [1929 *Nature* (*London*) **123** 494]
72. Magnetic behaviour of organic crystals [1929 *Nature* (*London*) **123** 605]
73. The relation between colour and molecular structure in organic compounds [1929 *Indian J. Phys.* **4** 57; with S Bhagavantam]
74. Diamagnetism and crystal structure [1929 *Nature* (*London*) **123** 945]
75. A new X-ray effect [1929 *Nature* (*London*) **124** 53; with P Krishnamurti]
76. Anomalous diamagnetism [1929 *Nature* (*London*) **124** 412]
77. Diamagnetism and molecular structure [1930 *Proc. Phys. Soc.* **42** 309]
78. A new type of magnetic birefringence [1931 *Nature* (*London*) **128** 758; with S W Chinchalkar]
79. Atoms and molecules as Fitzgerald oscillators [1931 *Nature* (*London*) **128** 795]
80. Evidence for the spin of the photon from light-scattering [1931 *Nature* (*London*) **128** 114; with S Bhagavantam]
81. The angular momentum of light [1931 *Nature* (*London*) **128** 545]
82. Experimental proof of the spin of the photon [1931 *Indian J. Phys.* **6** 353; with S Bhagavantam]
83. Experimental proof of the spin of the photon [1932 *Nature* (*London*) **129** 22; with S Bhagavantam]
84. Doppler effect in light-scattering [1931 *Nature* (*London*) **128** 636]
85. Nature of the thermal agitation in liquids [1935 *Nature* (*London*) **135** 761; with B V Raghavendra Rao]
86. Acoustic spectrum of liquids [1937 *Nature* (*London*) **139** 584; with B V Raghavendra Rao]
87. Light scattering and fluid viscosity [1938 *Nature* (*London*) **141** 242; with B V Raghavendra Rao]
88. New methods in the study of light scattering, Part I: Basic ideas [1941 *Proc. Indian Acad. Sci.* **A14** 228]

89. Determination of the adiabatic piezo-optic coefficient of liquids [1939 *Proc. R. Soc. London* **A171** 137; with K S Venkataraman]
90. Spectroscopic investigation of the solid and liquid states [1942 *Curr. Sci.* **11** 225]
91. The nature of the liquid state [1942 *Curr. Sci.* **11** 303]
92. The $\alpha - \beta$ transformation of quartz [1940 *Nature (London)* **145** 147; with T M K Nedangadi]
93. Lattice oscillations in crystals [1939 *Nature* (*London*) **143** 679; with T M K Nedungadi]
94. Scattering of light in crystals [1945 *Nature* (*London*) **155** 396]

# Volume II. Acoustics

## 1. Vibrations and Wave Motions

95. The small motion at the nodes of a vibrating string [1909 *Nature* (*London*) **82** 9]
96. The maintenance of forced oscillations of a new type (1909 *Nature* (*London*) **82** 156]
97. The maintenance of forced oscillations [1910 *Nature* (*London*) **82** 428]
98. Photographs of vibration curves [1911 *Philos. Mag.* **21** 615]
99. Remarks on a paper by J S Stokes on "Some curious phenomena observed in connection with Melde's experiment" [1911 *Phys. Rev.* **32** 307]
100. The small motion at the nodes of a vibrating string [1911 *Phys. Rev.* **32** 309]
101. The maintenance of forced oscillations of a new type [1912 *Philos. Mag.* **24** 513]
102. Some remarkable cases of resonance [1912 *Phys. Rev.* **35** 449]
103. Experimental investigations on the maintenance of vibrations [1912 *Bull. Indian Assoc. Cultiv. Sci.* **6**, 1]
104. Some acoustical observations [1913 *Bull. Indian Assoc. Cultiv. Sci.* **8** 17]
105. The maintenance of vibrations [1914 *Phys. Rev.* **4** 12]
106. On motion in a periodic field of force [1914 *Bull. Indian Assoc. Cultiv. Sci.* **11** 25]
107. On motion in a periodic field of force [1915 *Philos. Mag.* **29** 15]
108. The maintenance of vibrations by a periodic field of force [1917 *Philos. Mag.* **34** 129; with A Dey]
109. On the maintenance of combinational vibrations by two simple harmonic forces [1915 *Phys. Rev.* **5** 1]
110. On discontinuous wave-motion, Part I [1916 *Philos. Mag.* **31** 47; with S Appaswamaiyar]
111. On discontinuous wave-motion, Part II [1917 *Philos. Mag.* **33** 203; with A Dey]
112. On discontinuous wave-motion, Part III [1917 *Philos. Mag.* **33** 352; with A Dey]
113. An experimental method for the production of vibrations [1919 *Phys. Rev.* **14** 446]
114. A new method for the absolute determination of frequency [1919 *Proc. R. Soc. London* **A95** 533; with A Dey]
115. Whispering gallery phenomena at St. Paul's Cathedral [1921 *Nature* (*London*) **108** 42; with G A Sutherland]

116. On the whispering gallery phenomenon [1922 *Proc. R. Soc. London* **A100** 424; with G A Sutherland]
117. On whispering galleries [1922 *Bull. Indian Assoc. Cultiv. Sci.* **7** 159]
118. On the sounds of splashes [1920 *Philos. Mag.* **39** 145; with A Dey]
119. The nature of vowel sounds [1921 *Nature* (*London*) **107** 332]

## 2. Musical Instruments—The Violin and the Pianoforte

120. The dynamical theory of the motion of bowed strings [1914 *Bull. Indian Assoc. Cultiv. Sci.* **11** 43]
121. On the "wolf-note" of the violin and 'cello [1916 *Nature* (*London*) **97** 362]
122. On the "wolf note" in the bowed stringed instruments [1916 *Philos. Mag.* **32** 391]
123. On the alterations of tone produced by a violin "mute" [1917 *Nature* (*London*) **100** 84]
124. On the "wolf-note" in bowed stringed instruments [1918 *Philos. Mag.* **35** 493]
125. The "wolf-note" in pizzicato playing [1918 *Nature* (*London*) **101** 264]
126. On the mechanical theory of the vibrations of bowed strings and of musical instruments of the violin family, with experimental verification of the results—Part I [1918 *Bull. Indian Assoc. Cultiv. Sci.* **15** 1]
127. On the partial tones of bowed stringed instruments [1919 *Philos. Mag.* **38** 573]
128. The kinematics of bowed strings [1919 *J. Dept. of Sci.*, Univ. Calcutta **1** 15]
129. On a mechanical violin-player for acoustical experiments [1920 *Philos. Mag.* **39** 535]
130. Experiments with mechanically-played violins [1920 *Proc. Indian Assoc. Cultiv. Sci.* **6** 19]
131. The theory of the cyclical vibrations of a bowed string [1918 *Bull. Indian Assoc. Cultiv. Sci.* **5** 1]
132. The subjective analysis of musical tones [1926 *Nature* (*London*) **117** 450]
133. On Kaufmann's theory of the impact of the pianoforte hammer [1920 *Proc. R. Soc. London* **A97** 99; with B Banerji]

## 3. Musical Instruments of India

134. 'The Ectara', [1909 *J. Indian Math. Club* 170]
135. Oscillations of the stretched strings [1910 *J. Indian Math. Club* 14]
136. Musical drums with harmonic overtones [1920 *Nature* (*London*), **104** 500; with S Kumar]
137. The Indian musical drums [1935 *Proc. Indian Acad. Sci.* **A1** 179]
138. On some Indian stringed instruments [1921 *Proc. Indian Assoc. Cultiv. Sci.* **7** 29]
139. The acoustical knowledge of the ancient Hindus, Asutosh Mookerjee Silver Jubilee Volume, (Calcutta: University Press)[1922 **2** 179]

## 4. Monograph

140. Musical instruments and their tones [1927 *Handb. Phys.* **8** 354]

## 5. Ultrasonics

141. The diffraction of light by high frequency sound waves: Part I [1936 *Proc. Indian Acad. Sci.* **A2** 406; with N S Nagendra Nath]
142. The diffraction of light by sound waves of high frequency: Part II [1936 *Proc. Indian Acad. Sci.* **A2** 413; with N S Nagendra Nath]
143. The diffraction of light by high frequency sound waves: Part III, Doppler effect and coherence phenomena [1936 *Proc. Indian Acad. Sci.* **A3** 75; with N S Nagendra Nath]
144. The diffraction of light by high frequency sound waves: Part IV, Generalised theory [1936 *Proc. Indian Acad. Sci.* **A3** 119; with N S Nagendra Nath]
145. The diffraction of light by high frequency sound waves: Part V. General considerations—Oblique incidence and amplitude changes [1936 *Proc. Indian Acad. Sci.* **A3** 459; with N S Nagendra Nath]
146. Diffraction of light by ultrasonic waves [1936 *Nature* (*London*) **138** 616; with N S Nagendra Nath]
147. Nature of the thermal agitation in liquids [1935 *Nature* (*London*) **135** 761; with B V Raghavendra Rao]
148. Acoustic spectrum of liquids [1937 *Nature* (*London*) **139** 584; with B V Raghavendra Rao]
149. Light scattering and fluid viscosity [1938 *Nature* (*London*) **141** 242; with B V Raghavendra Rao]

# Volume III. Optics

150. Unsymmetrical diffraction-bands due to a rectangular aperture [1906 *Philos. Mag.* **12** 494]
151. Newton's rings in polarised light [1907 *Nature* (*London*) **76** 637]
152. Secondary waves of light [1908 *Nature* (*London*) **78** 55]
153. Historical note on the discovery of the ultramicroscopic method [1909 *Philos. Mag.* **17** 495]
154. The experimental study of Huygens's secondary waves [1909 *Philos. Mag.* **17** 204]
155. The photometric measurement of the obliquity factor of diffraction [1909 *Nature* (*London*) **82** 69]
156. The photometric measurement of the obliquity factor of diffraction [1911 *Philos. Mag.* **21** 618]
157. On intermittent vision [1915 *Philos. Mag.* **30** 701]
158. The colours of the striae in mica [1918 *Nature* (*London*) **102** 205; with P N Ghosh]
159. On the diffraction figures due to an elliptic aperture [1919 *Phys. Rev.* **13** 259]
160. The scattering of light in the refractive media of the eye [1919 *Philos. Mag.* **38** 568]
161. The "radiant" spectrum [1921 *Nature* (*London*) **108** 12]
162. The "radiant" spectrum [1922 *Nature* (*London*) **109** 175]
163. On the phenomenon of the "radiant spectrum" observed by Sir David Brewster [1922 *Philos. Mag.* **43** 357]
164. On the colours of mixed plates—Part I [1921 *Philos. Mag.* **41** 338; with B Banerji]
165. On the colours of mixed plates—Part II [1921 *Philos. Mag.* **41** 860; with B Banerji]

166. On the colours of mixed plates—Part III [1921 *Philos. Mag.* **42** 679; with K Seshagiri Rao]
167. On Quetelet's rings and other allied phenomena [1921 *Philos. Mag.* **42** 826; with G L Datta]
168. The colours of breathed-on plates [1921 *Nature* (*London*) **107** 714]
169. The colours of tempered steel [1922 *Nature* (*London*) **109** 105]
170. A method of improving visibility of distant objects [1921 *Nature* (*London*) **108** 242]
171. The spectrum of neutral helium [1922 *Nature* (*London*) **110** 700]
172. On the spectrum of neutral helium—Part I [1923 *Atrophys. J.* **57** 243; with A S Ganesan]
173. On the spectrum of neutral helium—Part II [1924 *Astrophys. J.* **59** 61; with A S Ganesan]
174. Anomalous dispersion and multiplet lines in spectra [1925 *Nature* (*London*) **115** 946; with S K Datta]
175. On Einstein's aberration experiment [1922 *Astrophys. J.* **56** 29]
176. Einstein's aberration experiment [1922 *Nature* (*London*) **109** 477]
177. On the convection of light (Fizeau effect) in moving gases [1922 *Philos. Mag.* **43** 447; with N K Sethi]
178. Conical refraction in biaxial crystals [1921 *Nature* (*London*) **107** 747]
179. On a new optical property of biaxial crystals [1922 *Philos. Mag.* **43** 510; with V S Tamma]
180. The effect of dispersion on the interference figures of crystals [1924 *Nature* (*London*) **113** 127]
181. The optical properties of amethyst quartz [1925 *Trans. Opt. Soc. London* **26** 289; with K Banerji]
182. On Brewster's bands—Part I [1925 *Trans. Opt. Soc.*,*Am* **26** 51; with S K Datta]
183. On the diffraction of light by spherical obstacles [1926 *Proc. Phys. Soc. London* **38** 350; with K S Krishnan]
184. On the nature of the disturbance in the second medium in total reflection [1925 *Philos. Mag.* **50** 812]
185. On the total reflection of light [1926 *Proc. Indian Assoc. Cultiv. Sci.* **9** 271]
186. Huygens's principle and the phenomena of total reflection [1927 *Trans. Opt. Soc. London* **28** 149]
187. The diffraction of light by metallic screens [1927 *Proc. R. Soc. London* **A116** 254; with K S Krishnan]
188. Diffraction of light by a transparent lamina [1927 *Proc. Phys. Soc. London* **39** 453; with I Ramakrishna Rao]
189. The diffraction of light by high frequency sound waves: Part I [1936 *Proc. Indian Acad. Sci.* **A2** 406; with N S Nagendra Nath]
190. The diffraction of light by sound waves of high frequency: Part II [1936 *Proc. Indian Acad. Sci.* **A2** 413; with N S Nagendra Nath]
191. The diffraction of light by high frequency sound waves: Part III, Doppler effect and coherence phenomena [1936 *Proc. Indian Acad. Sci.* **A3** 75; with N S Nagendra Nath]
192. The diffraction of light by high frequency sound waves: Part IV, Generalised theory [1936 *Proc. Indian Acad. Sci.* **A3** 119; with N S Nagendra Nath]
193. The diffraction of light by high frequency sound waves: Part V, General considerations—Oblique incidence and amplitude changes [1936 *Proc. Indian Acad. Sci.* **A3** 459; with N S Nagendra Nath]
194. Diffraction of light by ultrasonic waves [1936 *Nature* (*London*) **138** 616; with N S Nagendra Nath]

195. On the wave-like character of periodic precipitates [1939 *Proc. Indian Acad. Sci.* **A9** 455; with K Subba Ramaiah]
196. Interference patterns with Liesegang rings [1938 *Nature* (*London*) **142** 355; with K Subba Ramiah]
197. Haidinger's rings in curved plates [1939 *J. Opt. Soc. Am.* **29** 413; with V S Rajagopalan]
198. Haidinger's rings in soap bubbles [1939 *Proc. Indian Acad. Sci.* **A10** 317; with V S Rajagopalan]
199. Conical refraction in naphthalene crystals [1941 *Proc. Indian Acad. Sci.* **A14** 221; with V S Rajagopalan and T M K Nedungadi]
200. Conical refraction in naphthalene crystals [1941 *Nature* (*London*) **147** 268; with V S Rajagopalan and T M K Nedungadi]
201. The phenomena of conical refraction [1942 *Curr. Sci.* **11** 44]
202. The theory of the Christiansen experiment [1949 *Proc. Indian Acad. Sci.* **A29** 381]
203. The Christiansen experiment with spherical particles [1949 *Proc. Indian Acad. Sci.* **A30** 211; with S Ramaseshan]
204. Diffraction of light by transparent spheres and spheroids: The Fresnel patterns [1949 *Proc. Indian Acad. Sci.* **A30** 277; with S Ramaseshan]
205. The Christiansen experiment [1953 *Curr. Sci.* **22** 31; with M R Bhat]
206. The structure and optical behaviour of some natural and synthetic fibres [1954 *Proc. Indian Acad. Sci.* **A39** 109; with M R Bhat]
207. The theory of the propagation of light in polycrystalline media [1955 *Proc. Indian Acad. Sci.* **A41** 37; with K S Viswanathan]
208. A generalized theory of the Christiansen experiment [1955 *Proc. Indian Acad. Sci.* **A41** 55; with K S Viswanathan]
209. The Christiansen experiment with birefringent powders [1955 *Proc. Indian Acad. Sci.* **A41** 61; with M R Bhat]
210. The optical behaviour of polycrystalline solids [1957 *J. Madras Univ.* **B27** 1]
211. Christiaan Huygens's and the wave theory of light [1959 *Proc. Indian Acad. Sci.* **A49** 185]
212. The principle of Huygens's and diffraction of light [1959 *Curr. Sci.* **28** 267]
213. The optics of mirages [1959 *Proc. Indian Acad. Sci.* **A49** 251; with S Pancharatnam]
214. The scintillation of the stars [1964 *Curr. Sci.* **33** 355]
215. Lectures on Physical Optics, Part I [1959 Indian Academy of Sciences, Bangalore]

# Volume IV

## 1. Miscellaneous Papers

216. The curvature method of determining the surface tension of liquids [1907 *Philos. Mag.* **14** 591]
217. Some new methods in kinematical theory [1912–13 *Bull. Calcutta Math. Soc.* **4** 1]
218. On the summation of certain fourier series involving discontinuities [1913–14 *Bull. Calcutta. Math. Soc.* **5** 5]
219. The viscosity of liquids [1923 *Nature* (*London*) **111** 600]

220. A THEORY OF THE VISCOSITY OF LIQUIDS [1923 *Nature* (*London*) **111** 532]
221. THE PHOTOGRAPHIC STUDY OF IMPACT AT MINIMAL VELOCITIES [1918 *Phys. Rev.* **12** 442]
222. PERCUSSION FIGURES IN ISOTROPIC SOLIDS [1919 *Nature* (*London*) **104** 113]
223. ON SOME APPLICATIONS OF HERTZ'S THEORY OF IMPACT [1920 *Phys. Rev.* **15** 277]
224. THE OPTICAL STUDY OF PERCUSSION FIGURES [1926 *J. Opt. Soc. Am.* **12** 387]
225. PERCUSSION FIGURES IN CRYSTALS [1958 *Proc. Indian Acad. Sci.* **A48** 307]
226. PERCUSSION FIGURES IN CRYSTALS [1959 *Curr. Sci.* **28** 1]
227. INDIA'S DEBT TO FARADAY [1931 *Nature* (*London*) **128** 362]
228. NEWTON AND THE HISTORY OF OPTICS [1942 *Curr. Sci.* **11** 453]
229. ASTRONOMICAL RESEARCH IN INDIA: I [1943 *Curr. Sci.* **12** 197]
230. ASTRONOMICAL RESEARCH IN INDIA: II [1943 *Curr. Sci.* **12** 289]
231. ASTRONOMICAL RESEARCH IN INDIA: III [1943 *Curr. Sci.* **12** 313]
232. CENTENARY OF THE FARADAY EFFECT [1945 *Curr. Sci.* **14** 281]
233. SCIENCE IN EASTERN EUROPE: I [1958 *Curr. Sci.* **27** 371]
234. SCIENCE IN EASTERN EUROPE: II [1958 *Curr. Sci.* **27** 421]
235. ZONAL WINDS AND JET STREAMS IN THE ATMOSPHERE [1967 *Curr. Sci.* **36** 593]
236. THE ATMOSPHERE OF THE EARTH [1968 *Curr. Sci.* **37** 151]

## 2. Colour

237. THE ORIGIN OF THE COLOURS IN THE PLUMAGE OF THE BIRDS [1934 *Proc. Indian Acad. Sci.* **A1** 1]
238. ON IRIDESCENT SHELLS, PART I. INTRODUCTORY [1934 *Proc. Indian Acad. Sci.* **A1** 567]
239. ON IRIDESCENT SHELLS—PART II. COLOURS OF LAMINAR DIFFRACTION [1934 *Proc. Indian Acad. Sci.* **A1** 574]
240. ON IRIDESCENT SHELLS—PART III. BODY-COLOURS AND DIFFUSION-HALOES [1934 *Proc. Indian Acad. Sci.* **A1** 859]
241. THE STRUCTURE AND OPTICAL BEHAVIOUR OF IRIDESCENT SHELLS [1954 *Proc. Indian Acad. Sci.* **A39** 1; with D Krishnamurti]
242. THE STRUCTURE AND OPTICAL CHARACTERS OF IRIDESCENT GLASS [1939 *Proc. Indian Acad. Sci.* **A9** 371; with V S Rajagopalan]
243. COLOURS OF STRATIFIED MEDIA I: ANCIENT DECOMPOSED GLASS [1940 *Proc. Indian Acad. Sci.* **A11** 469; with V S Rajagopalan]
244. THE IRIDESCENT FELDSPARS [1950 *Curr. Sci.* **A19** 301]
245. THE STRUCTURE OF LABRADORITE AND THE ORIGIN OF ITS IRIDESCENCE [1950 *Proc. Indian Acad. Sci.* **A32** 1; with A Jayaraman]
246. THE STRUCTURE AND THE OPTICAL BEHAVIOUR OF THE CEYLON MOONSTONES [1950 *Proc. Indian Acad. Sci.* **A32** 123; with A Jayaraman and T K Srinivasan]
247. THE DIFFUSION HALOES OF THE IRIDESCENT FELDSPARS [1953 *Proc. Indian Acad. Sci.* **A37** 1; with A Jayaraman]
248. ON THE IRIDESCENCE OF POTASSIUM CHLORATE CRYSTALS—PART I. ITS SPECTRAL CHARACTERS [1952 *Proc. Indian Acad. Sci.* **A36** 315; with D Krishnamurti]
249. ON THE IRIDESCENCE OF POTASSIUM CHLORATE CRYSTALS—PART II. POLARISATION EFFECTS [1952 *Proc. Indian Acad. Sci.* **A36** 321; with D Krishnamurti]
250. ON THE IRIDESCENCE OF POTASSIUM CHLORATE CRYSTALS—PART III. SOME GENERAL OBSERVATIONS [1952 *Proc. Indian Acad. Sci.* **A36** 330; with D Krishnamurti]

251. On the polarisation and spectral character of the iridescence of potassium chlorate crystals [1952 *Proc. Indian Acad. Sci.* **A36** 419; with D Krishnamurti]
252. The structure and optical behaviour of iridescent crystals of potassium chlorate [1953 *Proc. Indian Acad. Sci.* **A38** 261; with D Krishnamurti]
253. The structure of opal and the origin of its iridescence [1953 *Proc. Indian Acad. Sci.* **A38** 101; with A Jayaraman]
254. The structure of optical behaviour of iridescent opal [1953 *Proc. Indian Acad. Sci.* **A38** 343; with A Jayaraman]
255. The structure and optical behaviour of pearls [1954 *Proc. Indian Acad. Sci.* **A39** 215; with D Krishnamurti]
256. Optics of the pearl [1954 *Curr. Sci.* **23** 173; with D Krishnamurti]
257. On the chromatic diffusion halo and other optical effects exhibited by pearls [1954 *Proc. Indian Acad. Sci.* **A39** 265; with D Krishnamurti]
258. The structure and optical behaviour of iridescent agate [1953 *Proc. Indian Acad. Sci.* **A38** 199; with A Jayaraman]
259. The structure and optical behaviour of iridescent calcite [1954 *Proc. Indian Acad. Sci.* **A40** 1; with A K Ramdas]
260. The structure and optical behaviour of jadeite [1955 *Proc. Indian Acad. Sci.* **A41** 117; with A Jayaraman]
261. Crystals of quartz with iridescent faces [1950 *Proc. Indian Acad. Sci.* **A31** 275]

## 3. Optics of Minerals

262. The optical anisotropy and heterogeneity of vitreous silica [1950 *Proc. Indian Acad. Sci.* **A31** 141]
263. Structural birefringence in amorphous solids [1950 *Proc. Indian Acad. Sci.* **A31** 207]
264. The lamellar structure and birefringence of plate glass [1950 *Proc. Indian Acad. Sci.* **A31** 359]
265. The Smoky quartz [1921 *Nature* (*London*) **108** 81]
266. The structure of amethyst quartz and the origin of its pleochroism [1954 *Proc. Indian Acad. Sci.* **A40** 189; with A Jayaraman]
267. The birefringence patterns of crystal spheres [1956 *Proc. Indian Acad. Sci.* **A43** 1]
268. Amethyst—its nature and origin [1954 *Curr. Sci.* **23** 379]
269. On the structure of amethyst and its genesis in nature [ 1954 *Proc. Indian Acad. Sci.* **A40** 221; with A Jayaraman]
270. On the optical behaviour of crypto-crystalline quartz [ 1954 *Proc. Indian Acad. Sci.* **A41** 1; with A Jayaraman]
271. X-ray study of fibrous quartz [1954 *Proc. Indian Acad. Sci.* **A40** 107; with A Jayaraman]
272. On the polycrystalline forms of gypsum and their optical behaviour [1954 *Proc. Indian Acad. Sci.* **A39** 153; with A K Ramdas]
273. X-ray studies on polycrystalline gypsum [1954 *Proc. Indian Acad. Sci.* **A40** 57; with A Jayaraman]
274. The luminescence of fluorspar [1962 *Curr. Sci.* **31** 361]
275. The two species of fluorite [1962 *Curr. Sci.* **31** 445]

### 4. Diamond

276. The physics of the diamond [1942 *Curr. Sci.* **11** 261]
277. The structure and properties of diamond [1943 *Curr. Sci.* **12** 33]
278. The four forms of diamond [1944 *Curr. Sci.* **13** 145]
279. The crystal symmetry and structure of diamond [1944 *Proc. Indian Acad. Sci.* **A19** 189]
280. The nature and origin of the luminescence of diamond [1944 *Proc. Indian Acad. Sci.* **A19** 199]
281. Birefringence patterns in diamonds [1944 *Proc. Indian Acad. Sci.* **A19** 265; with G R Rendall]
282. The crystal forms of diamond and their significance [1946 *Proc. Indian Acad. Sci.* **A24** 1; with S Ramaseshan]
283. The diamond and its teachings [1946 *Curr. Sci.* **15** 205]
284. New concepts of crystal structure [1946 *Curr. Sci.* **15** 329]
285. The luminescence of diamond and its relation to crystal structure [1950 *Proc. Indian Acad. Sci.* **A32** 65; with A Jayaraman]
286. The luminescence of diamond—I [1950 *Curr. Sci.* **19** 357]
287. The luminescence of diamond—II [1951 *Curr. Sci.* **20** 1]
288. The luminescence of diamond—III [1951 *Curr. Sci.* **20** 27]
289. The luminescence of diamond—IV [1951 *Curr. Sci.* **20** 55]
290. The diamond [1956 *Proc. Indian Acad. Sci.* **A44** 99]
291. The tetrahedral carbon atom and the structure of diamond [1957 *Proc. Indian Acad. Sci.* **A46** 391]

## Volume V. Crystal Physics

### 1. Diffuse X-ray Reflections

292. A new X-ray effect [1940 *Curr. Sci.* **9** 165; with P Nilakantan]
293. Reflection of X-rays with change of frequency Part I. Theoretical discussion [1940 *Proc. Indian Acad. Sci.* **A11** 379; with P Nilakantan]
294. Reflection of X-rays with change of frequency Part II. The case of diamond [1940 *Proc. Indian Acad. Sci.* **A11** 389; with P Nilakantan]
295. Reflection of X-rays with change of frequency Part III. The case of sodium nitrate [1940 *Proc. Indian Acad. Sci.* **A11** 398; with P Nilakantan]
296. Quantum theory of X-ray reflection and scattering Part I. Geometric relations [1940 *Proc. Indian Acad. Sci.* **A12** 83; with N S Nagendra Nath]
297. Reflection of X-rays with change of frequency Part IV. Rock salt [1940 *Proc. Indian Acad. Sci.* **A12** 141; with P Nilakantan]
298. The two types of X-ray reflection in crystals [1940 *Proc. Indian Acad. Sci.* **A12** 427]
299. Crystals and photons [1941 *Proc. Indian Acad. Sci.* **A13** 1]
300. The quantum theory of X-ray reflection: Basic ideas [1941 *Proc. Indian Acad. Sci.* **A14** 317]
301. Quantum theory of X-ray reflection: Mathematical formulation [1941 *Proc. Indian Acad. Sci.* **A14** 332]

302. Quantum theory of X-ray reflection: Experimental confirmation [1941 *Proc. Indian Acad. Sci.* **A14** 356; with P Nilakantan]
303. Dynamic X-ray reflections in crystals [1948 *Curr. Sci.* **17** 65]
304. X-rays and crystals [1955 *Curr. Sci.* **24** 395]
305. New concepts of the solid state [1942 *Proc. Indian Acad. Sci.* **A15** 65]

## 2. Dynamics of Crystal Lattices

306. The vibration spectrum of a crystal lattice [1943 *Proc. Indian Acad. Sci.* **A18** 237]
307. New paths in crystal physics [1947 *Curr. Sci.* **16** 67]
308. The vibration spectra of crystals Part I. Basic theory [1947 *Proc. Indian Acad. Sci.* **A26** 339]
309. The vibration spectra of crystals Part II. The case of diamond [1947 *Proc. Indian Acad. Sci.* **A26** 356]
310. The vibration spectra of crystals Part III. Rock salt [1947 *Proc. Indian Acad. Sci.* **A26** 370]
311. The vibration spectra of crystals Part IV. Magnesium oxide [1947 *Proc. Indian Acad. Sci.* **A26** 383]
312. The vibration spectra of crystals Part V. Lithium and sodium fluorides [1947 *Proc. Indian Acad. Sci.* **A26** 391]
313. The vibration spectra of crystals Part VI. Sylvine [1947 *Proc. Indian Acad. Sci.* **A26** 396]
314. The infra-red spectrum [1947 *Curr. Sci.* **16** 359]
315. The eigenvibrations of crystal structures [1948 *Curr. Sci.* **17** 1]
316. The scattering of light in crystals and the nature of their vibration spectra [1951 *Proc. Indian Acad. Sci.* **A34** 61]
317. The vibration spectra of crystals and the theory of their specific heats [1951 *Proc. Indian Acad. Sci.* **A34** 141]

## 3. Elasticity of Crystals

318. The elasticity of crystals [1955 *Curr. Sci.* **24** 325]
319. The elastic behaviour of isotropic solids [1955 *Proc. Indian Acad. Sci.* **A42** 1; with K S Viswanathan]
320. On the theory of the elasticity of crystals [1955 *Proc. Indian Acad. Sci.* **A42** 51; with K S Viswanathan]
321. Evaluation of the four elastic constants of some cubic crystals [1955 *Proc. Indian Acad. Sci.* **A42** 111; with D Krishnamurti]

## 4. Vibrational and Thermal Energy of Crystals

322. The nature of the thermal agitation in crystals [1955 *Proc. Indian Acad. Sci.* **A42** 163]
323. The thermal energy of crystals [1955 *Curr. Sci.* **24** 357]
324. Quantum theory and crystal physics [1956 *Curr. Sci.* **25** 377]

325. The physics of crystals [1956 *Proc. Indian Acad. Sci.* **A43** 327]
326. The specific heats of crystals: Part I. General theory [1956 *Proc. Indian Acad. Sci.* **A44** 153]
327. The specific heats of crystals: Part II. The case of diamond [1956 *Proc. Indian Acad. Sci.* **A44** 160]
328. The specific heats of crystals: Part III. Analysis of the experimental data [1956 *Proc. Indian Acad. Sci.* **A44** 367]
329. The heat capacity of diamond between 0–1000° K [1957 *Proc. Indian Acad. Sci.* **A46** 323]
330. The diffraction of X-rays by diamond: Part I [1958 *Proc. Indian Acad. Sci.* **A47** 263]
331. The diffraction of X-rays by diamond: Part II [1958 *Proc. Indian Acad. Sci.* **A47** 335]
332. The diffraction of X-rays by diamond: Part III [1958 *Proc. Indian Acad. Sci.* **A48** 1]
333. The infra-red absorption by diamond and its significance Part I. Materials and methods [1962 *Proc. Indian Acad. Sci.* **A55** 1]
334. The infra-red absorption by diamond and its significance Part II. A General survey of the results [1962 *Proc. Indian Acad. Sci.* **A55** 5]
335. The infra-red absorption by diamond and its significance Part III. The perfect diamonds and their spectral behaviour [1962 *Proc. Indian Acad. Sci.* **A55** 10]
336. The infra-red absorption by diamond and its significance Part IV. The non-luminescent diamonds [1962 *Proc. Indian Acad. Sci.* **A55** 14]
337. The infra-red absorption by diamond and its significance Part V. The composite diamonds [1962 *Proc. Indian Acad. Sci.* **A55** 20]
338. The infra-red absorption by diamond and its significance Part VI. The free vibrations of the structure [1962 *Proc. Indian Acad. Sci.* **A55** 24]
339. The infra-red absorption by diamond and its significance Part VII. The characteristic frequencies [1962 *Proc. Indian Acad. Sci.* **A55** 30]
340. The infra-red absorption by diamond and its significance Part VIII. Dynamical theory [1962 *Proc. Indian Acad. Sci.* **A55** 36]
341. The infra-red absorption by diamond and its significance Part IX. The activity of the normal modes [1962 *Proc. Indian Acad. Sci.* **A55** 42]
342. The infra-red absorption by diamond and its significance Part X. Evaluation of the specific heat [1962 *Proc. Indian Acad. Sci.* **A55** 49]
343. The infra-red behaviour of diamond [1962 *Curr. Sci.* **31** 403]
344. The diamond: Its structure and properties [1968 *Proc. Indian Acad. Sci.* **A67** 231]
345. Quantum theory and crystal physics [1956 *Proc. Indian Acad. Sci.* **A44** 361]
346. The specific heats of crystalline solids: Part I. [1957 *Curr. Sci.* **26** 195]
347. The specific heats of crystalline solids: Part II. [1957 *Curr. Sci.* **26** 231]
348. The specific heats of some metallic elements Part I. Analysis of the experimental data [1957 *Proc. Indian Acad. Sci.* **A45** 1]
349. The specific heats of some metallic elements Part II. Approximate theoretical evaluation [1957 *Proc. Indian Acad. Sci.* **A45** 7]
350. The specific heats of some metallic elements Part III. The characteristic frequencies [1957 *Proc. Indian Acad. Sci.* **A45** 59]
351. The specific heats of some metallic elements Part IV. The residual spectrum [1957 *Proc. Indian Acad. Sci.* **A45** 139]
352. The specific heats of crystals and the fallacy of the theories of Debye and Born [1957 *Proc. Indian Acad. Sci.* **A45** 273]

## Volume VI. Colour and its Perception

### 1. Light, Colour and Vision

384. The perception of light and colour and the physiology of vision, Part II. The visual pigments [1960 *Proc. Indian Acad. Sci.* **A52** 267]
385. The perception of light and colour and the physiology of vision, Part III. The carotenoid pigment [1960 *Proc. Indian Acad. Sci.* **A52** 281]
386. The perception of light and colour and the physiology of vision, Part IV. Ferroheme and ferriheme [1960 *Proc. Indian Acad. Sci.* **A52** 292]
387. The perception of light and colour and the physiology of vision, Part V. The colour of triangle [1960 *Proc. Indian Acad. Sci.* **A52** 305]
388. The perception of light and colour and the physiology of vision, Part VI. Defective colour vision [1960 *Proc. Indian Acad. Sci.* **A52** 314]
389. The perception of light and colour and the physiology of vision, Part VII. General summary [1960 *Proc. Indian Acad. Sci.* **A52** 324]
390. The role of the retina in vision [1962 *Curr. Sci.* **31** 315]
391. Light, colour and vision [1962 *Curr. Sci.* **31** 489]
392. Floral colours and their spectral composition [1963 *Curr. Sci.* **32** 147]
393. The visual pigments and their location in the retina [1963 *Curr. Sci.* **32** 389]
394. Floral colours and the physiology of vision [1963 *Curr. Sci.* **32** 293]
395. The trichromatic hypothesis [1963 *Curr. Sci.* **32** 245]
396. Visual acuity and its variations [1963 *Curr. Sci.* **32** 531]
397. The colours of gemstones [1963 *Curr. Sci.* **32** 437]
398. The green colour of vegetation [1963 *Curr. Sci.* **32** 341]

## 2. Visual Perception of Colour

Floral colours and the physiology of vision [1963 *Proc. Indian Acad. Sci.* **A58**]

399. Part I. Introductory 57
400. Part II. The green colour of leaves 62
401. Part III. The spectrum of the morning glory 67
402. Part IV. The queen of flowers 70
403. Part V. The blue of the Jacaranda 73
404. Part VI. Comparative study of three cases 76
405. Part VII. The Aster and its varied colours 81
406. Part VIII. The spectra of roses 84
407. Part IX. Hibiscus and bougainvillea 87
408. Part X. Flowers exhibiting band spectra 92
409. Part XI. A review of the results 96
410. Part XII. Some concluding remarks 106
411. The visual synthesis of colour [1964 *Curr. Sci.* **33** 97]
412. Fluctuations of luminosity in visual fields [1964 *Curr. Sci.* **33** 65]
413. Stars, nebulae and the physiology of vision [1964 *Curr. Sci.* **33** 293]

The new physiology of vision [1964 *Proc. Indian Acad. Sci.*]

414. Part I. Introductory [**60** 139]
415. Part II. Visual sensations and the nature of light [**60** 143]
416. Part III. Corpuscles of light and the perception of luminosity [**60** 211]

417. Part IV. Corpuscles of light and the perception of form [**60** 287]
418. Part V. Corpuscles of light and the perception of colour [**60** 292]
419. Part VI. Vision in dim light [**60** 369]
420. Part VII. The perception of colour in dim light [**60** 375]
421. Part VIII. The perception of polarised light [1965 **61** 1]
422. Part IX. The structure of the fovea [**61** 7]
423. Part X. The major visual pigments [**61** 57]
424. Part XI. The carotenoid pigments [**61** 65]
425. Part XII. Chromatic sensations at high luminosities [**61** 129]
426. Part XIII. Blue, indigo and violet in the spectrum [**63** 133]
427. Part XIV. The red end of the spectrum [**61** 187]
428. Part XV. The chromatic response of the retina [**61** 193]
429. Part XVI. Further studies of the retinal responses [**61** 267]
430. Part XVII. Location of visual pigments in the retina [**61** 335]
431. Part XVIII. The visual synthesis of colour [*Proc. Indian Acad. Sci.* **62** 1]
432. Part XIX. Perception of colour and the trichromatic hypothesis [**62** 10]
433. Part XX. Superposition and masking of colours [**62** 67]
434. Part XXI. The green colour of vegetation [**62** 73]
435. Part XXII. The colours of flowers [**62** 125]
436. Part XXIII. The colours of the roses [**62** 133]
437. Part XXIV. Floral pigments and the perception of colour [**62** 177]
438. Part XXV. The colours of natural and synthetic gemstones [**62** 183]
439. Part XXVI. Structural colours [**62** 237]
440. Part XXVII. The colours of interference [**62** 243]
441. Part XXVIII. Observation with a neodymium filter [**62** 307]
442. Part XXIX. The reproduction of colour [**62** 310]
443. Part XXX. The photomechanical reproduction of colour [1966 **63** 1]
444. Part XXXI. The integration of colour by the retina [**63** 5]
445. Part XXXII. Defects in colour vision [**63** 65]
446. Part XXXIII. The testing of colour vision [**63** 71]
447. Part XXXIV. The nature and origin of defects in colour vision [**63** 133]
448. Part XXXV. The faintest observable spectrum [**63** 138]
449. Part XXXVI. The postulated duality of the retina [**63** 207]
450. Part XXXVII. The spectrum of the night-sky [**63** 213]
451. Part XXXVIII. The adaptation of vision to dim light [**63** 263]
452. Part XXXIX. Daltonian colour vision [**63** 267]
453. Part XL. The colours of Iolite [**63** 321]
454. Part XLI. Photography in colour [**63** 325]
455. Part XLII. Further observations with the neodymium filter [**63** 329]
456. Part XLIII. The colours of fluorspar [**63** 333]

## 3. Floral colours

457. Floral colours and their origin [1969 *Curr. Sci.* **38** 179]
458. The florachromes: their constitution and optical behaviour [1969 *Curr. Sci.* **38** 451]
459. The colours of roses [1969 *Curr. Sci.* **38** 503]
460. Spectrophotometry of floral extracts [1969 *Curr. Sci.* **38** 527]
461. Blue delphiniums and the purple Bignonia [1969 *Curr. Sci.* **38** 553]
462. The varied colours of verbena [1969 *Curr. Sci.* **38** 579]
463. The pelargoniums [1970 *Curr. Sci.* **39** 1]
464. The red oleander and the purple petria [1970 *Curr. Sci.* **39** 25]

## 4. Monograph—Physiology of Vision

465. The physiology of vision, Indian Academy of Sciences, Bangalore [1968 p. 1–164]

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